the problem of investigation of these phenomena in a strong electric field appears to us one thought up especially for this occasion. In order to heat up the electron gas under conditions when primary current is absent, the author had to introduce artificially a strong electric field perpendicular to the primary temperature gradient (in the presence of a magnetic field in the direction of the latter). This immediately leads to a contradiction in the calculation of the electronic component of thermoconductivity, for instance. The calculation was carried out, as usual, with the assumption of absence of electric current in the specimen ($j = 0$). While doing this, however, the author did not account for the fact that a strong current is required in the semi-conductor in order to heat the electron gas.

Furthermore, in all formulas obtained, there enters a quantity χ^{ν}, which is dependent upon the electric and magnetic fields E and H, and, in the presence of a temperature gradient, also upon the coordinates r. Nevertheless, the calculations of χ^{ν} is carried out under the assumption that the symmetrical part of the distribution function f_0 does not depend on the magnetic field or on the coordinates, and the solution of Davydov is used for this case. We do not agree with Avak'iants, who states that "there is no necessity" for solving the equations of Davydov in the case in which f_0 depends on E, H and r. From the formulas of Davydov[3] it follows that for not very small magnetic fields (or small H at sufficiently low temperatures) the dependence of f_0 upon H cannot be neglected.

In calculating f_0, Avak'iants also neglects a term which accounts for the entrance of electrons into the zone of conductivity (or to local levels). This is justified only in those cases in which the concentration of electrons (holes) differs but little from the equilibrium condition. But, in a kinetic equation, under conditions where the semiconductor is in a strong electric field, not only the usual thermal ionization, but also the ionization by the field must be accounted for. Neglect of the terms expressing the ionization by the field, is, in our opinion, one of the basic causes of the disagreement between theory and experiment.

Thus, the papers of Avak'iants cannot interpret experimental results (for instance, the Pool* effect) and do not contribute, as it appears to us, anything new to the problem of behavior of semiconductors in strong electric fields.

Translated by M. G. Jacobson
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* *Translator's note:* Probably misprint; correct reference probably is to Suhl effect.

1 G. M. Avak'iants, J. Exper. Theoret. Phys. USSR **26**, 562, 668 (1954)
2 B. Y. Davydov, J. Exper. Theoret. Phys. USSR **6**, 471 (1936)
3 B. Y. Davydov, J. Exper. Theoret. Phys. USSR **7**, 1069 (1937)

The Velocity of the Wave Front in Nonlinear Electrodynamics

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In papers by Blokhintsev[1] and Blokhintsev and Orlov[2], it is shown that for nonlinear electrodynamics and mesodynamics, the propagation of a signal (defined as the surface of a weak discontinuity in the field strength) can take place with a velocity greater than the velocity of light in the vacuum*. Both papers are based on the method of characteristics of systems of partial differential equations, going into detail in the case of plane waves. In view of the importance of this question, it is interesting to investigate it further and to simplify the method.

Sommerfeld[4] has investigated the velocity of the signal and of the wave front (the group and phase velocities**) in Maxwell-Lorentz linear electrodynamics. He showed that, in linear electrodynamics, the velocity of the front is always (independent of the medium) equal to the velocity of light in the vacuum***. This result is particularly easy to get by making use of a method pointed out by Levi-Civita. We shall apply the same method to nonlinear electrodynamics, since the equation for the velocity of the wave front can be derived simply and intuitively****.

As is well known, the equations of electrodynamics are gotten by the use of the variational principle from a Lagrangian depending on the first and second invariants of the field, that is,

$$L = L(K, I^2),$$

where

$$K \equiv \frac{1}{2}(E^2 - H^2), \quad I^2 \equiv (\mathbf{E}, \mathbf{H})^2.$$

First let us investigate the use of a plane wave. Let $E = E_x(z, t)$, $H = H_y(z, t)$, $E_y = E_z = H_x = H_z = 0$.

Then the basic equations of the field take the form

$$\alpha \frac{\partial K}{\partial z} H + \alpha \frac{\partial K}{\partial t} E + \frac{\partial H}{\partial z} + \frac{\partial E}{\partial t} = 0; \quad (1)$$

$$\frac{\partial E}{\partial z} + \frac{\partial H}{\partial t} = 0.$$

(the trivial equations $\partial E_y \partial z = 0$ etc., are not written down). Above and in what follows we take the velocity of light $c = 1$,

$$\alpha \equiv \frac{\partial^2 L/\partial K^2}{\partial L/\partial K}, \ \beta \equiv \frac{\partial^2 L/\partial K \partial I}{\partial L/\partial K}, \ \gamma \equiv \frac{\partial^2 L/\partial I^2}{\partial L/\partial K}$$

In the case now under consideration, $\beta = \gamma = 0$.

Let the plane $z = \nu t$ be the surface of weak continuity of E_x and H_y moving with velocity ν. Denoting by $\dot{\Phi}$ the change in the value of any quantity $\dot{\Phi}$ on going through the surface of discontinuity, we have

$$[E] = 0, \ [H] = 0, \ [\partial E/\partial z] \equiv e \neq 0, \quad (2)$$

$$[\partial H/\partial z] \equiv h \neq 0.$$

According to the method we are adopting, we must form the differences on crossing the surface of discontinuity for each of the equations,

$$E(z + \Delta z, \ t + \Delta t) = E(z, \ t) + \frac{\partial E}{\partial z}\Delta z + \frac{\partial E}{\partial t}\Delta t, \quad (3)$$

$$H(z + \Delta z, \ t + \Delta t) = H(z, \ t) + \frac{\partial H}{\partial z}\Delta z + \frac{\partial H}{\partial t}\Delta t,$$

and then form the same differences for crossing the surface for each of Eqs. (1), where $\Delta z = \nu \Delta t$. Using Eqs. (2), we get the following formulas for substitution into Eqs. (1):

$$\frac{\partial}{\partial t} \to -v\frac{\partial}{\partial z}, \quad \frac{\partial E}{\partial z} \to e, \quad \frac{\partial H}{\partial z} \to h.$$

This gives

$$\alpha\, [\partial K/\partial z]\,(H - Ev) + h - ev = 0, \quad e = hv, \quad (4)$$

where $[\partial K/\partial z] = Ee - Hh$.

Solving Eq. (4) we find the velocity of the front (that of the surface of weak discontinuity),

$$v_{1,2} = \frac{\alpha EH \pm \sqrt{1 + \alpha(E^2 - H^2)}}{1 + \alpha E^2}. \quad (5)$$

where $\partial K/\partial z = 0$ (that is, $E^2 - H^2 = f(t)$, or in particular, $E^2 - H^2 = 0$), we get from Eq. (4)

$$v_{1,2} = \pm 1. \quad (6)$$

The velocity of a plane wave front is equal to the velocity of light in the vacuum in the following cases:

1) $\alpha = 0$, $\beta = 0$; i.e., for Lagrangians of the form

$$L = \text{const}\cdot K + f(I^2); \quad (7)$$

$$2)\ E^2 - H^2 = f(t) \ \ (\text{i.e. } \partial K/\partial z = 0). \quad (8)$$

It is easy to show that for constants $E_x = E_0$ and $H_y = H_0$ and for variable $E_y = \epsilon$ and $H_x = h$, if $\epsilon \ll E_0$, $h \ll H_0 (\epsilon^2 \sim h^2 \sim \epsilon h \sim 0)$ and if $\epsilon = h = 0$ on the surface of discontinuity of the derivatives $\partial E_y/\partial z$ and $\partial H_x/\partial z$, then the surface (the wave front of ϵh) is propagated with the velocity

$$v = \frac{-\gamma EH \pm \sqrt{\gamma(E^2 - H^2) - 1}}{1 - \gamma H^2}. \quad (9)$$

If one assumes that the principle of superposition is valid for weak disturbances of the field, then any plane wave $\vec{\epsilon}$, $\mathbf{h}$, propagated in a constant field $\mathbf{E}_0$, $\mathbf{H}_0$, ($\mathbf{E}_0 \perp \mathbf{H}_0$; $\epsilon \ll E$, $h \ll H$) and perpendicular to the constant field, breaks up into two linear polarized rays ϵ_x, h_y and ϵ_y, h_x, each of which moves with velocities (5) and (9)*, respectively.

In the general case of the presence of all the components of the field strength, the present method [by using an equation of the form of Eq. (3) for each component] allows one easily to derive the following set of equations from the basic equations of the field:

$$Se + \Phi h_x + \Pi h_y = 0, \quad (10)$$
$$Te + Dh_x + \Phi h_y = 0,$$
$$Re - Th_x - Sh_y = 0,$$

where

$$e \equiv \left[\frac{\partial E_z}{\partial z}\right] \neq 0, \ h_x \equiv \left[\frac{\partial H_x}{\partial z}\right] \neq 0, \ h_y \equiv \left[\frac{\partial H_y}{\partial z}\right] \neq 0;$$

$$S \equiv \alpha E_z H_z - \beta(E_y E_z - H_y H_z) - \gamma E_y E_z$$
$$- \{\alpha E_x E_z + \beta\ (E_x H_z + E_z H_x) + \gamma H_x H_z\}\, v,$$

$$\Phi \equiv \{\alpha E_x E_y + \beta(E_x H_y + E_y H_x) + \gamma H_x H_y\}\, v^2$$
$$+ \{\alpha(E_x H_x - E_y H_y) - \beta(E_x^2 - E_y^2 - H_x^2 + H_y^2)$$
$$+ \gamma(E_y H_y - E_x H_x)\}\, v - \alpha H_x H_y$$
$$+ \beta(E_x H_y + E_y H_x) - \gamma E_x E_y,$$

$$\Pi \equiv -(\alpha E_x^2 + 2\beta E_x H_x + \gamma H_x^2 + 1)v^2 + 2\{\alpha E_x H_y - \beta(E_x E_y - H_x H_y) - \gamma E_y H_x\}v - \alpha H_y^2 + 2\beta E_y H_y - \gamma E_y^2 + 1;$$

$$T \equiv \{\alpha E_y E_z + \beta(E_y H_z + E_z H_y) + \gamma H_y H_z\}v + \alpha E_z H_x - \beta(E_x E_z - H_x H_z) - \gamma E_x H_z;$$

$$D \equiv -(\alpha E_y^2 + 2\beta E_y H_y + \gamma H_y^2 + 1)v^2 + 2\{-\alpha E_y H_x + \beta(E_x E_y - H_x H_y) + \gamma E_x H_y\}v - \alpha H_x^2 + 2\beta E_x H_x - \gamma E_x^2 + 1,$$

$$R \equiv \alpha E_z^2 + 2\beta E_z H_z + \gamma H_z^2 + 1.$$

Equations (10) are homogeneous with respect to e, h_x, h_y. Therefore,

$$\begin{vmatrix} S & \Phi & \Pi \\ T & D & \Phi \\ R & -T & -S \end{vmatrix} = 0.$$

Expanding the determinant, we get

$$R(\Phi^2 - \Pi D) + S(T\Phi - SD) + T(S\Phi - T\Pi) = 0. \quad (11)$$

Substitution of the expressions for R, Φ, etc. into Eq. (11), leads to the equation for the velocity of the front [see Eq. (10) of Ref. 2]. Clearly, if $E = E_x$, $H = H_y$, $E_y = E_z = H_x = H_z = 0$, then Eq. (11) [just as Eq. (10) of Ref. 2] is unsuitable, for then $e = 0$, $h_x = 0$, $S = T = 0$, $R = 1$, and Eqs. (10) reduce to $\Pi = 0$, $\Phi = 0$, which give Eq. (5). Use of Eq. (11) would give $\Phi^2 - \Pi D = 0$ which leads to incorrect values of ν.

From the derived Eqs. (5) and (9), it is seen that nonlinear equations, generally speaking, give a velocity for the propagation of the wave front which is greater than the velocity of light in the vacuum (for a given choice of the Lagrangian). However, this leaves open the question of whether such a possibility really exists. We note that for a nonlinear Lagrangian, quantum electrodynamics gives a wave front velocity for a plane wave, which is not greater than the vacuum velocity of light.

Analogously, one can get an expression for the propagation velocity of a wave front in scalar or pseudoscalar mesodynamics. The field equation for $\phi = \phi(x, t)$ is

$$\phi'' - \ddot{\phi} - \alpha\phi'^2\phi'' - \alpha\dot{\phi}^2\ddot{\phi} + 2\alpha\phi'\dot{\phi}\dot{\phi}' + \beta\phi(\phi'^2 - \dot{\phi}^2) + \gamma\phi = 0$$

Here

$$\phi' \equiv \frac{\partial\phi}{\partial x}, \quad \dot{\phi} \equiv \frac{\partial\phi}{\partial t}, \quad \alpha \equiv \frac{\partial^2 L/\partial K^2}{\partial L/\partial K}, \quad \beta \equiv \frac{\partial^2 L/\partial K \partial I}{\partial L/\partial K},$$

$$\gamma \equiv \frac{\partial^2 L/\partial I^2}{\partial L/\partial K},$$

$$K \equiv -\frac{1}{2}\phi'^2 + \frac{1}{2}\dot{\phi}^2, \quad I^2 \equiv \frac{1}{2}\phi^2.$$

Making use of $[\phi] = 0$, $[\phi'] = [\dot{\phi}] = 0$, $[\phi''] \equiv \phi_{xx} \neq 0$ and the equations

$$\phi'(x + \Delta x, t + \Delta t) = \phi'(x, t) + \phi''\Delta x + \dot{\phi}'\Delta t,$$

$$\dot{\phi}(x + \Delta x, t + \Delta t) = \dot{\phi}(x, t) + \dot{\phi}'\Delta x + \ddot{\phi}\Delta t,$$

we get, by the same method,

$$(1 + \alpha\dot{\phi}^2)\nu^2 + 2\alpha\dot{\phi}\phi'\nu + \alpha\phi'^2 - 1 = 0.$$

From this,

$$\nu = (-\alpha\dot{\phi}\phi' \pm \sqrt{1 + 2\alpha K})/(1 + \alpha\dot{\phi}^2),$$

which follows from the formulas in Ref. 1.

All the results can be achieved by forming the differences of the divergence of the energy-momentum tensor across the surface of the discontinuity.

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Translated by E. Saletan
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* The question of the change of velocity of propagation of light in nonlinear electrodynamics was first investigated by Svirski [e.g., see Ref. 3, M. S. Svirski, Vestn. (Moscow State University) 3, 43 (1951)].

** It should be noted that there is a difference in terminology in Refs. 2 and 4 [e.g., see Ref. 2, D. I. Blokhintsev and V. V. Orlov, J. Exper. Theoret. Phys. USSR **25**, 513 (1953) and Ref. 4, A. Sommerfeld, Ann. d. Phys. **44**, 177 (1914)]: in Ref. 4, the velocity of the signal means the actual group velocity, whereas in Ref. 2, it means the wave front velocity.

*** Sommerfeld showed that it is impossible to verify experimentally the result that only "forerunners" of extremely weak intensities travel at the vacuum velocity. However, using modern photoelements and amplifiers, it is possible that one can construct an experiment, for instance, to detect weak light pulses traveling without refraction through a plane parallel plate or prism. One can make use of the fact that the "forerunners" are unpolarized by a polarizer.

**** See Eq. (10) in Ref. 2 [e.g., D. I. Blokhintsev

and V. V. Orlov, J. Exper. Theoret. Phys. USSR **25**, 513 (1953)]

* This situation was considered in a discussion with V. I. Skobelkin.

[1]D. I. Blokhintsev, Doklady Akad. Nauk SSSR **82**, 553 (1952)

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[3]M. S. Svirski, Vestn. (Moscow State University) 3, 43 (1951)

[4]A. Sommerfeld, Ann. d. Phys. **44**, 177 (1914)

The Problem of Obtaining a Metastable Modification of Thallium

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AN explanation as to the role played by the crystal lattice in the phenomenon of superconductivity may be found in various studies of the crystalline modification of one or another of the substances at low temperatures. In three well-known metals, thallium, titanium and zirconium, the α-modification exhibits superconductivity but β- modification has not been investigated at low temperatures.

One of the methods yielding a high temperature modification in metastable form is that of sudden quenching. This method of quenching pure substances has been treated by Sekito [1]. In this work an x-ray investigation was made of the modification of thallium (prepared by Kal'baum) which had received rapid cooling of the metal in ice water . As is known, at 235 °C, thallium undergoes allotropical changes in which the density due to hexagonal close packing changes to that of a body centered cubic[2] . Due to this quenching[1] the sample now exhibits a face centered lattice structure.

We have undertaken a low temperature study of the metastable modification of thallium (99.98% pure). The desired quenching may be achieved by several methods:

1. Thallium melted in a glass tube over a Bunsen-burner and plunged into ice water (method of reference 1).

2. To avoid crystallization of melted thallium in the α-modification, stable at 0 °C, the sample before quenching is slowly cooled in the oven from melting temperature (303 °C) to 290 °C. The sample is prepared by melting thallium in thin walled capillary tubes having a wall thickness of 0.1 mm.

3. For very rapid quenching the melted thallium is poured out under vacuum on a copper surface cooled to the temperature of liquid air.

Immediately after the preparation of the sample, x-ray analysis followed. It appears that x-ray analysis does not reveal any difference between the quenched sample and that of ordinary thallium. This likewise applies to the measured magnetic moment of the samples at the liquid temperature of helium. In all samples, in quenched as well as in unquenched, the transition to the superconducting state was observed at 2.38-2.4 °K. The marked absence of hysteresis (less than 1%) and the abrupt transition from superconductivity to the normal state is evidence of the absence of impurities occluded in the sample.

Analysis of the results of these methods shows that not one of the above methods lends itself to producing the thallium in metastable modification as in contrast of the statements found in reference 1. Thus the question of quenching pure thallium is left open.

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Translated by A. Andrews
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[2] H. Lipsona, A. R. Stoks. Nature **148**, 437 (1941)

Possible Methods of Obtaining Active Molecules for a Molecular Oscillator

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249-250 (February, 1955)

AS was shown in reference 1, one must use molecular beams in order to make a spectroscope with high resolving power. In this reference the possibility of constructing a molecular oscillator was investigated. Active molecules needed for self-excitation in the molecular oscillator were to be obtained by deflecting the molecular beam through inhomogeneous electric or magnetic fields. This method of obtaining active molecules has already been employed in the construction of a molecular oscillator [2]