Thermal and Galvanometric Effects in Strong Fields at Low Temperatures

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(Submitted to JETP editor July 17, 1954)
J. Exper. Theoret. Phys USSR **29**, 762-769 (December, 1955)

The thermoelectromotive force, resistance and Hall effect are considered on the basis of a two band model for a metal in a magnetic field at low temperatures.

THE kinetic energy of electrons in a magnetic field, in a plane perpendicular to the direction of H, is quantized[1]. This holds both for free electrons and for electrons in a metal[2-4], independently of their dispersion law[5]. Because of the quantization of the energy, all the physical quantities that depend on the distribution of the electrons in the quantized levels, will oscillate upon a change in H. The oscillation of the magnetic susceptibility has been studied in greatest detail [5-9]; however, a study of the behavior of other physical quantities (resistance, Hall effect, thermal emf, etc.) upon variation of H is of interest for the electron theory of metals. It was pointed out by Akhiezer[6], on the basis of the research work of Titeica[10], that the resistance of metals (in accord with published experiments[11,12]) must oscillate upon variation of the magnetic field. Blokhintsev and Nordheim[13] investigated the increase of resistance in a magnetic field. Davydov and Pomeranchuk[14], making use of the particular case of a two band model for bismuth, showed that, along with the oscillations, there was observed an unlimited increase in the resistance of bismuth in a magnetic field. Their expressions applied only to bismuth, and in view of their complicated character, were unsuitable for comparison with experiment. Kohler[15] obtained a simple expression for the resistance in a magnetic field, but he did not take into account the quantization of the energy of the electrons.

In the present research, we have considered the behavior of the resistance, Hall effect and thermal emf (in the isotropic case) for a two band model of a metal in a strong magnetic field at low temperatures.

In place of the inequality $\omega\tau \ll 1$ ($\omega = eH/mc$, τ is the free path time for the electron in the metal), which allows us to write down the kinetic equation in the usual way, we shall assume the inverse inequality to be valid: $\omega\tau \gg 1$. In this case the quantization is preserved, in spite of collisions with the lattice.

1. EIGENFUNCTIONS AND THE ENERGY OF THE ELECTRONS

For free electrons in a magnetic field $H = H_z$ (described by the vector potential $A_x = -H_y$, $A_y = A_z = 0$) and an electric field $F = F_y$, we have the equation

$$\mathcal{H}\psi = -\frac{\hbar^2}{2m}\Delta\psi + i\mu Hy\frac{\partial\psi}{\partial x} + \frac{e^2H^2y^2}{2mc^2}\psi + eFy\psi = E\psi, \tag{1}$$

where $\mu = e\hbar/mc$, e = absolute value of the electronic charge. The solution of this equation has the form

$$\psi_{k_1nk_3} = \frac{\exp\{i(k_1x+k_3z)\}}{2\sqrt{L_1L_3\alpha_0}}\varphi_n\left(\frac{y}{\alpha_0} - \frac{\hbar ck_1}{\alpha_0 eH} + \frac{Fmc^2}{\alpha_0 eH^2}\right), \tag{2}$$

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where $\alpha_0 = (\hbar c/eH)^{1/2}$; φ_n = Chebyshev-Hermite functions;

$$E_{k_1nk_3} = E_{nk_3} + \hbar c k_1 E/H, \tag{3}$$

$$E_{nk_3} = \mu H(n + 1/2) + \hbar^2 k_3^2/2m.$$

We note that $\hbar c k_1 F/H = eFy_0$, where y_0 is the equilibrium position. To the state (2) there corresponds the current

$$i_x = -ecF/H, \quad i_y = 0. \tag{4}$$

We now pass over to electrons in a lattice, wherein we shall make use of the approximation of strongly coupled electrons, employing the rather simplified method of Peierls[2]. If $\varphi_0(\mathbf{r})$ is the eigenfucntion of the electron in an atom located at the point $\mathbf{n} = 0$, then (see reference 4) we have for the electron in the atom at the point $\mathbf{n}$ (if the magnetic field directed along the axis oz is described by the potential $A_y = Hx$):

$$\varphi_{\mathbf{n}}(\mathbf{r}) = \exp\{-i\alpha_0^{-2} n_1 y\}\, \varphi_0(\mathbf{r} - \mathbf{n}). \tag{5}$$

For an electron in the periodic field of the lattice, we have

$$\frac{1}{2m}\left(\hat{\mathbf{p}} + \frac{e}{c}A\right)^2 \psi + (V_p + eFy)\psi = E\psi. \tag{6}$$

We seek the solution of this equation in the form

$$\psi = \sum_{\mathbf{n}} a_{\mathbf{n}} \varphi_{\mathbf{n}}(\mathbf{r}), \tag{7}$$

where the function $\varphi_{\mathbf{n}}(\mathbf{r})$, which is defined by Eq. (5), satisfies the equation

$$\frac{1}{2m}\left(\hat{\mathbf{p}} + \frac{e}{c}\mathbf{A}\right)^2 \varphi_{\mathbf{n}} + [V(\mathbf{r} - \mathbf{n}) + eFy]\varphi_{\mathbf{n}} = E_{\mathbf{n}}\varphi_n, \tag{8}$$

$V(\mathbf{r} - \mathbf{n})$ is the field in an isolated atom at the point $\mathbf{n}$. For the coefficients $a_{\mathbf{n}}$ we get (assuming the neighboring $\varphi_{\mathbf{n}}$ to be slightly overlapping)

$$\varepsilon a_{\mathbf{m}} = \sum_{\mathbf{n}} a_{\mathbf{n}} \varepsilon_{\mathbf{mn}}; \qquad \varepsilon = E - E_a \tag{9}$$

(E_a are the eigenvalues in the isolated atom),

$$\varepsilon_{\mathbf{mn}} = \exp\{i\alpha_0^{-2}(m_1 - n_1)n_2\}[A(\mathbf{m} - \mathbf{n}) + eFn_2\delta_{\mathbf{mn}}]. \tag{10}$$

The exchange integral $A(\mathbf{q})$ is equal to

$$A(\mathbf{q}) = \int \varphi_0^*(\mathbf{r} - \mathbf{q})\varphi_0(\mathbf{r})[V_p(\mathbf{r}) - V(\mathbf{r})]\,d\tau \tag{11}$$

It is not difficult to obtain the operator $\hat{\varepsilon}$ from the matrix elements (10). It has the form

$$\hat{\varepsilon} = \sum_{\mathbf{q}} A(\mathbf{q}) e^{-i\mathbf{q}\hat{\mathbf{R}}} + eFy, \tag{12}$$

where $\hat{\mathbf{R}} = [\hat{\mathbf{p}} + (e/c)\mathbf{A}]/\hbar$ (e is the absolute value of the electronic charge), while $A_x = -Hy$, $A_y = A_z = 0$, and $\mathbf{q}$ is the lattice vector.

Writing the operator $\hat{\varepsilon}$ in the form (12), we have neglected the noncommutability of the components of the operator $\hat{\mathbf{R}}$ (a = lattice constant):

$$a\hat{R}_x a\hat{R}_y - a\hat{R}_y a\hat{R}_x = -ia^2/\alpha_0^2, \tag{13}$$

inasmuch as $a^2/\alpha_0^2 \ll 1$, which takes place even in very strong magnetic fields (in order that this ratio be unity, the field must be of the order of 10^8–10^9 oersteds).

Since the exchange integral $A(\mathbf{q})$ increases very rapidly with q, we can write Eq. (12) for the cubic lattice in the form:

$$\hat{\varepsilon} = A_{00} + 2A_1(\cos a\hat{R}_1 + \cos a\hat{R}_2 + \cos a\hat{R}_3) + eFy. \tag{14}$$

We consider the field F small, so that the following inequality is satisfied: $eFN\alpha_0 \ll \mu H$, where $N = \sqrt{2n+1}$, $N\alpha_0$ is the amplitude of oscillation of the electron. Below the upper band we can expand in a cosine series, neglecting two terms. Then the operator (14) is identical with the operator (1), if we introduce the effective mass

$$m_{\text{eff}} = m_1 = -\hbar^2/2A_1a^2$$

These operators have identical eigenfunctions* [Eq. (2)] and, if we select the value of the energy at $k_1 = 0$ as zero, identical eigenvalues (3).

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* In the approximation of strong coupling of the electrons, for $ka \ll 1$, the function $U_k(\mathbf{r})$ in the expression $\psi = \exp\{i\mathbf{k}\cdot\mathbf{r}\}\sum\exp\{i\mathbf{k}\cdot(\mathbf{n} - \mathbf{r})\}\varphi_0(\mathbf{r} - \mathbf{n}) = \exp\{i\mathbf{k}\cdot\mathbf{r}\}U_k(\mathbf{r})$ depends very weakly on $\mathbf{k}$, since $|\mathbf{r} - \mathbf{n}| \lesssim a$, and therefore the rapidly oscillating factor of $U_{\mathbf{k}}(\mathbf{r})$ can be considered constant in the calculation of arbitrary matrix elements from functions in the region $ka \ll 1$. In the presence of a magnetic field, Eq. (2) appears in place of $e^{i\mathbf{kr}}$.

Above the lower band we can again expand Eq. (14) in a series, neglecting the quadratic terms. By means of a transformation of the eigenfunctions [multiplication of them by $\exp\{\pm(i\pi/a)(x+y+z)\}$] we can reduce the problem to Eq. (1) here also. However, the effective mass, $-m_2$, is now given by

$$m_{\text{eff}} = -m_2 = \hbar^2 / 2A_2a^2, \quad m_{\text{eff}} < 0.$$

The eigenfunctions will have the form

$$\psi_{k_1nk_3} = \sqrt{2}\, {\sin \atop \cos} \frac{\pi}{a}(x+y+z) \frac{\exp\{i(k_1x+k_3z)\}}{2\sqrt{L_1L_3\alpha_0}} \times \varphi_n\left(\frac{y}{\alpha_0} - \frac{\hbar ck_1}{\alpha_0 eH} - \frac{Fm_2c^2}{\alpha_0 eH^2}\right). \tag{15}$$

Here the square of the first factor (which is rapidly oscillating and identical for all $\psi_{k_1nk_3}$) can be replaced by ½ in the calculation of the matrix elements.

The energy of the electron can be written as

$$E_{k_1nk_3} = A_0 - \mu_2 H\left(n+\frac{1}{2}\right) - \frac{\hbar^2k_3^2}{2m^2} + \frac{\hbar ck_1F}{H} \tag{16}$$

Here A_0 is the overlap of the lower and upper bands.

The same current corresponds to the state (15) and to the state (2). The same result obtains in classical physics: the velocity of the electron has the mean value cF/H in the field $F = F_y$, $H = H_z$ and depends neither on its mass nor on its charge. The current density for all electrons in the first (upper) band will be $j_{x_1} = -(ecF/H)N_1$; N_1 = number of electrons in this zone per cm^3. The current density in the second (lower) zone will be equal to $j_{x_2} = (ecF/H)N_2$ if we take it into account that the current which corresponds to a filled band is zero. Here N_2 is the number of free states in the lower band (number of holes). The resulting current density along the ox axis will be

$$j_x = -(ecF/H)(N_1 - N_2). \tag{17}$$

The overlapping of the bands A_0 and the Fermi level E_0 (for $H = 0$, $T = 0$) are connected with the numbers N_1 and N_2 by the relations

$$N_1 = (8\pi/3h^3)(2m_1E_{00})^{3/2};$$
$$N_2 = (8\pi/3h^3)[2m_2(A_0 - E_{00})]^{3/2}.$$

2. THE CURRENT IN THE DIRECTION OF THE ELECTRIC FIELD

The current in the direction of the electric field F_y is brought about by transport of electrons because of collisions with the lattice[10]. At the very low temperatures of interest to us we can consider only scattering by distortions of the lattice.

The current I_y through the plane $y = 0$ is equal to the number of transfers across this plane per second, multiplied by the charge on the electron:

$$I_y = \frac{2\pi e}{\hbar}\left(\frac{L_1L_3}{\pi^2}\right)^2 \times \int_{k_1>0}\int_{\varkappa_1<0}\sum_{n,m}\int_{k_3\varkappa_3} |V_{k_1nk_3;\,\varkappa_1m\varkappa_3}|^2\,\delta(E_{k_1nk_3} - E_{\varkappa_1m\varkappa_3}) \times [f_{k_1}(E_{nk_3})(1 - f_{\varkappa_1}(E_{m\varkappa_3})) - f_{\varkappa_1}(E_{m\varkappa_3}) \times (1 - f_{k_1}(E_{nk_3}))]\,dk_1\,d\varkappa_1\,dk_3\,d\varkappa_3, \tag{18}$$

Here $V_{k_1nk_3\varkappa_1m\varkappa_3}$ is the matrix element of the perturbation V, produced by the transfer:

$$V_{k_1nk_3;\,\varkappa_1m\varkappa_3} = \frac{1}{4L_1L_3\alpha_0}\int \exp\{-i(k_1-\varkappa_1)x - i(k_3-\varkappa_3)z\} \times \varphi_n\left(\frac{y}{\alpha_0} - \frac{\hbar ck_1}{\alpha_0 eH} + \frac{Fmc^2}{\alpha_0 eH^2}\right) \times \varphi_m\left(\frac{y}{\alpha_0} - \frac{\hbar c\varkappa_1}{\alpha_0 eH} + \frac{Fmc^2}{\alpha_0 eH^2}\right) V\,d\tau. \tag{19}$$

The argument of the distribution function f will be E_{nk_3} [Eq. (3)], not $E_{k_1nk_3}$, inasmuch as $\hbar ck_1F/H$ plays the role of a potential energy in the electric field. The index k_1 of the function f is introduced for the case in which there is a weak temperature gradient in the metal.

In a way similar to that used in reference 10, we can show that the function f is identical, up to terms of order F^2, with the usual Fermi function, provided that the temperature gradient is proportional to the field F (thermo-emf). The proof is based on the fact that in Eq. (20) below [as also in Eq. (18)],

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = \frac{2L_1L_3}{\pi\hbar}\sum_m \int_{-\infty}^{+\infty} d\varkappa_1 \int_{-\infty}^{+\infty} d\varkappa_3 \, |V_{k_1nk_3;\,\varkappa_1m\varkappa_3}|^2 \, \delta(E_{k_1nk_3} - E_{\varkappa_1m\varkappa_3}) \times [f_{k_1}(E_{nk_3})(1 - f_{\varkappa_1}(E_{m\varkappa_3})) - f_{\varkappa_1}(E_{mk_3})(1 - f_{k_1}(E_{nk_3}))], \quad (20)$$

the integration over $\varkappa_1$ is effectively between the limits

$$-\alpha_0(M+N) \leqslant \alpha_0^2(\varkappa_1 - k_1) \leqslant \alpha_0(M+N), \quad (21)$$
$$N = \sqrt{2n+1}, \quad M = \sqrt{2m+1},$$

since the matrix element (19) differs from zero only in this interval. The integral (20) depends on the field F only in the combination $eF\alpha_0^2(k_1 - \varkappa_1)$, but in the expansion in powers of F and integration, such terms drop out, in view of the condition (21). This will be true even in the case in which there is a temperature gradient in the metal, a gradient proportional to the field. We can then write

$$f_{\varkappa_1} = f_{k_1} + (\varkappa_1 - k_1)\frac{\hbar c}{eH}\left[\frac{\partial T}{\partial y}\frac{\partial f_{k_1}}{\partial T} + \frac{\partial E_0}{\partial y}\frac{\partial f_{k_1}}{\partial E}\right], \quad (22)$$

where E_0 is the Fermi level.

In the calculation of Eq. (18) we make use of the smallness of the difference $E_{nk_3} - E_{m\varkappa_3}$ and of the expansion (22):

$$I_y = \frac{2\pi e d_0^2}{\hbar}\left(\frac{L_1L_2}{\pi^2}\right)^2 \Big\{\left(-eF - \frac{\partial E_0}{\partial y}\right) \int\limits_{k_1>0}\int\limits_{\varkappa_1<0}\sum_{m,n}\int\limits_{k_3\varkappa_3} |V_{k_1nk_3;\,\varkappa_1m\varkappa_3}|^2 \times \delta(E_{k_1nk_3} - E_{\varkappa_1m\varkappa_3})(k_1 - \varkappa_1)\frac{\partial f_{k_1}}{\partial E_{nk_3}}\, dk_1\, d\varkappa_1\, dk_3\, d\varkappa_3$$
$$- \frac{1}{T}\frac{\partial T}{\partial y}\int\limits_{k_1>0}\int\limits_{\varkappa_1<0}\sum_{m,n}\int\limits_{k_3\varkappa_3} |V_{k_1nk_3;\,\varkappa_1m\varkappa_3}|^2\,\delta(E_{k_1nk_3} - E_{\varkappa_1m\varkappa_3})(k_1 - \varkappa_1) \times (E_{nk_3} - E_0)\frac{\partial f_{k_1}}{\partial E_{nk_3}}\, dk_1\, d\varkappa_1\, dk_3\, d\varkappa_3\Big\}. \quad (23)$$

To calculate the matrix element (19) which determines the probability of elastic scattering on distortions of the periodic potential of the lattice, randomly located in the crystal, we assume that the difference V between the real and ideal periodic potential can be expressed in the form

$$V = V_0 \sum_{\mathbf{p}} \delta(\mathbf{r} - \mathbf{p}), \quad (24)$$

where the summation is carried out over all the nodes where distortion takes place. We assume such a form of V completely if the scattering of electrons located near the boundary of the band is considered.

Making use of the independence of the location of neighboring distortions, we can calculate the matrix element of the perturbation and carry out the integration over k_1, $\varkappa_1$, $\varkappa_3$, after which Eq. (23) takes the form

$$I_y = V_0^2 n_d \frac{L_1L_3 m e}{\alpha_0^2 6\pi^3\hbar^3}\Big\{\left(-eF - \frac{\partial E_0}{\partial y}\right)\sum_{m,n}\int\limits_{k_3}\frac{M^2 + N^2}{\sqrt{k_3^2 + 2\alpha_0^{-2}(n-m)}}\frac{\partial f}{\partial E_{nk_3}}\, dk_3$$
$$- \frac{1}{T}\frac{\partial T}{\partial y}\sum_{m,n}\int\limits_{k_3}\frac{M^2 + N^2}{\sqrt{k_3^2 + 2\alpha_0^{-2}(n-m)}}\frac{\partial f}{\partial E_{nk_3}}(E_{nk_3} - E_0)\, dk_3\Big\}. \quad (25)$$

Here n_d is the concentration of the distortions. Summation over m is obtained with the aid of Poisson's formula and yields

$$\sum_m \frac{M^2+N^2}{\sqrt{k_3^2+2\alpha_0^{-2}(n-m)}} \tag{26}$$

$$= \frac{2\sqrt{2}\hbar}{\sqrt{m_0}(\mu H)^2}\left[\frac{2}{3}E_{nk_3}^{3/2}+\mu H\left(n+\frac{1}{2}\right)E_{nk_3}^{1/2}\right]$$

$$+\frac{2\hbar\sqrt{2\pi}}{\sqrt{m_0}(\mu H)^{3/2}}\sum_{q=1}^{\infty}\frac{(-1)^q}{\sqrt{2\pi q}}\cos\left(\frac{2\pi q E_{nk_3}}{\mu H}-\frac{\pi}{4}\right)$$

$$\times\left(E_{nk_3}+\mu H\left(n+\frac{1}{2}\right)\right)$$

We substitute this in Eq. (25), sum over n and integrate over k_3, thus obtaining the final expression for I_y. This expression is very cumbersome, but it becomes appreciably simpler if the following inequalities are satisfied:

$$\exp\{2\pi^2 kT/\mu H\} \gg 1,\quad kT \not< \mu H,\quad \mu H/E_0 \ll 1,\quad kT/E_0 \ll 1. \tag{27}$$

These conditions are usually satisfied in the theory of diamagnetism at low temperatures.

For the current density in the direction OY we get, for electrons in the lower levels of the upper band:

$$j_{y_1} = \frac{m_1^4 C}{H^2}\left\{\left(eF+\frac{\partial E_0}{\partial y}\right)\left[\frac{8}{3}E_0^2+0{,}6\mu_1 H E_0-\frac{40\pi^2\sqrt{2}}{3}\frac{kTE_0^{3/2}}{\sqrt{\mu_1 H}}e^{-\gamma_1}\cos\left(\varepsilon_1-\frac{\pi}{4}\right)+\ldots\right]\right.$$
$$\left.+k^2T\frac{\partial T}{\partial y}\left[\frac{16\pi^2}{9}E_0+\frac{\pi^2}{5}\mu_1 H-\frac{40\pi}{3\sqrt{2}}\frac{\sqrt{\mu_1 H}}{kT}E_0^{3/2}e^{-\gamma_1}(1-\gamma_1)\sin\left(\varepsilon_1-\frac{\pi}{4}\right)+\ldots\right]\right\}; \tag{28}$$

$$C = V_0^2 n_d \frac{2c^2}{3\pi^3 e\hbar^7};\quad \gamma_1=\frac{2\pi^2 kT}{\mu_1 H};\quad \varepsilon_1=\frac{2\pi E_0}{\mu_1 H}; \tag{29}$$

and for the electrons in the upper part of the lower band:

$$j_{y_2}=\frac{m_2^4 C}{H^2}\left\{\left(eF+\frac{\partial E_0}{\partial y}\right)\left[\frac{8}{3}(A_0-E_0)^2+0{,}6\mu_2 H(A_0-E_0)\right.\right.$$
$$\left.-\frac{40\pi^2\sqrt{2}}{3}\frac{kT(A_0-E_0)^{3/2}}{\sqrt{\mu_2 H}}e^{-\gamma_2}\cos\left(\varepsilon_2-\frac{\pi}{4}\right)+\ldots\right]-$$
$$-k^2T\frac{\partial T}{\partial y}\left[\frac{16\pi^2}{9}(A_0-E_0)+\frac{\pi^2}{5}\mu_2 H-\right.$$
$$\left.\left.-\frac{40\pi}{3\sqrt{2}}\frac{\sqrt{\mu_2 H}}{kT}(A_0-E_0)^{3/2}e^{-\gamma_2}(1-\gamma_2)\sin\left(\varepsilon_2-\frac{\pi}{4}\right)+\ldots\right]\right\}; \tag{30}$$

$$\gamma_2 = 2\pi^2 kT/\mu_2 H,\qquad \varepsilon_2 = 2\pi(A_0-E_0)/\mu_2 H. \tag{31}$$

3. THERMOELECTRIC FIELD

The theory of the thermal emf in a magnetic field, based on the kinetic equation[15], shows that, upon unbounded increase in H, the longitudinal electric field (in the direction of the temperature gradient) becomes significantly greater than the transverse (their ratio is of the order of $\omega\tau$). We make the assumption that this conclusion is qualitatively correct in the case of interest to us: $\omega\tau \gg 1$, although the kinetic equation, strictly speaking, is valid only for $\omega\tau < 1$. An independent argument in favor of our assumption is the fact that a thermoelectric field exists in the longitudinal direction, even in the absence of a magnetic field, while, under such conditions, there is no such field in the transverse direction. Moreover, the change of thermal emf is always small in a magnetic field (experiments have not yet been carried out at low temperatures).

If our assumption is correct, we can obtain the thermoelectric field F by making use of Eqs. (28) and (30), and the equation

$$j_{y_1}+j_{y_2}=0.$$

In this case the H^{-2} in front of Eqs. (28) and (30) drops out, as a result of which the terms which increase rapidly with field are absent in the thermal emf case, in contradistinction to the cases of resistance and the Hall coefficient. If the above mentioned assumption is not correct, then such terms can appear in the thermal emf. We get

$$eF=-\frac{\partial E_0}{\partial y}-\frac{2}{3}\frac{\partial T}{\partial y}\frac{\pi^2k^2T^2}{m_1^4E_0^2+m_2^4(A_0-E_0)^2}\Big\{m_1^4E_0-m_2^4(A_0-E_0)$$
$$\frac{9}{80}(\mu_1Hm_1^4-\mu_2Hm_2^4)+\frac{15}{2\pi\sqrt{2}}\frac{m_1^4\sqrt{\mu_1H}E_0^{3/2}}{kT}e^{-\gamma_1}(1-\gamma_1)\sin\left(\varepsilon_1-\frac{\pi}{4}\right) \tag{32}$$
$$+\frac{15}{2\pi\sqrt{2}}\frac{m_2^4\sqrt{\mu_2H}(A_0-E_0)^{3/2}}{kT}e^{-\gamma_2}(1-\gamma_2)\sin\left(\varepsilon_2-\frac{\pi}{4}\right)+\ldots\Big\}.$$

In the single band model we have

$$eF=-\frac{\partial E_0}{\partial y}-\frac{2}{3}\frac{\partial T}{\partial y}\frac{1}{E_0}\left[1-\frac{9}{80}\frac{\mu H}{E_0}+\frac{15}{2\pi\sqrt{2}}\frac{\sqrt{\mu HE_0}}{kT}e^{-\gamma}(1-\gamma)\sin\left(\varepsilon-\frac{\pi}{4}\right)+\ldots\right];$$
$$\frac{\partial E_0}{\partial y}=\frac{\partial E_0}{\partial T}\frac{\partial T}{\partial y}=k\frac{\partial T}{\partial y}\left[-\frac{\pi^2}{6}\frac{kT}{E_0}+\pi^{3/2}(1-\gamma)\sqrt{\frac{\mu H}{2\pi E_0}}e^{-\gamma}\sin\left(\varepsilon-\frac{\pi}{4}\right)+\ldots\right].$$

Thus the thermal emf must oscillate upon change of H, like the other physical quantities. The oscillating term has the factor $\sqrt{\mu HE_0/kT}$, of the same order of magnitude as the corresponding term $(\sqrt{\mu HE_0/kT})\,(kT/\mu H)^2$ in the expression for the magnetic susceptibility.

4. RESISTANCE IN A TRANSVERSE MAGNETIC FIELD AND THE HALL EFFECT

The relative positions of the components of the current and field in the specimen are shown in Fig. 1. In the absence of a temperature gradient, the entire current is directed along the ξ axis.

The resistance in the magnetic field is $\rho_H=F_\xi/j=(j_y/j^2)F$, where $j_y=j_{y_1}+j_{y_2}$. Making use of Eqs. (28), (30) we obtain

$$\rho_H=\frac{CH^2(f_1+f_2)}{ec^2H^2(N_1-N_2)^2+cC^2(f_1+f_2)^2}, \tag{33}$$

where

$$f_1=\frac{8}{3}E_0^2m_1^4\left(1+\frac{9}{40}\frac{\mu_1H}{E_0}-5\pi^2\sqrt{2}\frac{kT}{\sqrt{\mu_1HE_0}}e^{-\gamma_1}\cos\left(\varepsilon_1-\frac{\pi}{4}\right)+\ldots\right) \tag{34}$$

and f_2 is obtained from f_1 by replacing m_1 by m_2, and E_0 by A_0-E_0. Equation (33) is valid only for $\omega\tau\gg 1$ (in strong fields). For $N_1=N_2$, ρ_H increases proportionally to H^2, while for $N_1\neq N_2$, saturation is observed.

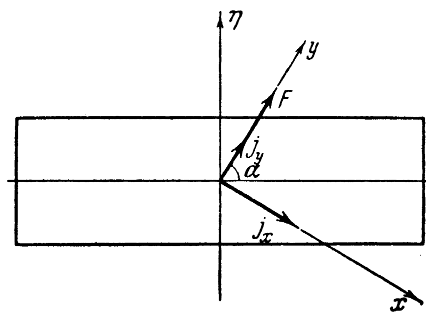

FIG. 1

The ratio F_η/F_ξ, introduced by Borovik[12] as a characteristic of the Hall effect at low temperatures, is equal to

$$\frac{F_\eta}{F_\xi}=\frac{j_x}{j_y}=\frac{-H(N_1-N_2)}{C(f_1+f_2)}. \tag{35}$$

Equations (33) and (34) are in conformity with the classification of metals[12] according to the type of dependence of the resistance and the Hall field on the direction of the magnetic field. If $N_1\neq N_2$, the resistance increases up to saturation and the Hall field is proportional to H. If $N_1=N_2$, the resistance increases in proportion to H^2, and the Hall field is very small. It follows from Eq. (35) that in this case $F_\eta=0$; this is dependent on our approximation (in the expression for the current j_x, there must appear terms of much higher order relative to $(\omega\tau)^{-1}$, which leads to a decrease of F_η in strong fields; this has also been observed[12]).

The oscillations of the resistance and the Hall field must differ in phase by π; this has been observed in the experiments of Borovik[12]. The mobility of the electrons enters only into the current j_y; therefore, if the resistance decreases under external pressure (as a consequence of the increase in the exchange integral A and, consequently, the decrease of the effective mass m), then the Hall field must increase. This has also been observed by Alekseevskii and Brandt[16].

The expression (28) for the current density (for $dT/dy = 0$) can be written in the form

$$j_y = \sigma F(\omega\tau)^{-2}, \tag{36}$$

which follows from the theory of Kohler[15] for $\omega\tau \gg 1$. In such a case, if we assume $\sigma = e^2 n\tau/m$, we can estimate the mean free time of the electrons. If we keep in Eq. (28) only the principal term $(8/3)E_0^2$, we get

$$1/\tau = V_0^2 n_d 2m \sqrt{2mE_0}/3\pi\hbar^4. \tag{37}$$

while for the mean free path we get

$$1/l = (1/\tau)\sqrt{m/2E_0} = 2V_0^2 n_d\, m^2/3\pi\hbar^4, \tag{38}$$

and $1/l$, as always, in scattering on impurities or distortions, is proportional to the concentration of the impurities and does not depend on the concentration of electrons.

If we also keep all the other terms in Eq. (28), then the free time of the electrons on the Fermi surface oscillates upon change of the magnetic field:

$$\frac{1}{\tau} = \frac{2V_0^2 n_d\, m}{3\pi\hbar^4}\sqrt{2mE_0}\Big(1 + \frac{9}{40}\frac{\mu H}{E_0} - 5\pi^2\sqrt{2}\frac{kT}{\sqrt{\mu H E_0}} e^{-\gamma}\cos\Big(\varepsilon - \frac{\pi}{4}\Big) + \ldots\Big) \tag{39}$$

Here we assume E_0 to be a constant quantity, since the corrections will be of a higher order of smallness than the terms considered.

The author takes this occasion to express his thanks to Professor I. M. Lifshitz for his discussions of the work.

[16] N. E. Alekseevskii and N. G. Brandt, Proceedings of the Conference on Low temperature Magnetism, Kharkov, 1954.

Translated by R. T. Beyer
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