Diffraction Splitting of Fast Nonrelativistic Deuterons

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A calculation of diffraction splitting is carried out for deuterons of energies of order 100 mev interacting with nuclei whose dimensions are large compared with those of the deuteron. It is shown that the total cross section for the process has the behavior of the stripping cross section. The angular distributions of the outgoing proton and neutron in correlated pairs and their energy distribution have the same behavior as in deuteron stripping when either a neutron or a proton is scattered out. Because of the indeterminacy of the structure of the nuclear surface (and its transparency for fast nucleons) and the lack of knowledge about the behavior of the deuteron wave function for small neutron-proton separations, the formulae obtained are good only for some of the processes; namely, the processes in which a small amount of momentum is transferred to the nucleus. The total cross section is only estimated. The angles and the energy spectrum for diffraction splitting are different from those of the electric (coulomb) splitting. The cross section of the diffraction splitting is independent of energy and increases with the atomic number as $A^{1/3}$.

1. INTRODUCTION

As was shown by one of us previously[1], fast deuterons should undergo in collisions with nuclei a characteristic process of diffraction splitting due to the action of nuclear forces. This process would not, as a rule, be accompanied by a nuclear reaction and would take place outside the nucleus. In this work we calculate the cross section for diffraction splitting of deuterons of energies of the order 100 mev by sufficiently heavy nuclei. It will become evident that an exact calculation cannot be made for all conditions uniquely, because: a) the quantitative result is determined by the nature of the deuteron wave function that is known only for large separations between the particles; b) the result depends also on the transparency of the nucleus for a nucleon passing at various distances from its center. This dependence causes complications due to the lack of knowledge of the outer regions of the nucleus.

For the above reasons the differential cross section will be strictly obtained only for a group of the processes, namely, the cases in which the momentum q transferred to the nucleus is smaller than the reciprocal of the radius of nuclear forces $1/\mu$ (μ is the π-meson mass and we shall set $\hbar = c = 1$).

This range of processes is the most important one for the following reasons. From physical considerations it is clear that for effective splitting the important q are those up to near the reciprocal dimensions of deuteron, i.e., $q \sim 2\sqrt{M\epsilon} \approx 0.6\ \mu$ (M is the mass of a nucleon and ϵ the binding energy of the deuteron). For $q \gtrsim 0.6\ \mu$ the cross section for the process should vanish rapidly (the corresponding matrix elements will contain rapidly oscillating multipliers; this conclusion follows also from purely optical wave analogies). Considering the boundary of the nucleus sharp we shall not make any error by limiting ourselves to that part of the effect due to small q, $q \ll \mu$. For the full effect we shall obtain an order of magnitude estimate by extrapolation to $q \sim \mu$. In accord with this incompleteness of the result we can allow other approximations in the calculations. However, the result seems to us to deserve attention even with such approximations. It turns out that the estimate results in a substantial value of the cross section — of the order of magnitude of the stripping cross section. The process has its peculiar characteristics which allow us to separate it from, for example, the process of electric splitting. Furthermore, even though the diffraction splitting of deuterons is only a small part of the total (for small q), it may be considered as a model for processes involving the diffraction splitting in other systems (for example, of other nuclei or, in particular, in cosmic rays). Therefore it has a methodological importance.

The cross section of the process in question, together with the process of electric splitting can be computed for sufficiently heavy nuclei from the Born approximation formula

$$d\sigma = 2\pi \,|<\chi_{\rho}^{(-)}\,|\,U + W\,|\,\psi_{0a} + \psi_d^{\text{диф}}>|^2\,\delta(E_a - E_b). \qquad (1.1)$$

Here U is the interaction potential of the proton-neutron system, which we shall consider depen-

[1] E. L. Feinberg, J. Exper. Theoret. Phys. USSR 29, 115 (1955); Soviet Phys. JETP 2, 58 (1956).

dent on the separation $r = r_p - r_n$ between the proton and the neutron; $U = U(r)$; W is the electrostatic interaction potential of the proton with the nucleus: $W = Ze^2/r_p$; ψ_{0a} is the wave function of the incident deuteron which may be considered nonrelativistic for the region of the deuteron momenta of interest and Ω is the normalization volume:

$$\psi_{0a} = \Omega^{1/2} \exp\{ip_d(\mathbf{r}_p + \mathbf{r}_n)/2\}\,\varphi(|\mathbf{r}_p - \mathbf{r}_n|), \quad (1.2)$$

where φ is the wave function of a stationary deuteron written similarly for the potential $U(r)$. $\psi_d^{\text{dif.}}$ is the wave function for the deuteron scattered as a whole as a result of the diffraction on the nucleus; χ_0^- is the product of wave functions of mutually noninteracting outgoing neutron and proton, of momenta $\mathbf{p}_n$ and $\mathbf{p}_p$. It should be noted that χ_0^- takes into account the interaction of these particles with the field of the nuclear forces of the nucleus. The wave function of the deuteron φ is also needed for the calculations. Outside of the radius of action of the nuclear forces which is of the order $1/\mu$,

$$\varphi = Ne^{-\alpha r}/r, \quad \alpha = \sqrt{M\varepsilon}. \quad (1.3)$$

Since we shall consider only transferred momentum not exceeding μ, i.e., distances between deuteron and nucleus greater than $1/\mu$, then we may use just this function φ. As is well known[2], with the proper normalization we obtain

$$N = \sqrt{3\alpha/4\pi}. \quad (1.4)$$

Furthermore, we introduce the average radius of the deuteron

$$R_d = 1/2\alpha = 1/2\sqrt{M\varepsilon} \approx 1.6/\mu. \quad (1.5)$$

In the present we shall compute the cross section for pure diffraction splitting, neglecting the influence of the electrostatic field of the nucleus since, as we shall see, the diffraction process takes place mainly in a definite energy range. Therfeore we set $W = 0$ and in $\psi_d^{\text{dif.}}$ we shall include only diffraction by the nuclear forces of the nucleus. This latter approximation is possible only for deuterons of sufficiently high energies. ($Ze^2/\hbar v < 1$). In particular, for medium heavy nuclei the deuteron energy should be greater than several tens of mev. In such a case the criterion of the external character of the process (cf. reference 1, Eq. (2.5)] is known to be satisfied.

[2] A. I. Akhiezer and I. Ia. Pomeranchuk, *Some Problems in Nuclear Theory*, 2nd ed., 1950, Sec. 2.

Furthermore, the wavelength of all particles is much smaller than the radius of the nucleus. The wave functions of the particles diffracted by the field V of the nuclear forces will be found with the assumption that the nucleus is completely opaque to neutrons and protons. This assumption is not desirable, of course, for nucleon energies greater than 70 mev, i.e., for deuterons whose energy E_d exceeds $\sim$ 150 mev. However, the generalization does not encounter any fundamental difficulties and we shall not yet discard it. Finally, in the range of energies E_d under consideration (40-150 mev), it is possible to neglect the exchange of the nucleons of the deuteron with the nucleons of the nucleus.

We shall see that under the described conditions diffraction splitting actually does take place and that the total cross section is of the order RR_d (R is the radius of the nucleus), and that the particles are scattered within angles of the order $\sqrt{M\epsilon}/p_d$, just as in deuteron stripping, and that the energies of the particles are equal to each other with accuracy of the order of $\sqrt{E_d\epsilon}$. The nucleus receives a small recoil (the recoil momentum is of the order $\sqrt{4M\epsilon}$. The energy transferred is smaller than the binding energy of the nucleon in the nucleus (even on recalculation for one nucleon); in reality, the momentum is absorbed by the surface layer of the nucleus – the "skin layer" – and the excitation energy is still smaller. Therefore, as a rule, the nucleus should remain undisturbed.

2. WAVE FUNCTIONS AND MATRIX ELEMENTS

The function χ_p^{-1} describes the noninteracting nucleons with momenta $\mathbf{p}_p$ and $\mathbf{p}_n$, which leaves the interaction region of a nucleus R (which is considered to be absolutely opaque). In this case χ_b^{-1} has the form (cf, for example, reference 3).

$$\chi_b^{(-)} = \Omega^{-1/2}\left(e^{i\mathbf{p}_n\mathbf{r}_n} + \frac{p_n}{2\pi i}\int \frac{\exp\{-ip_n|\mathbf{r}_n - \mathbf{s}_n|\}}{|\mathbf{r}_n - \mathbf{s}_n|}\,d\mathbf{s}_n\right) \times \Omega^{-1/2}\left(e^{i\mathbf{p}_p\mathbf{r}_p} + \frac{p_p}{2\pi i}\int \frac{\exp\{-ip_p|\mathbf{r}_p - \mathbf{s}_p|\}}{|\mathbf{r}_p - \mathbf{s}_p|}\,d\mathbf{s}_p\right) = \frac{1}{\Omega}(\psi_0^{(n)}(\mathbf{r}_n) + \psi_1^{(n)}(\mathbf{r}_n))(\psi_0^{(p)}(\mathbf{r}_p) + \psi_1^{(p)}(\mathbf{r}_p)), \quad (2.1)$$

Here S_n, S_p are two-dimensional vectors in the plane passing through the cross section of the nucleus over which the integration is carried out. The section is chosen perpendicular to the vectors

[3] L. L. Landau and I. Ia. Pomeranchuk, J. Exper. Theoret. Phys. USSR **24**, 505 (1953).

p_p and p_n (i.e., the region of integration is $s_n < R$, $s_n \perp p_n$; $s_p < R$, $s_p \perp p_p$), and Ω is the normalizing volume.

The wave function of the deuteron turns out to be somewhat more complicated since it is necessary to take into account the final (larger) dimensions of the deuteron (it appears that they have a limiting value). Fundamentally, it is necessary to take into account the different effective radii of the nucleus in respect to the nucleon and to the deuteron. This can be done in the following manner. "Absorption" of the deuteron (i.e., its disappearance from the incident beam) will take place when the center of gravity of the deuteron passes outside of the nucleus ($S_d > R$) and if at the same time even only one nucleon of the deuteron falls within the limits of the section of the nucleus. As soon as $S_d - R \gg R_d$ this effect can be neglected. In this way we obtain for the general case:

$$\psi_d = \Omega^{-1/2}\left(e^{ip_d \mathbf{r}_d} - \frac{p_d}{2\pi i}\int\limits_{\mathbf{s}_d \perp \mathbf{p}_d} f\left(\frac{s_d - R}{R_d}\right)\frac{\exp\{ip_d|\mathbf{r}_d - \mathbf{s}_d|\}}{|\mathbf{r}_d - \mathbf{s}_d|}\,d\mathbf{s}_d\right)\varphi(|\mathbf{r}_p - \mathbf{r}_n|), \tag{2.2}$$

where the "function of transparency" f is some function dependent on the structure of the deuteron and the opacity of the nucleus for nucleons at various distances from its center. For a completely opaque nucleus f is equal to unity for $S_d < R$ and it vanishes for $S_d - R \gg R_d$.

Let us assume that the center of mass of the deuteron moves along the z axis and passes at distance S_d ($S_d > R$) from the center of the nucleus. Since the distance of one nucleon, say the neutron, from the center of mass is a half of its distance from the second nucleon (the proton) then the probability density that the nucleon is at a distance $\sqrt{\rho^2 + z^2}$ from the center of mass (ρ, χ are the cylindrical coordinates in a plane perpendicular to z) is . The total probability that one nucleon is absorbed by the nucleus when the center of mass of the deuteron is at a distance S_d (in the section plane of the nucleus) is

$$\int\limits_{-\infty}^{+\infty} dz \int\limits_{s_d - R}^{\infty} \rho\, d\rho \int\limits_{0}^{\arccos\frac{s_d - R}{\rho}} d\chi\, |\varphi(2\sqrt{\rho^2 + z^2})|^2. \tag{2.3}$$

(We consider the surface of the nucleus to be flat at $R \gg R_d$. The probability of absorption of the second nucleon is the same. The function f is then equal to the square of Eq. (2.3)for $S_\alpha > R$. Inserting φ from Eq. (1.3) changing the variables $\rho = (s_d - R)x$, $\rho^2 + z^2 = \zeta^2\rho^2$, and Eq. (2.3) results in

$$\frac{1}{4} N^2 (s_d - R)\int\limits_1^{\infty} \arccos\frac{1}{x}\, dx \times \int\limits_1^{\infty} \frac{d\zeta}{\zeta}\, \frac{\exp\{-4\alpha(s_d - R)x\zeta\}}{(\zeta^2 - 1)^{1/2}}.$$

In this manner the characteristic parameter which determines the exponential vanishing of f for $S_d - R \gg R_d$ is $a = 1/4\alpha \approx 0.8/\mu$.

The following calculations will be carried out with the assumption that all distances are large compared with $1/\mu$. Therefore we simply set

$$f\left(\frac{s_d - R}{R_d}\right) = \begin{cases} 1 & \text{for } s_d < R, \\ e^{-(s_d - R)/a} & \text{for } s_d > R, \end{cases} \tag{2.4}$$

$$a \approx R_d/2 \approx 0{,}8/\mu.$$

Now, omitting W in Eq. (1.1) and substituting Eq. (2.1) into Eq. (2.2), we obtain:

$$\begin{aligned} <\chi_b^{(-)}|U|\psi_d> = \Omega^{-3/2}\int d\mathbf{r}_p\, d\mathbf{r}_n\, (\psi_0^{(n)*}(\mathbf{r}_n) \\ + \psi_1^{(n)*}(\mathbf{r}_n))(\psi_0^{(p)*}(\mathbf{r}_p) \\ + \psi_1^{(p)*}(\mathbf{r}_p))\, U(|\mathbf{r}_p - \mathbf{r}_n|)\left(\psi_0^{(d)}\left(\frac{\mathbf{r}_p + \mathbf{r}_n}{2}\right)\right. \\ \left. - \psi_1^{(d)}\left(\frac{\mathbf{r}_p + \mathbf{r}_n}{2}\right)\right)\varphi(|\mathbf{r}_p - \mathbf{r}_n|), \end{aligned} \tag{2.5}$$

where

$$\psi_0^{(n)*}(\mathbf{r}_n) = \exp\{-i\mathbf{p}_n\mathbf{r}_n\},$$

$$\psi_0^{(p)*}(\mathbf{r}_p) = \exp\{-i\mathbf{p}_p\mathbf{r}_p\},$$

$$\psi_0^{(d)}\left(\frac{\mathbf{r}_p + \mathbf{r}_n}{2}\right) = \exp\{i\mathbf{p}_d(\mathbf{r}_p + \mathbf{r}_n)/2\},$$

$\psi_1^{(n)*}$, $\psi_1^{(p)*}$ are the complex conjugates of the second terms in the brackets of Eq. (2.2) taken with a plus sign. Multiplying out the quantities in parenthesis of Eq. (2.5) we obtain eight integrals, of which only some are different from zero.

We denote the integrals by symbols Y_{ijk}, $i, j, k = 0$ or 1, the indices 0 and 1 denoting which one of the terms is taken from the given parenthesis, the indices being written in the same order as the functions for n, p and d in Eq. (2.5). First of all Y_{000}, containing the product of plane waves, is different from zero only if the momentum of the center of mass is observed. This condition cannot be reconciled with the law of conservation of energy. Therefore Y_{000} vanishes. Furthermore, the diffraction cone of deuterons is distributed in the direction of the shadow formed by the nucleus, i.e., if deuterons are moving along the z axis from the region $z < 0$, then ψ_1^d is different from zero only for $z > 0$. Conversely, ψ_1^p and ψ_1^n are different from zero only for $z < 0$. Therefore, the integrals Y_{101}, Y_{011} and Y_{111} are vanishingly small. The remaining four integrals will be computed.

Introducing the coordinates of the center of mass: $\mathbf{r}_0 = (\mathbf{r}_p + \mathbf{r}_n)/2$, $\mathbf{r} = \mathbf{r}_p - \mathbf{r}_n$, we have:

$$Y_{100} = -\frac{1}{\Omega^{3/2}}\frac{p_n}{2\pi i}\int d\mathbf{r}_0 \int d\mathbf{r} \int d\mathbf{s}_n \exp\{i(\mathbf{p}_d - \mathbf{p}_p, \mathbf{r}_0 + p_n|\mathbf{r}_0 - \mathbf{r}/2 - \mathbf{s}_n|)\}$$
$$\times \exp\{i(\mathbf{p}_d - \mathbf{p}_p)(\mathbf{r}/2 + \mathbf{s}_n)\}\frac{U(r)\varphi(r)}{|\mathbf{r}_0 - (\mathbf{r}/2) - \mathbf{s}_n|}.$$

Integrating first over $\mathbf{r}_0$ (introducing the variables $\mathbf{r}_0 - \mathbf{r}/2 - \mathbf{s}_n$), and then over $\mathbf{s}_n$ (with limits $s_n < R$, where R is the radius of the nucleus), we obtain

$$Y_{100} = \frac{4\pi i p_n}{(\mathbf{p}_d - \mathbf{p}_p)^2 - p_n^2}\frac{RJ_1(q_n R)}{q_n}\frac{1}{\Omega^{3/2}} \quad (2.6)$$
$$\times \int \exp\left\{-\frac{i}{2}(\mathbf{p}_d - 2\mathbf{p}_p, \mathbf{r})\right\} U(r)\varphi(\mathbf{r})\, d\mathbf{r}. \quad (2.6)$$

Similarly

$$Y_{010} = \frac{4\pi i p_p}{(\mathbf{p}_d - \mathbf{p}_n)^2 - p_p^2}\frac{RJ_1(q_p R)}{q_p}\frac{1}{\Omega^{3/2}} \quad (2.7)$$
$$\times \int \exp\left\{-\frac{i}{2}(\mathbf{p}_d - 2\mathbf{p}_n, \mathbf{r})\right\} U(r)\varphi(r)\, d\mathbf{r}.$$

At this point the momentum transferred to the nucleus appears for the first time

$$\mathbf{q} = \mathbf{p}_p - \mathbf{p}_n - \mathbf{p}_p. \quad (2.8)$$

and its projections q_p and q_n on the plane are perpendicular to $\mathbf{p}_p$ and $\mathbf{p}_n$, respectively.

The integral Y_{110} is somewhat more complicated but it belongs to the type of integrals obtained by Pomeranchuk[4]. First we notice that according to the law of conservation of energy,

$$\frac{p_d^2}{4M} - \varepsilon = \frac{p_n^2}{2M} + \frac{p_p^2}{2M},$$

Introducing the quantity $p = p_n - p_p$ and taking into account that $p \ll p_n, p_p, p_d$ as is verified by the last equation, and that it has in the important region a value of the order of $\sqrt{4M\epsilon}$, we obtain the following approximate relations:

$$p_d \approx 2p_n - p + (p^2 + 4M\varepsilon)/4p_n, \quad (2.9)$$
$$(p_d - p_n)^2 - p_p^2 = {}^1/_2\, p^2 + 2M\varepsilon = (p_d - p_p)^2 - p_n^2.$$

Because of the small angles θ_p and θ_n (relative to the direction of the primary deuteron beam) at which the proton and neutron emerge, and also the smallness of the angle θ_{np} between the proton and the neutron (see below), we can also write:

$$(\mathbf{p}_d - \mathbf{p}_p)^2 - \mathbf{p}_n^2 = {}^1/_2\,(p^2 + 4M\varepsilon + 2p_p p_d \vartheta_p^2), \quad (2.10)$$

$$(\mathbf{p}_d - \mathbf{p}_n)^2 - p_p^2 = {}^1/_2\,(p^2 + 4M\varepsilon + 2p_n p_d \vartheta_n^2), \quad (2.11)$$

$$(\mathbf{p}_n + \mathbf{p}_p)^2 - p_d^2 = -(p^2 + 4M\varepsilon + {}^1/_4\, p_d^2 \vartheta_{np}^2). \quad (2.12)$$

Expanding expressions of the type e^{ipt}/t in Fourier integrals, using the above formulae and the method of integration due to Pomeranchuk[4] we can reduce the integral Y_{110} to the form

$$Y_{110} = \frac{p_n p_p}{(2\pi i)^2 \Omega^{3/2}} \int ds_n \int ds_p \int d\mathbf{r} U(r) \varphi(r) \exp\left\{-\frac{i}{2}\left(\mathbf{p}_d \frac{\mathbf{r}}{2} + p_n \sigma_d\right)\right\}$$

$$\times \frac{i}{p_d} \int_{-\infty}^{+\infty} \exp\{-b\sigma_\perp \operatorname{ch}\theta\}\, d\theta,$$

where σ_d and $\sigma_\perp$ are the projections of the vectors $\mathbf{s}_p - \mathbf{s}_n - \mathbf{r}$ along and perpendicular to the direc-tion of $\mathbf{P}_d$. The variable r is limited by the value of the interaction of p and n, and s_p and s_n vary within the limits of 0 to R with the effective value lue $|s_p - s_n| \sim R$ much larger than r_{eff} accord-ing to our assumptions. Therefore it is possible to disregard r in the integral over θ in $\sigma_\perp$ and then the integrals over $\mathbf{s}_n$ and $\mathbf{s}_p$ separate from the integral over $\mathbf{r}$ and we obtain:

$$Y_{110} = \frac{4\pi i}{\Omega^{3/2}} \frac{p_d R}{(\mathbf{p}_n + \mathbf{p}_p)^2 - p_d^2} \frac{J_1(q_d R)}{q_d} \tag{2.13}$$

$$\times \int \exp\left\{-i\left(\frac{1}{2} - \frac{p_n}{p_d}\right)\mathbf{p}_d\mathbf{r}\right\} U(r)\varphi(r)\, d\mathbf{r}$$

(q_d is the projection of q on a plane perpendicular to p_d). Finally, we have in a similar way for Y_{001}

$$Y_{001} = \frac{4\pi i}{\Omega^{3/2}} \frac{p_d}{(\mathbf{p}_n + \mathbf{p}_p)^2 - p_d^2} \tag{2.14}$$

$$\times \int \exp\left\{-\frac{i}{2}(\mathbf{p}_n - \mathbf{p}_p, \mathbf{r})\right\}$$

$$\times U(r)\varphi(r)\, d\mathbf{r} \int f\left(\frac{s_d - R}{a}\right) \exp\{-i\mathbf{q}\mathbf{s}_d\}\, d\mathbf{s}_d$$

We separate the integral over $\mathbf{s}_d$ into two parts: from 0 to R, where $f = 1$, and from R to ∞ where, from Eq. (2.4), $f = e^{-(s_d - R)/a}$. The first integral is carried out in the same way as the previous one. In the second we note that those q are important that are close to $1/R_d$ and $R \gg R_d$, and consequently $qR \gg 1$. Therefore $qs_d \gg 1$.

Performing the angular part of the integration over s_d we obtain the function $J_0(qs_d)$ which we replace by its asymptotic value. In the expression preceeding the exponential in the integral over s_d instead of R. The result is

$$\int f\left(\frac{s_d - R}{a}\right) e^{-i\mathbf{q}\mathbf{s}_d}\, d\mathbf{s}_d = 2\pi \frac{RJ_1(q_d R)}{q_d} \tag{2.14a}$$

$$+ \sqrt{\frac{8\pi R}{q_d}} \frac{a}{1 + a^2 q_d^2} \left\{\cos\left(q_d R - \frac{\pi}{4}\right) - q_d a \sin\left(q_d R - \frac{\pi}{4}\right)\right\}.$$

The integrals that remain in Eqs. (2.6), (2.7), (2.13) and (2.14) have the meaning of form factors. Their exact value depends on the form of the wave function. Indeed, if the deuteron equation

$$-(1/M)\nabla^2\varphi + U(r)\varphi = -\varepsilon\varphi$$

is multiplied by $e^{i\lambda\cdot r}$ and integrated over all space, we obtain, by integrating the first term by parts:

$$\int e^{i\vec{\lambda}\mathbf{r}}\, U(r)\varphi(r)\, d\mathbf{r} \tag{2.15}$$

$$= -(\varepsilon + \lambda^2/M)\int \exp\{i\vec{\lambda}\mathbf{r}\}\varphi(r)\, d\mathbf{r}.$$

If we take for φ the functionof Eq. (1.3), then

$$\int \exp\{i\vec{\lambda}\mathbf{r}\}\, U(r)\varphi(r)\, d\mathbf{r} = -4\pi N/M. \tag{2.16}$$

Of course, the independence of $\bar{\lambda}$ is obtained only for this special choice of φ. Since the Eq. (1.3) is not correct at distances smaller than the range of the nuclear forces, the Eq. (2.16) is not correct for $\lambda \gg \mu$ (there should be present an additional decay with increasing λ). However, in all cases considered by us, λ has the magnitude $q < \mu$. One can convince himself of that fact by noticing that, according to the Eq. (2.8), $\bar{\lambda}$ is equal in Y_{100} Y_{010}, Y_{001} and Y_{110} respectively to $-1/2$ $(\mathbf{p}_p - \mathbf{p}_n - \mathbf{q})$, $+\frac{1}{2}(\mathbf{p}_n - \mathbf{p}_p - \mathbf{q})$, $-\frac{1}{2}(\mathbf{p}_n - \mathbf{p}_p)$ and $\mathbf{p}_d(p_n - p_p)/2p_n$, and also the following relation is valid (see below):

[4] I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR 96, 265 (1954).

$$p_n - p_p < 2\sqrt{M\varepsilon} \approx 0.6\ \mu.$$

Collecting the formulae of Eqs. (2.5)-(2.7), (2.13), (2.14) and (2.16), setting $q_n = q_p = q_d = q$ (because of the small scattering angles), and using the approximate relations (2.9)-(2.12), we obtain [in several cases we shall set $p_n \approx p_p \approx 1/2\, p_d$, forseeing the formula of Eq. (3.1)]:

$$\langle \chi_b^{(-)} | U | \psi_{0a} + \psi_d^{\text{диф}} \rangle = \frac{4\pi i}{\Omega^{3/2}}\left(-\frac{4\pi N}{M}\right)\left\{\frac{RJ_1(qR)}{q}\left[\frac{p_n}{(\mathbf{p}_d - \mathbf{p}_p)^2 - p_n^2} + \frac{p_p}{(\mathbf{p}_d - \mathbf{p}_n)^2 - p_p^2} + \frac{2p_d}{(\mathbf{p}_n + \mathbf{p}_p)^2 - p_d^2}\right] + \frac{p_d}{(\mathbf{p}_n + \mathbf{p}_p)^2 - p_d^2}\sqrt{\frac{2R}{\pi q}}\frac{a}{1 + a^2q^2} \times \left[\cos\left(qR - \frac{\pi}{4}\right) - qa\sin\left(qR - \frac{\pi}{4}\right)\right]\right\} \approx -\frac{16\pi^2 iN}{M\Omega^{3/2}}\frac{p_d}{(p_p - p_n)^2 + 4M\varepsilon} \tag{2.17}$$

$$\times\left\{\frac{RJ_1(qR)}{q}\left[\frac{1}{1 + A^2\vartheta_n^2} + \frac{1}{1 + A^2\vartheta_p^2} - \frac{2}{1 + {}^1/_4 A^2\vartheta_{np}^2}\right] - \frac{1}{1 + {}^1/_4 A^2\vartheta_{np}^2}\sqrt{\frac{2R}{\pi q}}\frac{a}{1 + a^2q^2}\left[\cos\left(qR - \frac{\pi}{4}\right) - qa\sin\left(qR - \frac{\pi}{4}\right)\right]\right\},$$

$$A^2 = \frac{p_d^2}{(p_p - p_n)^2 + 4M\varepsilon}, \qquad N^2 = \frac{3}{4\pi}\sqrt{M\varepsilon}.$$

It is understood that $J_1(qR)$ may be replaced everywhere by its asymptotic function.

3. THE LATERAL CROSS SECTION

According to Eqs. (1.1) and (2.17), squaring the matrix element, dividing by the current of the incident deuterons $p_d/2M\Omega$ and multiplying by the density of the final states

$$\frac{p_p^2\, dp_p\, d\omega_p\, p_n^2\, d\omega_n\, \Omega^2}{(2\pi)^6\,(dE/dp_n)}, \quad \frac{dE}{dp_n} = \frac{p_n}{M},$$

we obtain

$$d\sigma = \frac{12}{\pi^2}\sqrt{M\varepsilon}\,\frac{p_d}{[(p_p - p_n) + 4M\varepsilon]^2}\left\{\quad\right\}^2 p_n\, p_p^2\, dp_p\, d\omega_p\, d\omega_n, \tag{3.1}$$

where the quantity in parenthesis is the same as in Eq. (2.17). In squaring we, following Landau[3], shall replace $J_1(qR)$ by the average value of its asymptotic expression:

$$J_1^2(qR) \approx \frac{2}{\pi qR}\cos^2\left(qR - \frac{3\pi}{4}\right) \to \frac{1}{\pi qR};$$

and the product

$$\sin\left(qR - \frac{\pi}{4}\right)\cos\left(qR - \frac{\pi}{4}\right)$$

will also be replaced by its average value, i.e., zero, etc. Therefore

$$\left\{\quad\right\}^2 = \frac{R}{\pi q^3}\left\{L^2 + \frac{a^2q^2}{1 + a^2q^2}\left[\frac{1}{(1 + {}^1/_4 A^2\vartheta_{np}^2)^2} + \frac{2L}{1 + {}^1/_4 A^2\vartheta_{np}^2}\right]\right\}, \tag{3.2}$$

$$L = (1 + A^2\vartheta_n^2)^{-1} + (1 + A^2\vartheta_p^2)^{-1} - 2\,(1 + {}^1/_4 A^2\vartheta_{np}^2)^{-1}.$$

The denominator of the Eq. (3.1) shows that the neutron and proton produced by the splitting actually have very similar energies. The difference of the momenta is of the order

$$p = p_n - p_p \lesssim 2\sqrt{M\varepsilon}.$$

The difference of energies is

$$|E_p - E_n| \tag{3.3}$$

$$= \left|\frac{p_p^2}{2M} - \frac{p_n^2}{2M}\right| \approx \frac{2p_n}{2M}\left|p_p - p_n\right| \lesssim 2\sqrt{E_d\varepsilon},$$

where $E_d = p_d^2/4M$ is the kinetic energy of the incident deuteron. The scattering angles are of the order $(p^2 - 4M\epsilon)/p_d^2$, i.e.,

$$\vartheta_n \sim \vartheta_p \sim \vartheta_{np} \sim 8M\varepsilon / p_d^2 \sim \varepsilon / E_d. \tag{3.4}$$

Consequently, neutrons and protons emerge with practically the same energies (a few with somewhat greater difference in energy) and at the same angles as for the stripping process. However, the main difference rests in the fact that for each deuteron stripped by the nucleus, either a neutron or a proton emerges and the nucleus absorbs the remaining nucleon, undergoing a transmutation. In the diffraction splitting for each process both a neutron and a proton emerge from the interacting region. The nucleus, meanwhile, experiences only a recoil by absorbing a small momentum $q \sim \sqrt{M\epsilon}$.

The total cross section is obtained by integration over p_p and the angles. We shall introduce two-dimensional vectors in place of the angles,

$$\vec{\alpha}_p = A\vec{\vartheta}_p,\ \vec{\alpha}_n = A\vec{\vartheta}_n,\ \vec{\alpha}_{np} = {}^1/_2\, A\vec{\vartheta}_{np}, \tag{3.5}$$

$$\vec{\vartheta}_{np} = \vec{\vartheta}_n - \vec{\vartheta}_p.$$

We also take into account that the component of q, the momentum transferred to the nucleus, relative p_d is equal [cf. Feinberg[1], Eq. (2.4a)]:

$$q_\parallel \sim 2M\varepsilon / p_d,$$

and the perpendicular component $q_\perp$ has the order of magnitude

$$p_n \vartheta_n \sim \sqrt{M\varepsilon},$$

i.e., much larger than the component along p_d. Therefore, setting $q \approx q_\perp$, we have, according to Eq. (2.8), any one of the forms:

$$\mathbf{q} \approx (p_n \vec{\vartheta}_n + p_p \vec{\vartheta}_p) \approx (p_d/2)\,(\vec{\vartheta}_n + \vec{\vartheta}_p)$$

$$= ({}^1/_2\,\vec{\vartheta}_{np} + \vec{\vartheta}_p)\, p_d \tag{3.6}$$

$$= (\vec{\vartheta}_n - {}^1/_2\,\vec{\vartheta}_{np})\, p_d.$$

If we introduce new variables to facilitate integration,

$$\mathbf{q}_1 = {}^1/_2\,(\vec{\alpha}_n + \vec{\alpha}_p) = \mathbf{q}/c, \tag{3.7}$$

$$\mathbf{q}_2 = {}^1/_2\,(\vec{\alpha}_n - \vec{\alpha}_p) \equiv \vec{\alpha}_{np},$$

$$d\vec{\vartheta}_n\, d\vec{\vartheta}_p = 4A^{-4}\, d\mathbf{q}_1 d\mathbf{q}_2, \tag{3.8}$$

$$q = p_d\, q_1 A^{-1} = cq_1,\ c^2 = (p_p - p_n)^2 + 4M\varepsilon,$$

then the total cross section is:

$$\sigma = \frac{6\sqrt{M\varepsilon}R}{\pi^3} \tag{3.9}$$

$$\times \int \frac{dp_p}{[(p_p - p_n)^2 + 4M\varepsilon]^{3/2}} \frac{d\mathbf{q}_1 d\mathbf{q}_2}{q_1^3}\left\{L^2 + \frac{\beta^2 q_1^2}{1 + \beta^2 q_1^2}\right.$$

$$\left.\times\left[\frac{1}{(1 + q_2^2)^2} + \frac{2L}{1 + q_2^2}\right]\right\},$$

$$L = \frac{1}{1 + (\mathbf{q}_1 + \mathbf{q}_2)^2} + \frac{1}{1 + (\mathbf{q}_1 - \mathbf{q}_2)^2} - \frac{2}{1 + q_2^2},$$

$$\beta = ac = a\,[(p_p - p_n)^2 + 4M\varepsilon]^{1/2}.$$

carrying out the integration over $\mathbf{q}_2$ and the angles of the vector $\mathbf{q}_1$ we obtain

$$\sigma = \frac{24}{\pi}\sqrt{M\varepsilon}R \int \frac{dp_p}{[(p_p - p_n)^2 + 4M\varepsilon]^{3/2}} \int_0^{q_{1\max}} \frac{dq_1}{q_1^2}\left\{3 + \frac{1}{2q_1\sqrt{1 + q_1^2}} \ln\frac{\sqrt{1 + q_1^2} + q_1}{\sqrt{1 + q_1^2} - q_1}\right.$$

$$- \frac{4}{q_1\sqrt{4 + q_1^2}} \ln\left(1 + q_1\sqrt{4 + q_1^2}\,\frac{\sqrt{4 + q_1^2} + q_1}{\sqrt{4 + q_1^2} - q_1}\right) \tag{3.10}$$

$$\left. + \frac{1}{2}\frac{\beta^2 q_1^2}{1 + \beta^2 q_1^2}\left[\frac{4}{q_1\sqrt{4 + q_1^2}} \ln\left(1 + q_1\sqrt{4 + q_1^2}\,\frac{\sqrt{4 + q_1^2} + q_1}{\sqrt{4 + q_1^2} - q_1}\right) - 3\right]\right\}.$$

The integration over q_1 is carried out up to some maximum value where the momentum transferred to the nucleus $q = q_0$ (less than μ), the reciprocal of the magnitude of the smeared surface layer of the nucleus, i.e., up to

$$q_{1\,\max} = [(p_p - p_n)^2 + 4M\varepsilon]^{-1/2}\, q_0. \qquad (3.11)$$

Since $\sqrt{4M\epsilon} \approx 0.6\ \mu$, it means that $q_{1\ \max} < 1$. Therefore the function inside the integral can be expanded in powers of q_1. In doing so an important problem becomes clear. We write the cross section in the form of a sum of a part independent of the deuteron dimensions, and a part, determined by the difference in the radii of the nucleus due to the interaction with the nucleon and the deuteron: $\sigma = \sigma' + \sigma''$ (so that, for example, for $\alpha = \beta = 0$ in Eq. (3.9) $d\sigma = d\sigma''$). Then the mentioned expansion yields (if the integral over $p_p - p_n$ is taken from $-\infty$ to $+\infty$):

$$\sigma' \approx \frac{24}{\pi}\sqrt{M\varepsilon}\, R \int_{-\infty}^{+\infty} \frac{dp_p}{[(p_p - p_n)^2 + 4M\varepsilon]^{3/2}} \qquad (3.12)$$

$$\int_0^{q_{1\,\max}} dq_1 \frac{2}{5} q_1^2$$

$$= \frac{3}{80}\frac{Rq_0^3}{(M\varepsilon)^2} = \frac{3}{5} RR_d (R_d\mu)^3 \left(\frac{q_0}{\mu}\right)^3,$$

where q_0 is the maximum allowed value of the momentum absorbed by the nucleus, and $R_d = 1/2\sqrt{M\epsilon}$ is the average deuteron radius.

The second term dependent on β gives, after expansion in powers of q_1 ($aq_0 \ll 1$):

$$\sigma'' \approx \frac{12}{\pi}\sqrt{M\varepsilon}\, R \int \frac{dp_p\, a^2 q_0}{[(p_p - p_n)^2 + 4M\varepsilon]^3} = 6Ra^2 q_0. \qquad (3.13)$$

Setting $a \approx 1/2R_d$ according to Eq. (2.4), we obtain

$$\sigma'' \approx {}^3/_2\, RR_d (q_0 R_d) = {}^3/_2\, RR_d R_d \mu (q_0/\mu). \qquad (3.14)$$

and then

$$\sigma'/\sigma'' \approx {}^2/_5 (q_0 R_d)^2 = {}^2/_5 (R_d \mu)^2 (q_0/\mu)^2 \ll 1. \qquad (3.15)$$

Consequently, the main part of the spectrum for small q is given by σ'', which is due to the difference in the effective cross sections of absorption of the nucleon and the deuteron by the nucleus.

The absolute magnitude of the cross section is not well determined. Because we have always neglected the smearing of the nuclear surface, q_0 should be much smaller than μ (as was already mentioned, it is clear from physical arguments that the differential cross-section becomes smaller at $q \gtrsim 0.6\ \mu$).

Furthermore, in computation of the main term σ'' the form of Eq. (2.4) was used for the transparency function. Because of the insufficient knowledge of the deuteron wave function this calculation cannot be trusted at $r \sim 1/\mu$. Therefore the result obtained for the integrated cross section can be good only within an order of magnitude. If we set $q_0/\mu \sim 1/3 / 1/2$, we obtain

$$\sigma \approx \sigma'' \sim RR_d. \qquad (3.16)$$

If we attempt to extend the above calculations to all values of q, i.e., if we use the deuteron wave function of Eq. (1.3) for all separations, in which case we also have to change correspondingly the normalization constant N^2 by a factor of 2/3, then, integrating numerically Eq. (3.10) over q from 0 to ∞, we obtain an extremely large value for the cross section.

The energy distribution for the principal term is

$$d\sigma = \text{const}\, \frac{\sqrt{E_p}\, dE_p}{(E_p - E_n)^2 + 4\varepsilon E_d} \qquad (3.17)$$

$$\sim \frac{\sqrt{E_p}\, dE_p}{(E_p - {}^1/_2 E_d)^2 + \varepsilon E_d}.$$

For stripping (transparent model) the formula has the same character (however, it should be remembered that Eq. (3.17) is strictly correct only for that part of the effect with $q \ll \mu$).

DISCUSSION OF RESULTS

The Eqs. (3.13)-(3.16) show that diffraction splitting is a real effect. Since the cross sections for production of neutrons and for production of protons by stripping are equal to $1/2\, \pi RR_d$ (for an opaque nucleus), the number of neutrons and protons emerging in correlated pairs due to diffraction splitting should be comparable with the number emerging singly due to stripping. The scattering angles and the energy distribution should be similar to those observed for stripping. Thanks to these circumstances these neutrons and protons can be separated from those produced by Coulomb stripping: the cross section of the Coulomb stripping is not only smaller than the cross section of the diffraction splitting, but it also has a different energy spectrum (two maxima).

We are not aware of any experiments in which correlated neutron-proton pairs were observed after bombardment of nuclei by deuterons with energies of the order of 100-150 mev. However, Gol'danskii L'ubimov and Medvedev[5] have pointed out that the neutron yield for these processes (at energy ~ 200 mev) is about $1\frac{1}{2} - 2$ times greater than the predictions of the stripping theory and that this discrepency can be only partially removed by the inclusion of the Coulomb stripping. Recent experiments[6] have determined the proton yield from deuterons of energy 190 mev bombarding both the light elements (Be, C) and the heavy ones (U). The results showed a considerably larger cross section than one computed for stripping by an opaque nucleus: for Be and C $\sigma_{exp} = 0.35 \pm 0.03$ barns, for U$\sigma_{exp} = 1.4 \pm 0.2$ barns[7], while the formula $\sigma_{strip}^{opaque} = 1/2\pi RR_d$ gives 0.12 and 0.43 barns, respectively. The cross section for the electric stripping is 0.1 barn for U.

Various authors bring up for consideration an additional stripping process which is due to the transparency of the U nucleus. In this process the deuteron is wholly absorbed by the nucleus — the cross section for this absorption is $\pi R^2 \sim 2.5$ barns - undergoes stripping inside the nucleus and the proton can escape from the nucleus due to the small but still present transparency of the U nucleus ~ 0.12. The exact theoretical calculation of this last effect is very difficult since small changes in the transparency factor and variations in the description of the details of the process cause sharp changes of its contribution to the total cross section.

It is not impossible that diffraction splitting takes place in this case. More definite results could be obtained only from experiments at somewhat smaller energies, at which the transparency of the nuclei does not come into effect, particularly because for a semitransparent nucleus the yield of correlated pairs may be due to stripping inside the nucleus (it is true that a simultaneous emergence of both particles is comparatively improbable.

An interesting facet of the discussed process is its independence of energy. That fact allows us to expect that it should have still a measurable magnitude even in the relativistic energy range.

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Translated by M. J. Stevenson
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