

a plasma is, as we have said, an extremely simple one. However, in principle it is possible to extend our proposed approach to the consideration of phenomena in an inhomogeneous plasma and to more complicated cases, in which there occur dif-

ferences in the electron temperatures as well as in their concentrations.

Translated by M. G. Gibbons
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Paramagnetic Resonance in Synthetic Ruby

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The paramagnetic resonance absorption spectrum in monocrystals of synthetic ruby containing from 0.1 to 0.01% Cr is investigated at a frequency of 9.58×10^9 cycles. A theoretical explanation of the observed spectrum is given. By comparison of the results of theory and experiment there are found: the splitting of the basic level of energy in the electrical field of the crystal ($\delta = 0.38 \text{ cm}^{-1}$) and the factors of spectroscopic splitting: $g_{\parallel} \approx g_{\perp} = 1.98$.

IN the majority of investigations on the paramagnetic resonance of elements of the iron group, the hydrated salts of these elements were studied. In compounds of this type the internal electrical field causing splitting of the basic condition of the magnetic ion is created mainly by the molecules of water surrounding the ion in the form of an octahedron. For this reason the electrical field acting on the ion is divided into a strong cubic field created by the molecules of water and a weak field of much lesser symmetry caused by the remaining atoms of the lattice.

The paramagnetic properties of the investigated specimens of synthetic ruby are due to Cr^{+++} , which at low concentrations isomorphically displaces Al^{+++} in the lattice of corundum.¹ Closely surrounding the Cr^{+++} ion in the ruby is the octahedron of oxygen; here the chromium ion is not located at the center of the octahedron, but is displaced from the center along its trigonal axis, which is the optical axis of the crystal. On the whole, the individual cell of the ruby possesses a trigonal symmetry with respect to this self-same axis of the oxygen octahedron. Consequently, the internal electrical field within the ruby, created basically by the ions of oxygen, must be a powerful trigonal field.

* Experimental portion of the work carried out by Iu. Ia. Shamonin.

¹ E. S. Rudnitskaia, Works (Trudy) of the Institute of Crystallography, Academy of Sciences, USSR **8**, 1953, p. 13.

1. RESULTS OF MEASUREMENT

The fine structure of paramagnetic resonance absorption in monocrystals of synthetic ruby containing from 0.1 to 0.01% Cr was investigated at room temperature at a frequency of 9.58×10^9 cycles.

The investigated specimens were placed in a cavity resonator which was excited by a magnetic H_{112} wave. The power through the resonator served as the magnitude being measured. The external constant magnetic field which varied from 0 to 5500 oersteds was modulated by a frequency of 50 cycles with an amplitude of modulation of $A_{\text{max}} \approx \pm 350$ oersteds. Due to such broad modulation, the resonance curves were traced out completely on the screen of the oscilloscope.

The optical axis of the monocrystals had been determined previously at the Institute of Crystallography of the Academy of Sciences, U.S.S.R. However, for our purposes, the accuracy of this determination was inadequate. For more precise establishment of the optical axis, the great position sensitivity of several lines of the magnetic spectrum of ruby with respect to the magnitude of the angle between the directions of the optical axis and the constant field was employed. Coincidence of the optical axis with the axis of rotation of the crystal, perpendicular to the constant field, was established by the constancy of position of all lines of the spectrum on the oscilloscope screen.

The paramagnetic resonance of the spectrum of a monocrystal of ruby was studied with the orientation of the optical axis a) parallel and b) perpendicular to the constant magnetic field. Photographs of the spectra for both instances are reproduced in Fig. 1.

Given below are the resonance values of the magnetic field (in oersteds) which were determined with the aid of an instrument, which had been graduated by proton resonance, with an accuracy of 0.5%:

	Case a)			Case b)			
Experiment	650	3465	not observed	1178	1562	1967	5425
Theory	650	3465	7580	1174	1555	1952	5451

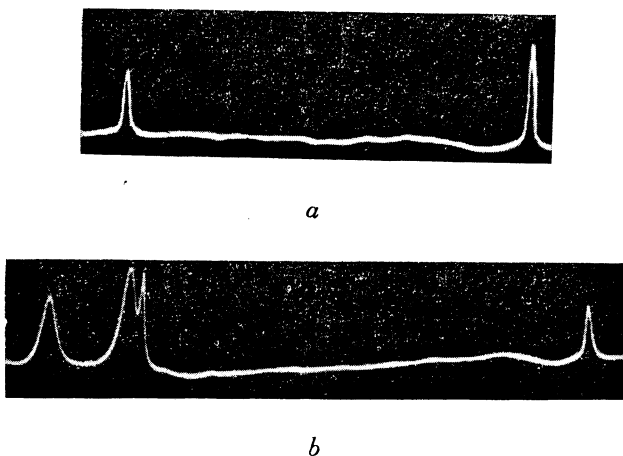


FIG. 1. Paramagnetic resonance spectrum of ruby at a frequency of 9.86×10^9 cycles: a) constant magnetic field parallel to the optical axis of the crystal; b) constant magnetic field perpendicular to the optical axis of the crystal.

2. THEORETICAL INTERPRETATION OF THE SPECTRUM

Having formulated the task of a theoretical explanation of the magnetic spectrum of ruby, it is necessary to examine the following Hamiltonian:

$$\mathcal{H} = \mathcal{H}_0 + V + \lambda(\mathbf{L}\mathbf{S}) + (\mathbf{L} + 2\mathbf{S})\mathbf{H}, \quad (1)$$

where \mathcal{H}_0 is the Hamiltonian of the free ion of chromium, V is the energy of the electrons of this ion in the electrical field created by all the remaining atoms in the crystal lattice. The third member of the Hamiltonian represents the operator of spin-orbital interaction, and the fourth is the energy of the electrons in an external magnetic field of intensity \mathbf{H} .

It is well known that the splitting caused by the internal electrical field of the crystal in elements

of the iron group is of at least two orders higher multiplicity. For this reason, applying the method of perturbation, we will at first examine the influence of the electrical field, and then the remaining members of the Hamiltonian (1).

The potential of the electrical field of trigonal symmetry with the aid of equivalent operators² can be represented in the following form:

$$\begin{aligned} V = & B_0^0 + B_2^0 \bar{r}^2 \alpha \{3L_z^2 - L(L+1)\} \\ & + B_4^0 \beta \bar{r}^4 \{35L_z^4 - 30L(L+1)L_z^2 \\ & + 25L_z^2 - 6L(L+1) + 3L^2(L+1)^2\} \\ & + 1/4 B_4^3 \bar{r}^4 \beta \{L_z(L_+^3 - L_-^3) \\ & + (L_+^3 - L_-^3)L_z\}. \end{aligned} \quad (2)$$

² K. W. H. Stevens, Proc. Phys. Soc. (London) A65, 209 (1952).

The basic condition of Cr^{+++} is 4F . From theoretico-group considerations it is known³ that in a field of trigonal symmetry the F -level splits into 3 simple and 2 double-degenerate levels. And it is a fact that after computation of the matrix elements of perturbation (2) we obtain a series equation of the seventh order, which breaks down into one cubic equation:

$$\begin{array}{c|ccc} L_z & 3 & 0 & -3 \\ 3 & A - W & B & 0 \\ 0 & B & -W & -B \\ -3 & 0 & -B & A - W \end{array} = 0 \quad (3)$$

and two quadratic equations:

$$\begin{array}{c|cc} L_z & 2 & -1 \\ 2 & C - W & D \\ -1 & D & E - W \end{array} = 0; \quad (4)$$

$$\begin{array}{c|cc} L_z & 1 & -2 \\ 1 & E - W & -D \\ -2 & -D & C - W \end{array} = 0,$$

where

$$\begin{aligned} A &= \frac{18}{35} \bar{r}^2 B_2^0 - \frac{8}{7} \bar{r}^4 B_4^0, & B &= -\frac{8\sqrt{7}}{7} \bar{r}^4 B_4^3, \\ C &= -\frac{8}{35} \bar{r}^2 B_2^0 - \frac{8}{21} \bar{r}^4 B_4^0, & D &= -\frac{8\sqrt{14}}{21} \bar{r}^4 B_4^3. \end{aligned}$$

The solution of Eq. (3) results in three simple equations of energy

$$W_1 = A, \quad W_{2,3} = \frac{1}{2}(A \pm \sqrt{A^2 + 8B^2}),$$

having their own corresponding functions:

$$\Psi_1 = 2^{-1/2} (\Phi_3 + \Phi_{-3}), \quad (5)$$

$$\Psi_i = [2(1 + 2\alpha_i^2)]^{-1/2} (\Phi_3 + 2\alpha_i \Phi_0 - \Phi_{-3}),$$

where

$$i = 2, 3, \text{ and } \alpha_i = B / W_i.$$

Equations (4) result in two doubly degenerate equations of energy:

$$W_{4,5} = \frac{1}{2}(C + E \pm \sqrt{(C + E)^2 + 4D^2})$$

with their own functions:

$$\Psi_i^{(1)} = \alpha_i \Phi_2 + \beta_i \Phi_{-1}, \quad \Psi_i^{(2)} = -\alpha_i \Phi_{-2} + \beta_i \Phi_1,$$

where $i = 4, 5$; $\alpha_i = -D / \sqrt{(C - W_i)^2 + D^2}$ and $\beta_i = \sqrt{1 - \alpha_i^2}$. Analysis of the experimentally obtained spectrum of magnetic resonance demonstrates that the basic level must be simple; consequently, no matter what the sign of A is, the lowest level will be W_3 .

In ref. 4 it is shown that, if the basic level is simple and non-magnetic, then (with an accuracy up to magnitudes of the second order) the energy levels are determined as the intrinsic values of the following matrix, depending only on the spin variables:

$$W_0 + 2\beta(\delta_{ij} - \lambda_{ij}) S_i H_j \quad (6)$$

$$- \lambda^2 \Lambda_{ij} S_i S_j - \beta^2 \Lambda_{ij} H_i H_j,$$

$$\Lambda_{ij} = \sum_{n \neq 0} \frac{\langle 0 | L_i | n \rangle \langle n | L_j | 0 \rangle}{W_n - W_0}.$$

The subscripts i, j refer to the cartesian coordinates, and $0, \dots, n$ to the orbital levels of energy. Note that in our problem $\Lambda_{ij} = 0$ when $i \neq j$. The last member in matrix (6) is not taken into consideration, because the correction to the energy introduced by this member with the magnetic fields employed at present is less than 0.001 cm^{-1} .

In conjunction with the completed experiment, we will examine the following two cases: 1) constant magnetic field parallel to the optical axis of the crystal, 2) magnetic field perpendicular to this axis.

In the first case, formula (6) results in the matrix:

$$C + g_{\parallel} \beta H S_z + \frac{1}{2} \delta S_z^2, \quad (7)$$

³ H. A. Bethe, Ann. Physik 3, 133 (1929).

⁴ A. Abragam and M. H. L. Pryce, Proc. Roy. Soc. (London) A205, 135 (1951).

where

$$g_{\parallel} = 2(1 - \lambda\Lambda_{zz}), \quad C = W_3 - \lambda^2\Lambda_{xx} S(S + 1), \quad \delta = 2\lambda^2(\Lambda_{xx} - \Lambda_{zz}),$$

$$\Lambda_{xx} = \frac{3}{4} \frac{1}{1 + 2\alpha_3^2} \left[\frac{(\alpha_4\sqrt{3} + 4\alpha_3\beta_1)^2}{W_4 - W_3} + \frac{(\alpha_5\sqrt{5} + 4\alpha_3\beta_5)^2}{W_5 - W_3} \right],$$

$$\Lambda_{zz} = \frac{9}{1 + 2\alpha_3^2} \frac{1}{W_1 - W_3}.$$

Rejecting the inapplicable constant, we obtain the following equations of energy from (7):

$$g_{\parallel} \beta H m + \frac{1}{2} \delta m^2 = E_m (m = 3/2, 1/2, -1/2, -3/2),$$

where m is the projection of the spin moment. Designating $2E_m/\delta = \varepsilon_m$, $g_{\parallel} \beta H/\delta = x$, we obtain the final expression in the form:

$$\varepsilon_{\pm 1/2} = 2 \pm 3x, \quad \delta_{\pm 1/2} = \pm x. \tag{8}$$

A graphic representation of (8) is given in Fig. 2.

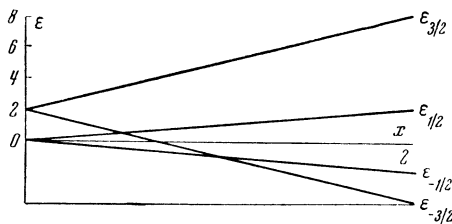


FIG. 2. Schematic of the splitting of the basic level of the Cr⁺⁺⁺ ion in ruby when the magnetic field is directed along the axis of the crystal.

The number and position of the line of paramagnetic resonance are easily determined with the aid of (8) if it is remembered that the probability of magnetic dipolar transitions is noticeably large only for those pairs of equations whose spin-magnetic quantum numbers differ by unity. These probabilities for the transitions $1/2 \leftrightarrow 3/2, 1/2 \leftrightarrow -1/2, -3/2 \leftrightarrow -1/2$ as a first approximation are related in the ratio 3:4:3. Identification of the experimental and theoretical spectra with the aid of (8) results in $g_{\parallel} = 1.98$ and $\delta = 0.38 \text{ cm}^{-1}$. One should note, however, that at the frequency of the variable field employed, a third resonance absorp-

tion line at $H \simeq 7580$ oersteds was to be expected.

In the case in which the constant magnetic field is perpendicular to the optical axis of the crystal, formula (6) results in the matrix:

$$C + g_{\perp} \beta H S_x + \frac{1}{2} \delta S_z^2, \tag{9}$$

where $g_{\perp} = 2(1 - \lambda\Lambda_{xx})$. The series equation of matrix (9) has the roots

$$\varepsilon_{1,2} = 1 + x \pm \sqrt{(1-x)^2 + 3x^2}, \tag{10}$$

$$\varepsilon_{3,4} = 1 - x \pm \sqrt{(1+x)^2 + 3x^2},$$

where $\varepsilon_i = 2E_i/\delta$ and $x = g_{\perp} \beta H/\delta$. The graphic representation of (10) is given in Fig. 3.

In order to compare the spectra obtained experimentally and theoretically, the probabilities of

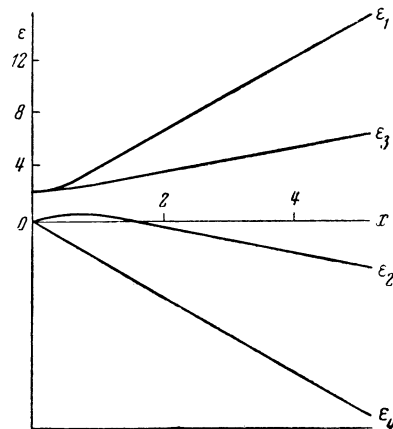


FIG. 3. Schematic of the splitting of the basic level of the Cr⁺⁺⁺ ion in ruby when the magnetic field is directed perpendicular to the axis of the crystal.

transitions between the various levels were calculated. The theoretical values of the resonance

fields, noted above, were computed according to formulas (10) using the values of $g_{||}$ and δ obtained previously.

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The Inelastic Scattering of Electrons from a Copper Oxide Surface

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An electrical differentiation method in a spherical condenser is used to study the inelastic scattering of electrons from a copper oxide surface. Inelastically scattered electrons with discrete energy losses were observed from both monocrystalline and polycrystalline surfaces. The discrete energy losses are equal to the energies required to transfer an electron of the crystal lattice from a filled band to an allowed one. It is shown that the inelastically scattered electrons, with discrete energy losses, are produced by the same mechanism as accounts for the group of genuinely secondary electrons, of discrete energies, which we had discovered previously.

INTRODUCTION

WHEN a group of electrons enters matter, individual electrons undergo changes in their direction of motion, and also lose energy. Little is known about the mechanism by which electrons of low and medium energies (tens and hundreds of eV) lose energy as they pass through a solid body. A study of this mechanism is both interesting for its own sake, and relevant to the understanding of many phenomena, in particular the emission of secondary electrons.

In the course of investigations on the distribution of secondary electrons emitted from metal and metal oxide surfaces, several authors¹⁻⁴ have observed inelastically scattered primary electrons with small, discrete energy losses. The primary electrons had energies of approximately 100 volts in these investigations and the inelastically scattered groups were observed at energies close to

the primary energy. The fundamental method used in the study of spontaneous energy losses was that of magnetic analysis. This method has high resolving power, but suffers from the disadvantage that only a small fraction of the electrons can be analyzed: namely those traveling in a small solid angle determined by the slit of the analyzer. The retarding field and spherical condenser method proposed by Lukirskii⁵ avoids this drawback, but does not have high enough resolving power to investigate the fine structure in the energy distribution of the electrons. As our experiments on the energy distribution of genuinely secondary electrons⁷ have shown, the use of electric differentiation in the spherical condenser method⁶ increases the resolution considerably and allows one to study the fine structure.

In the present work we examine the fine structure in the energy distribution of secondary electrons at energies close to the primary electron energy. As samples to be investigated, we used copper

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⁶N. B. Gornyi and L. M. Rakhovich, J. Exptl. Theoret. Phys. U.S.S.R. 26, 454 (1954).

⁷N. B. Gornyi, J. Exptl. Theoret. Phys. U.S.S.R. 27, 171 (1954).