

Nuclear Shells and the Classification of Atomic Nuclei*

A. V. SAVICH

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A nuclear shell model is proposed, on the basis of which classification is carried out of the spins and magnetic moments of the ground and metastable levels of nuclei with odd mass number. Classification is also made of the masses and of data on β -decay of nuclei with odd mass number.

1. CONSTRUCTION OF A SHELL SYSTEM IN ATOMIC NUCLEI

In the research of Mayer,¹ the three dimensional harmonic oscillator was used as a model for the construction of nuclear shells. The number of states N_v of a three dimensional oscillator with vibration quantum number v for particles of spin $\frac{1}{2}$ is given by the formula

$$N_v = (v + 1)(v + 2)(v = 0, 1, 2, \dots)$$

(the number of the shell is designated as $v + 1$). In the successive filling of such shells, the following numbers are obtained for closed "magic" configurations: 2, 8, 20, 40, 60, 112. Absent from this list is a series of "magic" numbers that are well known from the experiment: 28, 50, 82, 126.

We assume that both successive and non-successive filling of shells are possible in atomic nuclei; the latter is the case when one internal shell is entirely unfilled with nuclei (for example, the filling of shells 1, 2, 4 with a skipping of shell 3 gives the "magic" number $28 = 2 + 6 + 20$, etc.). We begin with the assumption that in certain atomic nuclei the process of non-successive filling of nuclear shells can be energetically more favorable than the process of their successive filling. For example, one can expect that in nuclei which have a number of nucleons of a particular kind close or equal to 28, non-successive filling of the levels will be more likely than successive filling, in which there are about 8 nucleons outside of the closed shell of 20 nucleons.

All the "magic" numbers which are obtained both in successive filling of the nuclear shells, and in the skipping of one of the interior shells are shown in Table 1. In each square of the table, there is placed, along with the magic number, the

list of filled levels which correspond to this magic number. The entire set of magic numbers of Table 1 can be characterized by two integers n_+ and n_- (laid out in the table along the vertical and horizontal axes). The number n_+ denotes the number of levels filled successively before the break, the number n_- denotes the number of levels filled successively after a single interior skipping (unfilled level).

We referred earlier to "subshells", i.e., to energetically less favorable configurations, all capable of filling, for which $n_- > n_+$.

In such configurations, relatively low energy levels are left unfilled. Below, we shall give additional arguments for the usefulness of such a subdivision. After separation of the "subshells," there remain 15 "magic" numbers on the left side of Table 1. Of these, 6 correspond to successive filling ($n_- = 0$): 2, 8, 20, 40, 70, 112; 5 correspond to an unfilled penultimate shell ($n_- = 1$): 14, 28, 50, 82, 126; 3 correspond to $n_- = 2$: 58, 92, 138; and, finally, one — 148 — corresponds to $n_- = 3$. Experimental data show that 7 magic numbers (2, 8, 20, 28, 50, 82, 126) stand out clearly.

There have been reports on the literature in the existence of four more shells: 40, 70, 14, 58^{2-3} ; there is some indication of the existence of the shells 112, 92, 138. However, there has been very little experimental data in the region of heavy nuclei; it is possible that the existence of the shells 112, 92, 138 and 148 will receive further confirmation. That some of the shells stand out less clearly than the others can be explained in part in the following manner.

The closed properties are most prominent in nuclei with shells that are filled both for protons and for neutrons (double magic numbers). Making use of the list of shells and subshells in Table 1, we can find 19 stable nuclei of this type. In ten of them (${}^4_2\text{He}$, ${}^{16}_8\text{O}$, ${}^{28}_{14}\text{S}$, ${}^{40}_{20}\text{Ca}$, ${}^{48}_{28}\text{Ca}$, ${}^{90}_{40}\text{Zr}$, ${}^{120}_{50}\text{Sn}$, ${}^{140}_{58}\text{Ce}$, ${}^{208}_{82}\text{Pb}$,

*In December, 1955, a supplementary paper was submitted which takes into consideration new data on nuclear masses and levels.

¹M. G. Mayer, Phys. Rev. 78, 16 (1950).

²H. E. Duckworth et al., Phys. Rev. 83, 1114 (1951).

³G. P. Dubex and S. Iha, Phys. Rev. 85, 1042 (1952).

TABLE I.

"Magic numbers"

n_+ \ n_-	0	1	2	3	4	5
0		2 6	2, 3 18	2, 3, 4 38	2, 3, 4, 5 68	2, 3, 4, 5, 6 110
1	1 2	1, 3 14	1, 3, 4 34	1, 3, 4, 5 64	1, 3, 4, 5, 6 106	
2	1, 2 8	1, 2, 4 28	1, 2, 4, 5 58	1, 2, 4, 5, 6 100		
3	1, 2, 3 20	1, 2, 3, 5 50	1, 2, 3, 5, 5.6 92	1, 2, 3, 5, 6, 7 148		
4	1, 2, 3, 4 40	1, 2, 3, 4, 6 82	1, 2, 3, 4, 6, 7 138			
5	1, 2, 3, 4, 5 70	1, 2, 3, 4, 5, 7 126				
6	1, 2, 3, 4, 5, 6 112					
7						

${}_{92}\text{U}_{138}^{230}$) both filled shells are ground shells; they ought then to be more clearly delineated in comparison with the others. These are shells with $N=2, 8, 14, 20, 28, 50, 70, 82, 126, 138$ and $Z=2, 8, 14, 20, 40, 50, 58, 82, 92$. The remaining shells ought to be less clearly marked—these are the shells with $Z=28, 70$ and $N=40, 58, 92, 112, 148$. Seven of the double magic nuclei (${}_{28}\text{Ni}_{34}^{62}$, ${}_{34}\text{Se}_{40}^{74}$, ${}_{50}\text{Sn}_{64}^{114}$, ${}_{50}\text{Sn}_{68}^{118}$, ${}_{64}\text{Gd}_{92}^{156}$, ${}_{70}\text{Yb}_{100}^{170}$, ${}_{70}\text{Yb}_{106}^{176}$) have a filled ground shell and one subshell; finally, in two nuclei (${}_{6}\text{C}_6^{12}$ and ${}_{64}\text{Er}_{100}^{168}$) both the filled shells are subshells.

Therefore, one should expect that the most clearly evident subshells will be $Z=6, 34, 68$ and $N=6,$

$34, 64, 68, 100, 106$. It is interesting to note that in tin, the element with the largest number of β -stable isotopes, there is the largest number of β -stable double magic isotopes: ${}_{50}\text{Sn}_{64}^{114}$, ${}_{50}\text{Sn}_{68}^{118}$, ${}_{50}\text{Sn}_{70}^{120}$.

It is more appropriate to compare the vibrational levels under consideration with the vibrational levels of polyatomic molecules, rather than with single particle levels, as is done in the quasi-atomic model of the nucleus. However, there exists a definite connection between these two models. Post⁴ has shown that the Schrödinger equation for

⁴H. R. Post, Proc. Phys. Soc. (London) 66A, 649 (1953).

a system of vibrating particles, interacting with each other in pairs according to Hooke's law, can lead to the Schrödinger equation for noninteracting particles located in the field of some effective oscillation potential.

3. CLASSIFICATION OF SPINS AND MAGNETIC MOMENTS OF NUCLEI WITH ODD A : NUCLEAR ISOMERS

In our model, a given atomic nucleus can have different configurations of filled levels, one of which (the energetically more favorable) corresponds to the ground level, the others to definite excited levels. It is appropriate to distinguish the different nuclear configurations with the numbers n_- and n_+ introduced above.

The spin and magnetic moment of a nucleus with odd mass number A is determined, as usual, by the configuration of shells with odd numbers of nucleons. The shell of a given nucleus with different configurations will correspond to different values of nuclear spin I and magnetic moment μ . If the two lowest energy levels of different configuration (one of which is the ground level) are close together in energy, then the phenomenon of isomerism will be observed for the large spin difference of these levels. In our treatment, isomeric transitions are accompanied by a rearrangement of the shells, which leads to additional prohibition rules in addition to those concerning spin and parity. In this connection, attention should be turned to the region of existence of even-even isomers; this is the region of rare earths with $A \approx 160-170$, where double magic configurations exist with filled subshells. The closeness of the levels corresponding to such configurations, and to configurations with successive filling of shells, also leads to the existence of even-even isomers. Furthermore, it has been repeatedly noted in the literature^{5,6} that systems of nuclear levels are observed in the decay schemes of certain radioactive isotopes between which no β -transitions occur, in spite of the fact that such transitions are in general not forbidden. We ascribe such systems of levels to the different shell configurations. It is of interest to note that they are connected with β -transitions and with various

isomeric levels of the parent nucleus.

We shall denote the magnetic moments of nuclei with odd A by μ^+ and μ^- for nuclei belonging to the Schmidt groups ($l+1/2$ and $l-1/2$, respectively (or simply by the sign $+$ or $-$ along with the value of the spin). In our treatment, nuclei with magnetic moment μ^+ correspond to configurations in which an odd number of weakly interacting nucleons lie outside of the closed shell; nuclei of the type μ^- correspond to configurations in which a sufficiently large number of nucleons, which are very strongly coupled, lie outside the closed shell, and in which only a small number of nucleons is needed to fill the shell.

Experimental data permit the establishment of the following empirical dependence of the spins of the lowest states of the nuclei (of a definite configuration) on n_- and n_+ :

$$I = (n_+ - n_-) + 1/2 \text{ for nuclei of type } \mu^+, \quad (1a)$$

$$I = (n_+ - n_-) - 1/2 \text{ for nuclei of type } \mu^-. \quad (1b)$$

The classification of spins and magnetic moments of the ground and metastable levels of nuclei with odd A which corresponds to these rules is given in Table 2. The Table is divided into four groups, corresponding to the four values of n_- : 0, 1, 2, 3; each group begins with the magic number for which $n_- = n_+$, i. e., the numbers 0, 14, 58, 148, respectively. Each group consists of two parts, which correspond to neutron and proton configurations. Within the limits of a single group, the spins of the nuclei, in accordance with the rules (1a) and (1b), change regularly, undergoing successive unit increases after the filling of a new shell. A given value of the spin and magnetic moment of the nucleus can arise in the lowest state of a certain configuration for a wide interval of odd N or Z , whereupon configurations of the type μ^+ ought to arise in the initial filling of the shell, and configurations of the type μ^- in the closing of the shell. The regions of possible appearance of nuclear isomers are immediately evident from the Table. These are the regions in which the lowest state of the nuclei of different configurations differ strongly in spin value; $N, Z = 41-49$, where the configurations $9/2^+, 0$ and $1/2^-, 1$ (the first number denotes the value of n), can compete with each other. Similar groupings are formed by the region of configuration $11/2^+, 0, 3/2^-, 1$ and $1/2^+, 2$ for $N, Z = 71-81$, and the region $13/2^+, 0, 5/2^-, 1; 3/2^+, 2$ and $1/2^-, 2$ for $N = 113-125$.

As is seen from Table 2, the experimental data on spins and magnetic moments of nuclei with odd

⁵L. K. Pekar, L. A. Sliv and A. V. Zolotavin, Dokl. Akad. Nauk SSSR 88, 781 (1953).

⁶L. A. Sliv and L. K. Pekar, Dokl. Akad. Nauk SSSR 92, 277 (1953).

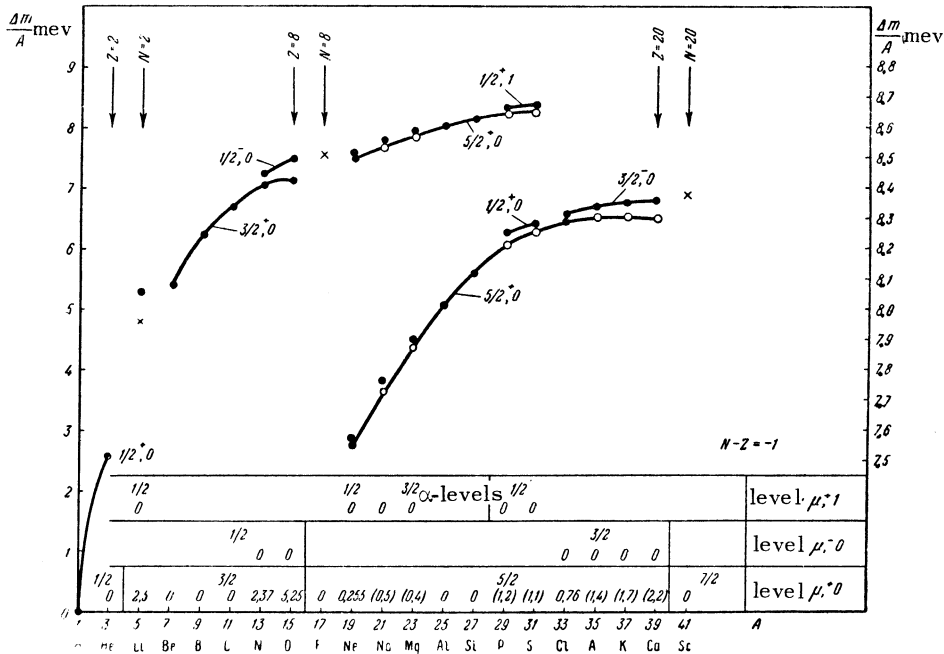


FIG. 1. Curves for the dependence of the packing fractions on the mass number for light nuclei with odd a and $N-Z=1$. The upper part of the curves is given at the right on the increased mass scale.

NOTATION: O—nucleus with known mass, energy level; O—nucleus with unknown mass or undiscovered level; X—intermediate nucleus with different configurations for neutron and proton shells.

A) on the mass number^{9,10}. Let us first consider Fig. 2 ($N-Z=+1$), where the data are most complete. The basic curve in this Figure links up the lowest level of configuration of consecutive filling of shells with the magnetic moment μ^+ . It consists of four smooth parts, which correspond to consecutive filling of $l=1/2, 3/2, 5/2$ and $7/2$. The following values of the spin are attached to the excited levels of the series of nuclei: $3/2^-$ — levels 3.09 C¹³, 5.28 N¹⁵; $5/2^-$ — levels 0.197 F¹⁹, 0.347 Ne²¹, 0.439 Na²³, 1.28 Si²⁹, 1.03 P³¹, 0.85S³³, 1.1 Cl³⁵, 1.46 A³⁷ (all levels in mev). Such a value of the spin does not contradict the experimental data. In K³⁹, in accordance with the data curve, one must expect the levels $5/2^+, 0$

with energies of the order of 3 mev. The ground levels of the nuclei C¹³, N¹⁵, S³³, Cl³⁵, A³⁷, K³⁹ correspond to the configurations $\mu^-, 0$ and possess larger binding energies than the excited levels $\mu^+, 0$; the latter are treated as weakly bound configurations. The levels of He⁵ (2.6 mev), to which is assigned the spin $3/2^+, 0^{17}(0) 5/2^+$ and Ca⁴¹ (0) $7/2^+$ lie in the transition region and are not included in the curves ($\mu^+, 0$) and ($\mu^-, 0$). They occupy an intermediate position, since they have a closed neutron shell (which corresponds in mass to the configuration $\mu^-, 0$), and a proton shell with the configuration $\mu^+, 0$.

Here we must emphasize the essential difference between the suggested order of the levels and those accepted in the single particle model of Mayer. In accordance with the latter, in a shell with certain orbital quantum number l , the levels with $j=l+1/2$ are filled first, and then $j=l-1/2$. For example, in the nucleus O¹⁷, for which an odd neutron is found in the shell $d_{5/2}$ in the ground state, there ought to be a low excited level in which the odd neutron would be found in the shell

⁹A. H. Wapstra, Physica 21, 367, 385 (1955); P. M. Endt and J. C. Kluver, Revs. Mod. Phys. 26, 95 (1955); G. F. Pieper, G. S. Stanford and P. V. Herrman, Phys. Rev. 98, 1185 (1955); H. J. Gerber, M. G. Munoz and D. Maeder, Helv. Phys. Acta 28, 478 (1955); H. Daniel, Z. Naturforsch. 9a, 974 (1954).

¹⁰F. Ajzenberg and L. Dauritsen, Revs. Mod. Phys. 27, 77 (1955).

TABLE 2²
Systematics of the spins and magnetic moments of stable and metastable levels of nuclei with odd *A*.

NOTATION: + or - on *I* conform to the proposed classification respectively for μ^+ or μ^- . The signs (+) or (-) for the excited levels of the isomers are given in parentheses. The figure in the parentheses denotes the number of nuclei with given (odd) *N* or *Z* for which the given value of *I* is measured (if it is greater than one). When the assumed classification does not agree with experiment, the experimental value of *I* is given in the table.

<i>N, Z</i> odd	<i>n₋ = 0</i>		<i>n₋ = 1</i>	
	<i>I_{theor}</i>	<i>I_{exp} for N</i>	<i>I_{theor}</i>	<i>I_{exp} for N</i>
1	$1/2^+$	+(2)	$1/2^-$	+
3	$3/2^+$	+	$3/2^-$	+
5	$1/2^-$	+	$5/2^-$	+
7	$1/2^-$	-	$3/2^-$	+
9	$5/2^+$	+	$1/2^-$	+
11	$3/2^+$	+	$3/2^-$	+
13	$1/2^-$	+	$5/2^-$	+
15	$3/2^-$	-	$1/2^-$	+
17	$1/2^-$	-	$3/2^-$	+
19	$3/2^-$	-	$5/2^-$	+
21	$7/2^+$	+	$1/2^-$	+
23	$5/2^+$	$5/2^-$	$3/2^-$	+
25	$3/2^+$	+	$5/2^-$	+
27	$1/2^-$	+	$7/2^-$	+
29	$5/2^-$	-	$1/2^-$	+
31	$3/2^-$	-	$3/2^-$	+
33	$1/2^-$	-	$5/2^-$	+
35	$3/2^-$	-	$7/2^-$	+
37	$5/2^-$	-	$1/2^-$	+
39	$3/2^-$	-	$3/2^-$	+

<i>n₋ = 0</i>	<i>n₋ = 1</i>	<i>n₋ = 2</i>
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Odd N	$n_- = 2$		$n_- = 3$		$n_- = 3$	
	I_{theor}	I_{exp}	I_{theor}	I_{exp}	I_{theor}	I_{exp}
41	$9/2^+$	+	$9/2^-$	(-)	$9/2^-$	(-)
43	$9/2^+$	+	$9/2^-$	(-)	$9/2^-$	(-)
45	$7/2$	(+)(2)	$5/2^+$	+	$7/2^+$	(-)(3)
47	$7/2$	+	$5/2^+$	+	$7/2^+$	(-)(2)
49	$7/2$	+	$5/2^+$	+	$7/2^+$	(-)(2)
51	$5/2^+$	-	$5/2^+$	+	$7/2^+$	+
53	$5/2^+$	-(2)	$5/2^+$	+	$7/2^+$	+
55	$5/2^+$	-(2)	$5/2^+$	+	$7/2^+$	+
57	$5/2^+$	-	$5/2^+$	+	$7/2^+$	+
59	$7/2^-$	-	$5/2^+$	+	$7/2^+$	+
61	$7/2^-$	-	$5/2^+$	+	$7/2^+$	+
63	$7/2^-$	(-)	$5/2^+$	+	$7/2^+$	+
65	$7/2^-$	-	$5/2^+$	+	$7/2^+$	+
67	$7/2^-$	-	$5/2^+$	+	$7/2^+$	+
69	$7/2^-$	-	$5/2^+$	+	$7/2^+$	+
71	$11/2^+$	$7/2$	$5/2^+$	+	$7/2^+$	+
73	$11/2^+$	$7/2$	$5/2^+$	+	$7/2^+$	+
75	$11/2^+$	(+)	$5/2^+$	+	$7/2^+$	+
77	$11/2^+$	(+)	$5/2^+$	+	$7/2^+$	+
79	$11/2^+$	(+)	$5/2^+$	+	$7/2^+$	+
81	$11/2^+$	(+)	$5/2^+$	+	$7/2^+$	+
83	$11/2^+$	-	$5/2^+$	+	$7/2^+$	+
85	$11/2^+$	-	$5/2^+$	+	$7/2^+$	+
87	$11/2^+$	(+)(2)	$5/2^+$	+	$7/2^+$	+
89	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
91	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
93	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
95	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
97	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
99	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
101	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
103	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
105	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
107	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
109	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
111	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
113	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
115	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
117	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
119	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
121	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
123	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
125	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
127	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
129	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
131	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
133	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
135	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
137	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
139	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
141	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
143	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
145	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
147	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
149	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+
151	$11/2^+$	+	$5/2^+$	+	$7/2^+$	+

⁷E. F. Kio¹ts, V. V. Kuntz and V. G. Khartman, Usp. Fiz. 55, 537 (1955).

A , and also on isomeric atomic nuclei, with few exceptions, agree excellently with the proposed classification.

We emphasize here that:

1) the experimental data verify that nuclei of the type μ^+ originate in the ground state at the beginning of filling of a shell, and nuclei of type μ^- at the end of filling of the shell;

2) the ground state of the nuclei always correspond to small numbers of nucleons outside the closed shell or to an insufficient number for closing. This rule is conspicuously violated among heavy nuclei;

3) Shells of successive filling ($n_-=0$) appear mainly among nuclei with small A ; shells of non-successive filling with $n_-=1$ predominate in the lower states of nuclei with medium values of A , and, finally, in heavy nuclei, the lower states correspond chiefly to configurations of the type $n_-=2$. This indicates that the spin of the nuclei is strongly dependent on its binding energy. Configurations with nucleons on the lower levels (successive filling) and even with a small number of nucleons outside of the closed shell, but with high spin ($11/2, 13/2$) are energetically less favorable than configurations with non-successive and less closed filling of shells, but with small spin ($1/2, 3/2$).

The possibility of a mutual effect of the proton and neutron shells on the order of nuclear levels can be shown in an example of a nucleus for which $N=40-50, Z=30-40$, and the ground and metastable levels correspond to the configurations $9/2^+, 0$ and $1/2^-, 1$ for neutrons, and $3/2^+, 1, 5/2^-, 0$ for protons. This is the region of nuclear isomers with odd A and with transitions of the type $M4$.

The question can be raised as to what combination of configurations of neutron and proton shells is energetically most favorable. Frequently it can be solved from a consideration of β -transitions. For example, it is evident from the decay scheme of ${}_{38}\text{Kr}_{49}^{85}$ (Ref. 8, p. 193) that β -decay proceeds from the level $1/2^- \text{Kr}^{85m}$ to the level $3/2^+ {}_{37}\text{Rb}_{48}^{85}$ (excited) and from the level $9/2^+$ of the ground state of Kr^{85} to the level $5/2^-$ (ground) of Rb^{85} . One can draw the conclusion from this that the neutron configuration $1/2^-, 1$ exists together with the proton configuration $3/2^+, 1$, and the neutron configuration $9/2^+, 0$ along with the proton $5/2^-, 0$. An addition forbiddenness is superposed on a β -decay between other pairs of levels, by

virtue of the configuration change.*

The spin of the ground level of the second isotope of rubidium ${}_{37}\text{Rb}_{50}^{87}$ is equal to $3/2$. In this nucleus, the neutron configuration $n=1$ is closed ($N=50$) and the additional gain in binding energy leads to a change in the order of the levels — the configuration with neutron shell $n_-=1$ and the proton configuration $3/2^+, 1$ correspond to the ground level. By considering the intercoupling of neutron and proton shells, it is possible to explain why certain configurations out of the number of those possible in the lowest state (that are in accord with the proposed model) are not observed experimentally. Continuing with this same line of argument, we can explain the absence of isomeric transitions for nuclei with $Z=47-49$. In this range, for two possible proton configurations $n_-=0$ and $n_-=1$, there do not exist two corresponding pairs of neutron configurations which are energetically nearly equally favorable; therefore, the isomeric levels are widely separated.

The assumed model does not correspond to the experimental data in the range of light and medium nuclei for ${}_{9}\text{F}_{10}^{19}$ ($I=1/2$), ${}_{11}\text{Na}_{12}^{23}$ ($3/2$), ${}_{22}\text{Ti}_{25}^{47}$ ($5/2$), ${}_{25}\text{Mn}_{30}^{55}$ ($5/2$), ${}_{34}\text{Se}_{41}^{75}$ ($5/2$), ${}_{34}\text{Se}_{45}^{79}$ ($7/2$). Further lack of correspondence is observed in the region of rare earths: ${}_{71}\text{Lu}_{104}^{175}$ ($7/2$), ${}_{73}\text{Ta}_{108}^{181}$ ($7/2$) and in the regions near uranium: ${}_{89}\text{Ac}_{138}^{227}$ ($3/2$), ${}_{31}\text{Pa}_{140}^{231}$ ($3/2$), ${}_{94}\text{Pu}_{145}^{239}$ ($1/2$), ${}_{95}\text{Am}_{146}^{241}$ ($5/2$).

A possible explanation of this lack of correspondence in the region of light nuclei will be given below. In the region of rare earths and uranium these noncorrespondences can be connected with the appearance in the nuclei of α -particles — structures which go beyond the framework of the shell model.

3. MASSES OF LIGHT NUCLEI WITH ODD A AND β DECAY

Curves are shown in Figs. 1-3 of the dependence of the packing fraction (binding energy divided by

*We note that in our treatment the change of spin ΔI coincides, as a rule, with the theory of Mayer, both for β and for γ -transitions; however, the change of parity frequently does not coincide, and together with this, new prohibition rules are introduced, due to the change of configurations, i.e., in Δn . Therefore the classification of β -transitions and nuclear isomers in the given treatment ought to be entirely different.

⁸M. Goldhaber and R. D. Hill, Revs. Mod. Phys. 24, 179 (1952).

TABLE 3

lg f_{β} for β -transitions of the type $\Delta I=0$, parity does not change, $\Delta \pi=0$, between configurations μ^+ , 0 and μ^- , 0 for light nuclei with odd A (data taken from Refs. 11-13).

Configura- tion	spin	$N-Z$ original and final	number of nucleons outside of closed shell									
			1	3	5	7	9	11	13			
0 + 1/2	1/2	-1, +1	n ¹ 3,21	H ³ 3,06								
	3/2	-1, +1		Be ⁷ 3,36			C ¹¹ 3,59					
	5/2	-1, +1 1,3		O ¹⁹ 5,55*			Ne ²³ ~ 5,2*	Al ²⁵ 3,47 Na ²⁵ 5,2		Si ²⁷ 3,55		Al ²⁹ 5,21*
	7/2	-1, +1 1,3 3,5	Sc ⁴¹ 3,40	Ti ⁴³ 3,40 Sc ⁴³ 5,31	Ti ⁴⁵ 4,6 Ca ⁴⁵ 5,65		V ⁴⁷ 4,7	Cr ⁴⁹ 5,0		Mn ⁵¹ 5,4		
0 + 3/2	1/2	-1, +1	O ¹⁵ 3,57	N ¹³ 3,67								
	3/2	-1, +1 1,3	Ca ³⁹ 3,49	K ³⁷ 3,51 Ar ³⁷ 4,9	A ³⁵ 3,53 S ³⁵ 5,07		Cl ³³ 3,60 P ³³ 5,4					
			number of nucleons lacking to complete shell									
			-1	-3	-5	-7						

*Decay occurs in the excited level, the energy of which is given in Fig. 2.

¹¹J. M. Hollander, I. Perlman and G. T. Seaborg, Revs. Mod. Phys. 25, 469 (1953).

¹²A. M. Feingold, Revs. Mod. Phys. 23, 10 (1951).

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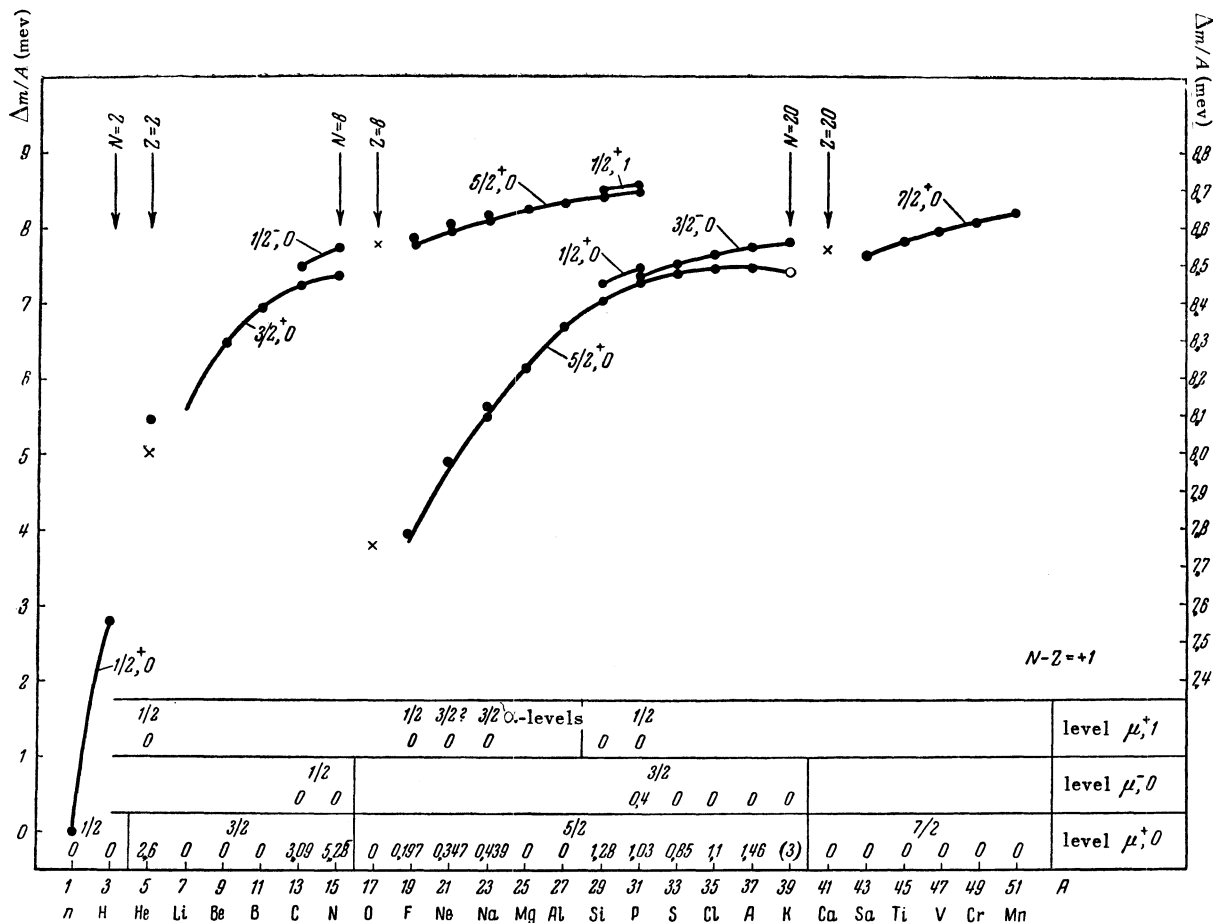


FIG. 2. Curves of the dependence of the packing fractions on the mass number for light nuclei with odd A, with N - Z = -1. Upper part of the curve is given at the right with increased mass scale. Notation same as Fig. 1.

$d_{3/2}$. In our treatment, these levels are classified as $5/2^+, 0$ and $3/2^-, 0$ and the level $3/2^-, 0$ cannot be observed in the 0^{17} nucleus, since configurations of the type μ^- exist only when the shell is close to being filled (in the given case, for $N, Z = 17, 19$). Furthermore, in our treatment, for nuclei with a ground state μ^- (for example, with $3/2^-, 0$), one should expect a neighboring excited level μ^+ (for example, $5/2^+, 0$). In Mayer's model on the other hand, when an odd nucleon is found in the $d_{3/2}$ shell in the ground state of the nucleus, it cannot undergo transition to the $d_{5/2}$ shell in the excited state, since the latter is already filled.

We turn our attention to a consideration of Fig. 2. The packing fractions of the ground state of the nuclei $^{14}_{15}\text{Si}^{29}$ and $^{15}_{16}\text{P}^{31}$ are separated on the curve since they correspond to a new configuration

$\mu^+, 1$, which is in the region $N, Z = 14$. In a series of nuclei where the rules, which connect the spins and configurations of nuclei (1a) and (1b), are not observed ($\text{F}^{19}, \text{Ne}^{21}, \text{Na}^{23}$), the ground levels must be related to α -configurations on the possibility of which in light nuclei has never been shown. These levels are drawn in the form of separate points under the curve $\mu^+, 0$. The lower states of He^5 must also be related to α -configurations, which is confirmed by the method of obtaining this level - resonance scattering of neutrons by α -particles. The level 2.6 mev, which lies close to the curve $\mu^+, 0$, is obtained from the reaction $\text{Li}(d, \alpha)\text{He}^5$, i.e., from the "shell" nucleus $\text{Li}^{7, 10}$. The curves of the packing fractions for nuclei with $N - Z = -1$ (Fig. 1) are analogous to the curves of Fig. 2. The levels of these nuclei are known much worse (the unknown levels are denoted by an open

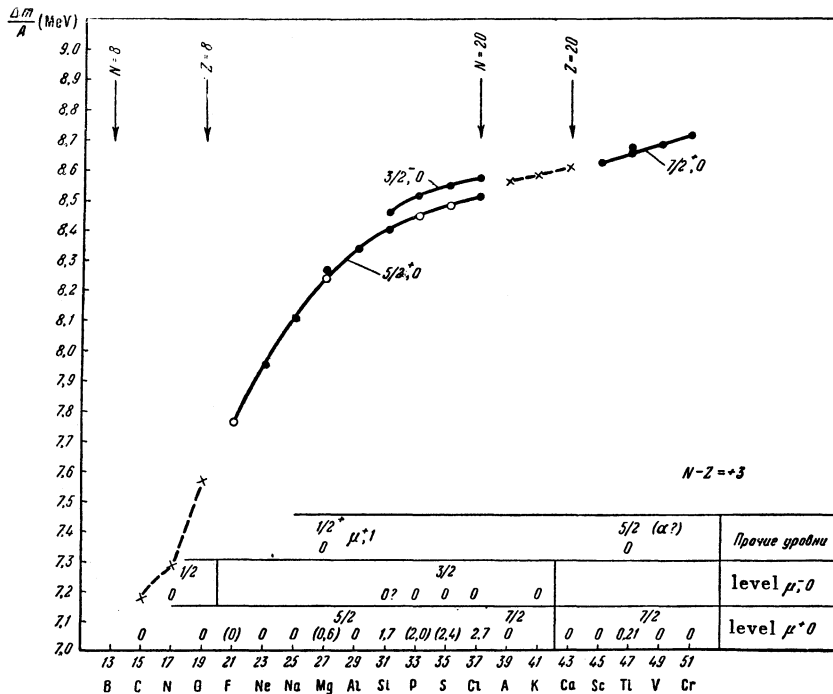


FIG. 3. The same as in Figs. 1 and 2 for nuclei with $N - Z=3$.

circle). It is difficult to doubt the existence of the assumed levels, since "similar" levels are always observed in mirror nuclei. The assumed energies of these levels are indicated in Fig. 1 in parentheses. For nuclei with $N - Z=3$, the packing fractions curve (Fig. 3) is similar to the curves of Figs. 1 and 2. We must regard the level treatment here less hopefully in view of the insufficiency of the experimental data. Thus the assumed classification of the levels makes it possible to systematize the masses and energies of the lowest levels of light nuclei with odd A .

For the systematics of the data on β -decay, we consider transitions between similar nuclear configuration $\mu^+, 0$ or $\mu^-, 0$ of the type $\Delta I=0, \Delta M=0$, with no change in the parity . Transitions in which the levels of other configurations enter in, are excluded from consideration.* The values of

$\lg ft$ for the transitions under consideration are given in Table 3, from which the following regularities are seen;

- 1) for nuclei of the type $\mu^+, 0$, $\lg ft$ increases systematically with increase in the number of nucleons outside of the closed shell (with increase in A). The nuclei $O^{19} + Se^{43}$ are excluded;
- 2) for nuclei of the type $\mu^-, 0$, $\lg ft$ increases systematically with increase in the number of nucleons needed to close the shell (with decreasing a).

Such a behavior of $\lg ft$ confirms the treatment of nuclei of the types μ^+ and μ^- as subshells and "holes" and indicates the dependence of the probability of transitions, above all, on the degree of closed-ness of the nuclear configurations between which this transition takes place.

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*For β -transitions with $\lg ft \sim 5$, the quantities ΔI cannot be determined unambiguously from β decay alone. The value $\Delta I=0$ is based on theoretical assumptions, and is confirmed by the location of the corresponding levels on the curves for the packing fractions (see Figs. 2 and 3)