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Concerning the Interaction Cross Sections of Neutrons with Nuclei

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There is proposed a new quasi-classical solution of the Schrödinger equation for $E>|V|$. This approximation is applied to the solution of the problem of the motion of a neutron in a potential with a complex component. An explanation is obtained for the behavior of total cross sections in the energy region 20-100 mev. For energies of 2-10 mev the Schrödinger equation is solved for a complex potential which goes to zero exponentially at infinity. It is shown that the absorption cross section for a well with fuzzy edges does not have sharp maxima as a function of R and E. This is in contrast to the results for the absorption cross sections for a rectangular potential well. The agreement of the calculated absorption cross sections at 2.5 and 4.3 mev with experimental values is satisfactory.

FOR a long time it had been assumed that if the wavelength λ of a neutron is less than the nuclear radius R, then the cross section for all inelastic processes of neutrons with a nucleus is equal to πR^2. However, experiments determining the total cross sections for neutrons of energy of the order of 100 mev[1] showed that not every neutron impinging on a nucleus leads to an inelastic event. In order to explain these high-energy interactions it was proposed[2] that fast neutrons have a finite mean free path in nuclear matter. At the same time it was considered that nuclei were black for low energy neutrons. The rough theory of cross sections based on the idea of black nuclei with a sharp boundary[3] led to monotonically decreasing cross sections as a function of energy. This turned out to be in disagreement with measurements. It was shown experimentally[4] that total nuclear interaction cross sections have maxima and minima as the energy of the neutron is varied.

In order to explain these experimental facts there was proposed[5] the model of the semi-transparent nucleus. According to this model, 1-3 mev neutrons have a large mean free path in nuclear matter. This arises because at low energies the collisions of the incoming neutrons with the nucleons in the nucleus are hindered by the Pauli principle which restricts the possible momentum transfers between the colliding particles.

In this way there have arisen two models. They both utilize the assumption of a finite absorption coefficient in a nucleus but differ in a series of important characteristics. For low energies the model involves a nucleus with sharp boundaries but for high energies the model is an optical one. In a nuclear model involving sharp boundaries there are additional refraction effects which do not exist in the optical model. Actually, a model with sharp boundaries is applicable only when $\lambda \gg R$, i.e., in the region where there is no inelastic scattering. The conclusions from this model were set forth in a previous communication by this author[6]. In this paper we examine a nuclear model with a finite coefficient of absorption and a fuzzy boundary which goes over into the optical model at high energies. At low energies it gives results significantly different from those given by a model with a rectangular potential.

1. STATEMENT OF THE PROBLEM

It is assumed that the interaction of a neutron with a nucleus can be represented by a complex potential $V + iW$, dependent only upon the coordinate r (neglecting spin orbit interactions). For simplicity it is assumed that the real and imaginary potentials depend on r in the same way. For sufficiently small values of $r (r < r_0)$ the function $V + iW$ goes over into a constant. The main variation of this function takes place in a thickness $1.5\text{-}2 \times 10^{-13}$ cm. In this case the Schrödinger equation for the neutron has the form:

$$\Delta\psi + (2m/\hbar^2)[E + V(r)(1 + i\zeta)]\psi = 0, \qquad (1)$$

where $W(r) = \zeta V(r)$. We now make the substitution $k = (2mE)^{1/2}/\hbar$ and $K_0 = [2mV(0)]^{1/2}/\hbar$. Then, writing the function ψ in the form:

$$\psi = \sum_{l,m} \psi_l(r)\,\Phi_{l,m}(\vartheta, \varphi), \qquad (2)$$

we obtain for the radial function $\psi_l(r)$ the equation:

$$\frac{d^2(r\psi_l)}{dr^2} + [k^2 + K_0^2 f(r)(1 + i\zeta)](r\psi_l) - \frac{l(l+1)}{r^2}(r\psi_l) = 0. \qquad (3)$$

When $r \leq r_0$, $f(r) = f(r_0) = 1$; when $r \to \infty$, $f(r) \to 0$

(it is assumed that $f(r)$ approaches zero sufficiently rapidly--more rapidly than any power law).

In order to solve Eq. (3) it is convenient to make the substitution*:

$$\psi_l(r) = r^{-1/2}[u_l^{(2)}(r) H_{l+1/2}^{(2)}(kr) + \eta_l u_l^{(1)}(r) H_{l+1/2}^{(1)}(kr)]. \qquad (4)$$

We then obtain the following equations for the functions $u_l^{(1)}$, $u_l^{(2)}$:

$$\frac{d^2 u_l^{(1,2)}}{dr^2} + 2\frac{d\ln r^{1/2} H_{l+1/2}^{(1,2)}}{dr}\frac{du_l^{(1,2)}}{dr} + K_0^2 f(r)(1+i\zeta)u_l^{(1,2)} = 0. \qquad (5)$$

When $r\to\infty$, Eq. (4) must approach the usual expression for the sum of incoming and outgoing waves, and thus:

$$\lim_{r\to\infty} u_l^{(1)}(r) = \lim_{r\to\infty} u_l^{(2)}(r) = 1.$$

It is more convenient to write Eq. (5) in terms of reduced coordinates $x = kr$:

$$\frac{d^2 u_l^{(1,2)}}{dx^2} + 2\frac{d\ln x^{1/2} H_{l+1/2}^{(1,2)}(x)}{dx}\frac{du_l^{(1,2)}}{dx} + K_0^2 f(x)(1+i\zeta)u_l^{(1,2)} = 0. \qquad (6)$$

The solution of Eq. (6) turns out to be simplest at high energies.

2. HIGH ENERGIES

If the energy is sufficiently high, then the functions u_l can be represented in the form:

$$u_l = \exp\left\{-\int_0^\infty S_l dx\right\}. \qquad (7)$$

We then obtain for the function S_l the equation:

$$\frac{dS_l^{(1)}}{dx} + S_l^{(1)^2} + 2\frac{d\ln x^{1/2} H_{l+1/2}^{(1)}(x)}{dx} S_l^{(1)} + \frac{k_0^2}{k^2} f(x)(1+i\zeta) = 0. \qquad (8)$$

Neglecting dS_l/dx by analogy with the similar quasi-classical problem we obtain:

$$u_l(x) = \exp\left\{\int_x^\infty \left(\frac{d\ln x'^{1/2} H_{l+1/2}^{(1)}(x')}{dx'} - \left[\left(\frac{d\ln x'^{1/2} H_{l+1/2}^{(1)}(x')}{dx'}\right)^2 - \frac{k_0^2}{a^2} f(x')(1+i\zeta)\right]^{1/2}\right)dx'\right\}. \qquad (9)$$

In order that the function ψ_l be finite at zero it is necessary and sufficient that:

$$\eta_l = u_l^{(2)}(0)/u_l^{(1)}(0). \qquad (10)$$

The function under the integral sign in Eq. (9) is finite everywhere on the real axis and therefore the evaluation of the integral presents no difficulty.

We now introduce the notation:

$$\mathrm{Re}\,\frac{d\ln x^{1/2} H_{l+1/2}^{(1)}(x)}{dx} = a_l, \qquad \mathrm{Im}\,\frac{d\ln x^{1/2} H_{l+1/2}^{(1)}(x)}{dx} = b_l. \qquad (11)$$

If it is assumed that the real part of the potential is much larger than the imaginary (which is the case for a transparent nucleus), then the expression under the square root sign can be expanded:

$$[(a_l^2 + ib_l^2)^2 - c^2(1+i\zeta)]^{1/2} = i[(b_l - ia_l)^2 + c^2]^{1/2} - {}^1/_2 c^2\zeta[(b_l - ia_l)^2 + c^2]^{-1/2}.$$

Then η has the form:

$$\eta = \exp\left\{-2i\int_0^\infty [b_l(x) - \mathrm{Re}((b_l - ia_l)^2 + c^2)^{1/2}]\,dx - \zeta\int_0^\infty c^2(x)\frac{\mathrm{Re}\,\{[b_l(x) - ia_l(x)]^2 + c^2\}^{1/2}}{[(b_l^2 - a_l^2 + c^2)^2 + 4a_l^2 b_l^2]^{1/2}}\,dx\right\}, \qquad (12)$$

* $H_{l+1/2}^{(1,2)}$ are the Hankel functions of the first and second order with half integral indices.

where

$$\mathrm{Re}\{(b_l - ia_l)^2 + c^2\}^{1/2} = 2^{-1/2}\{(b_l^2 + c^2 - a_l^2) + [(b_l^2 + c^2 - a_l^2)^2 + 4a_l^2 b_l^2]^{1/2}\}^{1/2}.$$

It is now easy to get expressions for the total cross section and absorption cross section:

$$\sigma_{\mathrm{abs}} = \sum_{l=0}^{\infty}(2l+1)\pi\lambda^2(1-|\eta_l|^2) = \sum_{l=0}^{\infty}(2l+1)\pi\lambda^2\Big(1-\exp\Big\{-\frac{2k_0^2}{k^2}2^{-1/2}\zeta \times \int_0^{\infty} f(x)[b_l^2+c^2-a_l^2+\{(b_l^2+c^2-a_l^2)^2+4a_l^2b_l^2\}^{1/2}]^{1/2}[(b_l^2+c^2-a_l^2)+4a_l^2b_l^2]^{-1/2}dx\Big\}\Big), \tag{13}$$

$$\begin{aligned}\sigma_{\mathrm{tot}} &= \sum_{l=0}^{\infty} 2(2l+1)\pi\lambda^2(1-|\eta_l|^2)\\ &= \sum_{l=0}^{\infty}2(2l+1)\pi\lambda^2\Big[1-\cos 2\int_0^{\infty}\Big\{b_l - 2^{-1/2}[b_l^2+c^2-a_l^2+\{(b_l^2+c^2-a_l^2)^2\\ &\quad + 4a_l^2b_l^2\}^{1/2}]^{1/2}\Big\}dx\,\exp\Big\{-\frac{k_0^2}{k^2}2^{-1/2}\zeta\int_0^{\infty}f(x)[b_l^2+c^2-a_l^2\\ &\quad + \{(b_l^2+c^2-a_l^2)^2+4a_l^2b_l^2\}^{1/2}]^{1/2}[(b_l^2+c^2-a_l^2)^2+4a_l^2b_l^2]^{-1/2}dx\Big\}\Big].\end{aligned} \tag{14}$$

Thus, the calculation of cross sections for any form of the potential can be carried out in this case by quadrature. However, the quadrature has to be carried out numerically and therefore it is convenient to expand the square root under the integral in a series, which is always possible if $k_0^2/k^2 \ll 1$.

The Eqs. (13) and (14) can then be put into the form, valid to order including k_0^4/k^4:

$$\sigma_{\mathrm{abs}} = \sum_{l=0}^{\infty}(2l+1)\pi\lambda^2\Big(1-\exp\Big\{-\frac{2k_0^2}{k^2}\zeta\int_0^{\infty}\frac{b_l}{b_l^2+a_l^2}f(x)\times\Big[1-\frac{k_0^2}{k^2}f(x)\frac{b_l^2-3a_l^2}{(b_l^2+a_l^2)^2}\Big]dx\Big\}\Big), \tag{15}$$

$$\sigma_{\mathrm{tot}} = \sum_{l=0}^{\infty}2(2l+1)\pi\lambda^2\Big(1-\cos\frac{k_0^2}{k^2}\int_0^{\infty}f(x)\frac{b_l}{b_l^2+a_l^2}\Big[1-\frac{k_0^2}{4k^2}\frac{b_l^2-3a_l^2}{(b_l^2+a_l^2)^2}\Big]dx \times \exp\Big\{-\frac{k_0^2}{k^2}\zeta\int_0^{\infty}\frac{b_l}{b_l^2+a_l^2}\Big[1-\frac{k_0^2}{2k^2}f(x)\frac{b_l^2-3a_l^2}{(b_l^2+a_l^2)^2}\Big]f(x)\,dx\Big\}\Big). \tag{16}$$

In the case when it is possible to neglect the term k_0^4/k^4 the expression under the integral sign for the harmonics with $l < kR$ is equal to the mean free path of a neutron with momentum l in a nucleus according to the optical model.

Let us assume that the potential at the nuclear boundary decreases exponentially according to $e^{-\alpha x}$. Then, using for the radius of the nucleus $R = r_0 + 1/\alpha$, we get from Eqs. (15) and (16) that the geometrical path of a neutron with momentum $l < kR$ is equal to $R - l(l+1)/2k^2R^2$ and does not depend on the magnitude of $1/\alpha$ for constant R. The boundary region is important only for $l > kR$.

When $1/\alpha \ll \lambda$, then at very high energies (in the approximation in which k_0^4/k^4 can be neglected) only waves with $l \ll kR$ penetrate into the nucleus. If $1/\alpha \geq \lambda$, then waves with $l > kR$ penetrate into the nucleus. In the first case the departure from geometrical optics consists only in the scattering of waves with $l = kR$; the contribution to the cross section of these tends to 0 like λ. If, however, the fuzzy region is larger than the wavelength, then the effect of the boundary on the magnitude of the cross section depends on the relative sizes of the wavelength and the nuclear radius.

For heavy nuclei having $R \sim 7 - 8 \times 10^{-13}$, the cross section increases by about 10% relative to that when $1/\alpha \ll \lambda$ and the same value of $r_0 + 1/\alpha$. For light nuclei the cross section increases very strongly. However, if it is assumed that $V(r)$ is proportional to the density of nuclear matter then, in this case, increasing the fuzziness (with an exponential form of the boundary) decreases the density of nuclear matter in the middle.

If the density of nuclear matter in the center is kept constant, then the absorption cross section depends very little on the form of the boundary (changing only by several percent). However, if it is assumed that $V(r)$ does not depend on nuclear density, there results a strong dependence of the cross section of light nuclei on the boundary behavior. At lower energies, when the second term in the exponent cannot be neglected, waves with $l > kR$ penetrate into the nucleus even for $1/\alpha \ll \alpha$*. This effect persists no matter how small the fuzziness. In this case the absorption cross section for sufficiently small path lengths of the neutron in the nucleus can be 15-20 percent greater than πR^2 even for heavy nuclei.

A more detailed study of the absorption coefficient cannot be carried out at present because of almost complete lack of experimental information on absorption cross section for energies with $E > V$.

An important conclusion follows from Eq. (16). The argument of the cosine decreases with increasing energy. At some energy it passes through the value 2π, and this corresponds to a minimum in the total cross section of a given l. At a still higher energy there appears a maximum corresponding to cosine $\varphi_l = 1$, that is, $\varphi_l = \pi$. After this, the cross section falls monotonically. The sum of the cross sections likewise has a maximum and minimum (although weaker). This maximum should be observed at higher energies for heavy nuclei than for light ones due to the longer path of the particle in the nucleus. This displacement of the maximum as a function of atomic weight is confirmed experimentally. From the position of the minimum in the cross section for Pb at $E = 55$ mev it is possible to evaluate the real part of the potential for $R = 1.2 \times 10^{-13} A^{1/3}$ (in the case of Pb the boundary shape has little influence on the cross section). This leads to $U(0) = 35$-40 mev.

It is worth mentioning that for $R = 1.2 \times 10^{-13} A^{1/3}$ the Fermi energy is equal to 31 mev and for this reason the potential U, taking into account the binding energy, should equal about 38 mev. Similar values are obtained also at lower energies.

At energies greater than 100 mev the simple model with a complex potential apparently is not applicable.

3. THE LIMITS OF APPLICABILITY OF THE QUASI-CLASSICAL METHOD

In order to make clear the limits of applicability of the quasi-classical solution, we return to Eq. (8). We write this equation in the form:

$$\frac{dS_l}{dx} + S_l^2 - S_l^{(0)2} + 2\frac{d \ln x^{1/2} H^{(1)}_{l+1/2}(x)}{dx} S_l - 2\frac{d \ln x^{1/2} H^{(1)}_{l+1/2}}{dx} S_l^{(0)} = 0. \quad (17)$$

Then, assuming that the approximate solution $S_l^{(0)}$ is close to the exact solution S_l, the difference $S_l^2 - S_l^{(0)2}$ can be replaced by $2(S_l - S_l^{(0)})S_l^{(0)}$. In this we are neglecting the quantity $S_l - S_l^{(0)}$ as compared to $2S_l^{(0)}$. Equation (17) then takes the form:

$$\frac{dS_l}{dx} + 2[S_l - S_l^{(0)}]\left(\frac{d \ln x^{1/2} H^{(1)}_{l+1/2}}{dx} + 2S_l^{(0)}\right) = 0. \quad (18)$$

This is a linear equation of the first order. After some transformations, the solution can be written in the form:

$$S_l = S_l^{(0)}(x) + \int_x^\infty dx' \frac{dS_l^{(0)}(x')}{dx'} \exp\left\{2\int_x^{x'}\left(\frac{d \ln x''^{1/2} H^{(1)}_{l+1/2}(x'')}{dx''} + S_l(x'')\right)dx''\right\}.$$

This solution is significantly more accurate than the second approximation of the usual quasi-classical method which can be obtained from it by using integration by parts:

* *Translator's note:* This looks like a typographical error. I think that it should be $1/\alpha \ll \lambda$.

$$S_l^{(1)} = S_l^{(0)} + \frac{1}{2}\frac{dS_l^{(0)}x}{dx}\left(\frac{a\ln x^{1/2}H^{(1)}_{l+1/2}}{dx} + S_l^{(0)}\right)^{-1} + I(x). \qquad (19)$$

For s-waves the second term of (19) gives a factor which drops out when the amplitude of the scattered wave is calculated. For $V = E$ this term contributes $\sim 20\%$. The magnitude of $I(X)$ is much smaller and for $E = V$ it is less than 1-2%. For $l \neq 0$ the second term of (18) does not drop out in the calculation of the scattered wave. However, when $l < kr_0$ the correction is small.

The correction is significant only when $l \sim kr_0$. When $U \sim E$ the contribution of the second approximation to the magnitude of η_l when $l = kR$ is smaller than $e^{-0.4i}$. The correction to the absorption cross section is not significant if the absorption is large. For this reason we can consider that Eqs. (13) and (14) are valid for $V < E$. However, it is not necessary to require the more stringent condition that $V \ll E$.

4. LOW ENERGIES

If $E < V(0)$, then the quasi-classical method of calculation is not satisfactory. In this case it is necessary to carry out calculations using an exact solution of Eq. (3) with a definite form for the boundary. The following calculations have been made on the assumption that $f(r) = e^{-\alpha(r-r_0)}$ when $r \geq r_0$. It is then necessary to satisfy the boundary condition when $r = r_0$:

$$\frac{1}{\psi_i}\frac{d\psi_i}{dr} = \frac{1}{\psi_a}\frac{d\psi_a}{dr}. \qquad (20)$$

When $r \leq r_0$ the potential V is a constant, and therefore the solution is written in the form:

$$\psi = r^{-1/2} J_{l+1/2}(\varkappa r),\ \varkappa = [k^2 + k_0^2(1 + i\zeta)]^{1/2}. \qquad (21)$$

In the region of changing potential $r \geq r_0$ a solution is sought, as earlier, in the form

$$\psi_a^{(1)(2)} = r^{-1/2} u_l^{(1)(2)} H^{(1)(2)}_{l+1/2}(kr).$$

Here we sought a solution of Eq. (6) in the form of a series involving an expansion in powers of k_0^2/α^2.

Let us make use of the condition $u_l = 1$ as $r \to \infty$. Then as a first approximation let us place $u_l = 1$ in Eq. (6), getting an equation of the first order for du_l/dr:

$$\frac{d^2u_l}{dy^2} + 2\frac{d\ln y^{1/2}H^{(1)}_{l+1/2}(ky/\alpha)}{dy}\frac{du_l}{dy} + \frac{k_0^2}{\alpha^2}f(y)(1 + i\zeta) = 0. \qquad (22)$$

Finding du_l/dy from Eq. (22) and integrating a second time we get:

$$u_l^{(1)}(y) = 1 - \frac{k_0^2}{\alpha}(1 + i\zeta)\int_y^\infty [y'^{1/2}H^{(1)}_{l+1/2}(ky'/\alpha)]^{-2} \times \int_{y'}^\infty f(y'')[y''^{1/2}H^{(1)}_{l+1/2}(ky''/\alpha)]^2\,dy'\,dy''. \qquad (23)$$

Equation (23) can be transformed into a more convenient form:

$$u_l^{(1)}(y) = 1 - \frac{k_0^2}{\alpha^2}(1 + i\zeta)\int_y^\infty y'[H^{(1)}_{l+1/2}(ky'/\alpha)]^2 f(y') \times \int_y^{y'}[H^{(1)}_{l+1/2}(ky''/\alpha)]^{-2}\frac{dy'dy''}{y''}.$$

Evaluating the inside integral we then find finally:

$$u_l^{(1)}(y) = 1 - \frac{i}{2}\frac{k_0^2}{\alpha^2}(1 + i\zeta)\frac{\alpha}{k}\left\{\int_y^\infty \frac{|P_l(ky'/\alpha)|^2}{(ky'/\alpha)^{2l}} f(y')\,dy' - \right. \tag{24}$$
$$\left. - \frac{P_l^x(ky/\alpha)}{P_l(ky/\alpha)}\int_y^\infty \exp\left\{2i\frac{k}{\alpha}(y'-y)\right\}(ky'/\alpha)^{-2l}\{P_l(ky'/\alpha)\}^2 f(y')\,dy'\right\}.$$

In Eq. (24)

$$P_l(ky/\alpha) = \exp\left\{- i\frac{k}{\alpha}y\right\}(ky/\alpha)^{l+1/2}H^{(1)}_{l+1/2}(ky/\alpha).$$

If it is assumed that $1/\alpha = 1.5 - 2\times 10^{-13}$, then the first approximation turns out to be insufficient and it is necessary to take 3-4 terms of the series for E = 3-10 mev. However, the integrals depend only on the single parameter k/α, and this significantly simplifies the calculations (the parameters k_0, ζ and r_0 can be varied at will).

The equation for the nth approximation can be obtained from Eq. (6) by introducing into it the (n-1)st approximation, and has the form:

$$u_l^{(1)[n]}(y) = 1 - \frac{i}{2}\frac{k_0^2}{\alpha k}(i + i\zeta)\left\{\int_y^\infty (ky'/\alpha)^{-2l}\,|P_l(ky'/\alpha)|^2\, u_l^{(1)[n-1]}(y') f(y')\,dy'\right. \tag{25}$$
$$\left. - \frac{P_l^x(ky/\alpha)}{P_l(ky/\alpha)}\int_y^\infty \exp\left\{2i\frac{k}{\alpha}(y'-y)\right\}[P_l(ky'/\alpha)]^2(ky'/\alpha)^{-2l} f(y') u_l^{(1)[n-1]}(y')\,dy'\right\}.$$

It should be pointed out that such complicated expressions are needed only for waves with small l (less than 5-6), for which quasi-resonances can exist.

The conditions for the existence of a quasi-resonance can be obtained quasi-classically starting from the condition that as $E \to \infty$ the argument of the cosine in Eq. (14) approaches $(2n+1)\pi$. Thus, for a potential with $1/\alpha \ll R$ we obtain:

$$(k_0^2R^2 - l^2)^{1/2} = (2n+1)\pi/2 \tag{26}$$
$$+\, l\,\mathrm{arctg}[(k_0^2R^2 - l^2)^{1/2}/l]$$

This is in excellent agreement with the exact solution.

It should be pointed out that for a washed-out boundary having $1/\alpha \sim R$ the agreement is poorer. However, for calculations, especially when the nucleus is heavy (i.e., $1/\alpha < R$), the quasi-classical conditions can be used.

5. COMPARISON WITH EXPERIMENT

The data on inelastic scattering cross sections for energies of a few million volts are relatively meager. Moreover, at energies less than 2-3 mev the number of nuclear levels that can be excited is small, and therefore, especially in the case of magic nuclei, the decomposition of the intermediate nucleus can go by way of elastic scattering.

We have used the data of Pasechnik[7] on the inelastic scattering of 2.5 and 4.3 mev neutrons. In order to compare the theory with these experiments, the absorption cross section was calculated for neutrons of these energies as a function of nuclear radius. In this an exponential form of the boundary $e^{-\alpha r}$ was assumed with $1/\alpha = 1.45\times 10^{-13}$ cm. The depth of the well was taken as 20 mev; however, the picture is little changed even if $V(0)$ = 40 mev. At E = 2.5 mev the absorption coefficient $V(0)\zeta$ was taken as $0.053\,V(0)$; at E = 4.3 mev the absorption coefficient was taken as $0.05\,V(0)$ and $0.11\,V(0)$.

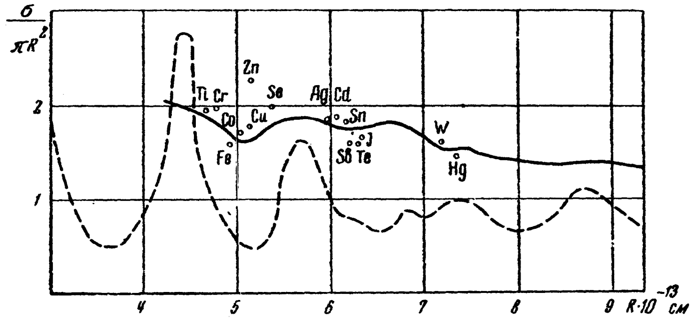

FIG. 1. Dependence of the inelastic scattering cross section on the nuclear radius for 2.6 mev; $\zeta = 0.05$. The continuous curve was computed for the model with diffuse boundaries, the broken curve for the model with sharp boundaries; the experimental points are marked by circles.

Figure 1 shows the results of the calculations of the absorption cross section $\sigma/\pi R^2$ as a function of $R = r_0 + 1/\alpha$ (solid curve). There are also presented the results of a calculation using a model with sharp boundaries and the same values of V and ζ (dashed curve). The circles denote the experimental points of Pasechnik, obtained using a threshold detector (Al). For these the nuclear radius was taken equal to $R = 1.25 \times 10^{-13} A^{1/3}$ As is seen the absorption cross sections from a model with a fuzzy boundary are about twice as large as those from a model with sharp boundaries. Also, the maxima are rather weak (the ratio of cross sections at maxima to that at minima is 1.3, whereas on the model with sharp edges this ratio exceeds 2). The experimental points scatter on both sides of the curve with fuzzy boundary. The magic nuclei have not been included in the data presented, since for these the inelastic scattering cross section is small because of the small number of levels available. The point for Zn is quite a bit off, its experimentally determined cross section being too high. In general, the data at 2.5 mev are less sensitive for comparison with theory than are the data at 4.3 mev.

Figure 2 shows the results of calculations for 4.3 mev neutrons. The solid curves give the cross sections for nuclei with fuzzy boundaries for the two absorption coefficients 0.05 $V(r)$ and 0.11 $V(r)$. The dashed curve corresponds to absorption cross sections for nuclei with a sharp boundary and $\zeta = 0.05$. For comparison there are given the experimental points of Pasechnik using $R = 1.25 \times 10^{-13} A^{1/3}$ cm. As is seen these points agree better with a curve for $\zeta = 0.11$ than with the curve for $\zeta = 0.05$.

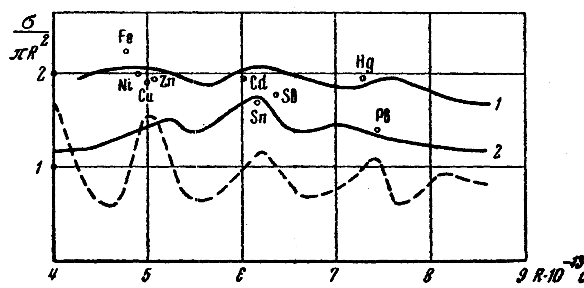

FIG. 2. Dependence of the inelastic scattering cross section on the nuclear radius for 4.3 mev. The continuous curves were computed for the model with diffuse boundaries, the broken curves for the model with sharp boundaries; the experimental points are marked by circles. For curve 1, $\zeta = 0.1$, for the other curves, $\zeta = 0.05$.

Thus, the experimental data indicate that the mean free path for neutrons of low energies is large (of the order of 2×10^{-12} cm), but that it rapidly decreases with increasing energy.

The nuclear model with a fuzzy boundary makes possible the reconciliation of nuclear radii obtained from other experiments (scattering of fast electrons) with those obtained from neutron cross sections. On the other hand, the model with sharp boundaries, even with a large absorption coefficient, gives cross sections equal to $1\text{-}1.2\pi R^2$, which by comparison with experimental data leads to very large radii: $R = 1.6 \times 10^{-13} A^{1/3}$. These are in sharp disagreement with results from all other experiments.

The maxima in absorption cross sections as a function of radius disappear in the model with washed-out boundaries at an energy of about 5 mev. At larger energies the absorption cross section is a monotonic function of the radius. The available data on inelastic scattering at $E = 14$ mev are not in contradiction with $R = 1.25 \times 10^{-13} A^{1/3}$, except for possibly the lightest nuclei. In this case the mean free path should be of the order of 3-4 $\times 10^{-13}$ cm, i.e., the nucleus should be black for waves with $l < kR$.

In contrast with absorption cross sections, the total cross sections will have maxima and minima for all values of l for which the nucleus is black because the phase of the scattered wave changes depending on R and E.

CONCLUSIONS

We have proposed a method of modulated spherical waves for studying the scattering of neutrons. This method, for $V(0) < E$ appears to be a generalization of the quasi-classical method.

In contrast to the usual quasi-classical approach, the proposed one satisfies exact boundary conditions. Higher approximations can easily be obtained, and thus a limiting transition to the exact phase analysis solution of the problem can be obtained.

The interaction cross sections of neutrons in all energy regions can be described using a model with fuzzy boundaries.

When $E > V(0)$ there is obtained a simple explanation of the maxima in total cross sections and likewise, of the deviations of total cross sections from $2\pi R^2$. At low energies the absence of sharp maxima in the absorption cross sections as a function of radius and energy, which are obtained from a model with sharp boundaries, is explained. Likewise the values of the cross sections are reconciled with the values of nuclear radii obtained from other experiments.

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