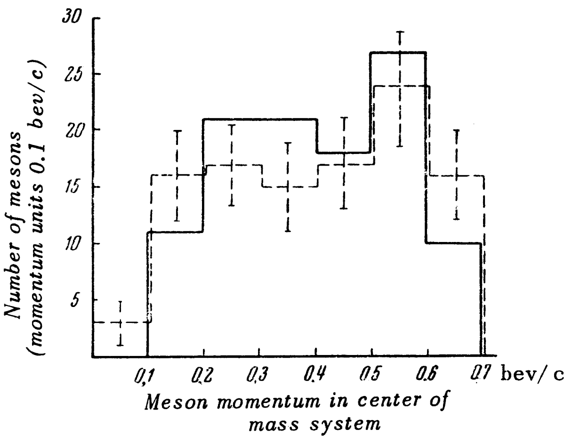

FIG. 1. Meson momentum distribution in the center of mass system for the reactions ($p-0$) and ($n+-$). Dashed line – Eisberg et al, experimental distribution; solid line–distribution according to the statistical theory including isobars.

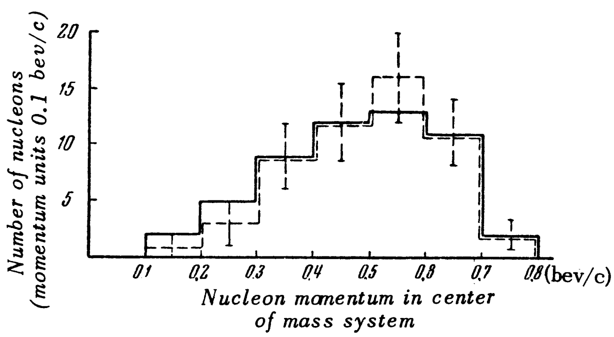

FIG. 2. Nucleon momentum distribution in the center of mass system for the reactions ($p-0$) and ($n+-$). Dashed line– Eisberg et al., experimental distribution; solid line– distribution according to the statistical theory including isobar states.

it includes isobar states.

We also note that according to the above calculation the meson momentum distribution is in contradiction with the statistical theory when isobar states are neglected, but is not in contradiction with the assumption that particle creation can only take place through isobaric states. However, the marked spread of Q (isobar decay energy) can be interpreted as being due to the fact that a considerable part is played by creation without immediate isobaric states.

In conclusion I wihh to express my gratitude to Professor S. Z. Belen'kii for interesting discussions and for his continued interest.

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On the Interaction between Nucleon and Antinucleon

Iu. A. TARASOV
Moscow State University

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WE investigate by means of the "old" Tamm method the interaction between nucleon and antinucleon. The Schrödinger equation for the wave functional of the system has the form:

$$(H_0 + H')\Psi = W\Psi, \quad (1)$$

where W is an eigenvalue of the total energy of the system. Let us expand Ψ in a series of eigenfunctions of H_0 :

$$\Psi = \sum_{\lambda, m, n} a_\lambda^{mn} \psi_\lambda^{mn}. \quad (1a)$$

Here m is the number of mesons in a free state, n is the number of nucleon pairs, λ denotes momenta, spins and isotopic spins of the particles, a_λ^{mn} is the probability amplitude of finding the system in the state(λ, m,n). From Eq. (1), we obtain an integral equation for the amplitude a_λ^{mn}

$$[W - E_\lambda^{mn}]\, a_\lambda^{mn} = \sum_{q=n-1}^{n+1} \sum_{p=m\pm1} \sum_\mu \langle \lambda mn \mid H' \mid \mu pq \rangle\, a_\mu^{pq}. \quad (1b)$$

From this equation it is possible to obtain an equation for the amplitude a_λ^{01} corresponding to the state in which only a nucleon and an antinucleon are present. We have

$$[W - E_\lambda^{01}]\, a_\lambda^{01} = \sum_\mu [\langle \lambda 01 \mid H' \mid \mu 11 \rangle\, a_\mu^{11} + \langle \lambda 01 \mid H' \mid \mu 10 \rangle\, a_\mu^{10} + \langle \lambda 01 \mid H' \mid \mu 12 \rangle\, a_\mu^{12}]. \quad (2)$$

The second term corresponds to the transformation of the nucleon and antinucleon into a meson. Let us confine ourselves to the case that only one virtual meson is present and there are no virtual pairs. Then it is only necessary to form the amplitudes a^{11}_{μ} and a^{10}_{μ} corresponding to the presence of a meson and a pair, and one meson. The equations, cut off according to the number of particles, have the form

$$[W - E^{11}_{\mu}]\, a^{11}_{\mu} = \sum_{\rho} \langle \mu\, 11 \,|\, H' \,|\, \rho\, 01 \rangle\, a^{01}_{\rho}, \quad (3)$$

$$[W - E^{10}_{\mu}]\, a^{10}_{\mu} = \sum_{\nu} \langle \mu\, 10 \,|\, H' \,|\, \nu\, 01 \rangle\, a^{01}_{\nu}.$$

Substituting (3) into (2), we obtain an equation for a^{01}_{λ}

$$[W - E^{01}_{\lambda}]\, a^{01}_{\lambda} = \sum_{\mu,\nu} \left[\frac{\langle \lambda\, 01 \,|\, H' \,|\, \mu\, 11 \rangle \langle \mu\, 11 \,|\, H' \,|\, \nu\, 01 \rangle}{W - E^{11}_{\mu}} + \frac{\langle \lambda\, 01 \,|\, H' \,|\, \mu\, 10 \rangle \langle \mu\, 10 \,|\, H' \,|\, \nu\, 01 \rangle}{W - E^{10}_{\mu}} \right] a^{01}_{\nu}. \quad (4)$$

Let $\mathbf{p}'_n i', \alpha'$ denote momentum, spin and isotopic spin of the nucleon, $\mathbf{p}'_{\bar{n}}, j', \beta'$ momentum, spin and isotopic spin of the antinucleon before meson exchange and $\mathbf{p}_n, i, \alpha, \mathbf{p}_{\bar{n}}, j, \beta$ the corresponding quantities after meson exchange. Then Eq. (4) takes the form:

$$[W - E^{01}(\mathbf{p}_n, \mathbf{p}_{\bar{n}})]\, a^{01}(\mathbf{p}_n \mathbf{p}_{\bar{n}}) = \int \Delta_2 (\mathbf{p}_n, \mathbf{p}_{\bar{n}}, \mathbf{p}_{n'}, \mathbf{p}_{\bar{n}'}, \mathbf{k}, W)\, a^{01}(\mathbf{p}'_n, \mathbf{p}'_{\bar{n}})\, d\mathbf{p}'_n d\mathbf{p}'_{\bar{n}}\, d\mathbf{k}, \quad (5)$$

$$\Delta_2 = \frac{\langle \mathbf{p}^{i\alpha}_n \,|\, H' \,|\, \mathbf{p}^{i'\alpha'}_{n'} \rangle \langle \mathbf{p}^{j\beta}_{\bar{n}} \,|\, H' \,|\, \mathbf{p}^{j'\beta'}_{\bar{n}'} \rangle}{W - E_{\mathbf{p}_{n'}} - E_{\mathbf{p}_{\bar{n}}} - \omega_{\mathbf{p}_{\bar{n}} - \mathbf{p}_{\bar{n}'}}} + \frac{\langle \mathbf{p}^{i\alpha}_n \mathbf{p}^{j\beta}_{\bar{n}} \,|\, H' \,|\, \mathbf{k} \rangle \langle \mathbf{k} \,|\, H' \,|\, \mathbf{p}^{i'\alpha'}_{n'} \mathbf{p}^{j'\beta'}_{\bar{n}'}}{W - \omega_{\mathbf{k}}}.$$

We take the interaction Hamiltonian in the charge-symmetric form: $H' = ig \bar{\Psi} \gamma_0 - (\tau\varphi)\, \Psi$. Calculating the matrix elements and integrating with respect to $d'\mathbf{p}_n$ and $d\mathbf{p}'_{\bar{n}}$, Eq. (5) reduces to the form

$$[W - E^{01}(\mathbf{p}_n, \mathbf{p}_{\bar{n}})]\, a^{01}(\mathbf{p}_n, \mathbf{p}_{\bar{n}}) = \int \Delta^{11}_2 a^{01}\, d\mathbf{k} + \int \Delta^{10}_2 (\mathbf{p}_n, \mathbf{p}_{\bar{n}}, \mathbf{p}'_n, \mathbf{p}_n + \mathbf{p}_{\bar{n}} - \mathbf{p}'_n)\, a^{01} d\mathbf{p}_n, \quad (6)$$

where

$$\Delta^{11}_2 = \frac{g^2}{8\pi^3} \frac{[\bar{U}^{i\alpha}(\mathbf{p}_n)\, \gamma_5 \tau^{\sigma}_{\alpha\alpha'} U^{i'\alpha'}(\mathbf{p}_n - \mathbf{k})]\, [\bar{V}^{j'\beta'}(-\mathbf{p}_{\bar{n}} - \mathbf{k})\, \gamma_5 \tau^{\sigma}_{\beta'\beta} V^{j\beta}(-\mathbf{p}_{\bar{n}})]}{\omega_{\mathbf{k}} (W - E_{\mathbf{p}_n - \mathbf{k}} - E_{\mathbf{p}_{\bar{n}}} - \omega_{\mathbf{k}})} \quad (6a)$$

$$\Delta^{10}_2 = \frac{g^2}{8\pi^3} \frac{[\bar{U}^{i\alpha}(\mathbf{p}_n)\, \gamma_5 \tau^{\alpha\beta} V^{j\beta}(-\mathbf{p}_{\bar{n}})]\, [V^{j'\beta'}(\mathbf{p}'_n - \mathbf{p}_n - \mathbf{p}_{\bar{n}})\, \gamma_5 \tau^{\sigma}_{\beta'\alpha'} \bar{U}^{i'\alpha'}(\mathbf{p}'_n)]}{\omega_{\mathbf{k}} (\omega_{\mathbf{k}} - W)}$$

Passing to the center-of-mass system ($\mathbf{p}_n - \mathbf{k} = \mathbf{p}'$, $\mathbf{p}_n = \mathbf{p}$, $\mathbf{p}_{\bar{n}} + \mathbf{k} = -\mathbf{p}'$, $\mathbf{p}_{\bar{n}} = -\mathbf{p}$) and carrying out the calculations, Eq. (6) reduces to the form

$$[W - 2E(\mathbf{p})]\, a^{01}(\mathbf{p}) = \int \Delta_2 (\mathbf{p}\mathbf{p}' W)\, a^{01}(\mathbf{p}')\, d\mathbf{p}', \quad (7)$$

where

$$\Delta_2 = \frac{g^2}{8\pi^3} \frac{(E_{\mathbf{p}} + M)(E_{\mathbf{p}'} + M)}{4E_{\mathbf{p}'}E_{\mathbf{p}}} \frac{\tau_1 \tau^T_2 (\sigma_1 \mathbf{R})(\sigma^T_2 \mathbf{R})}{\omega_{\mathbf{p}-\mathbf{p}'} (\omega_{\mathbf{p}-\mathbf{p}'} + E_{\mathbf{p}'} + E_{\mathbf{p}} - W)} + \frac{g^2}{8\pi^3} \frac{\tau^{\beta'\alpha'} \tau^{\alpha\beta} \delta_{ij} \delta_{j'i'}}{\mu (W - \mu)}; \quad (8)$$

$$\mathbf{R} = \mathbf{p}'/(E_{\mathbf{p}'} + M) - \mathbf{p}/(E_{\mathbf{p}} + M).$$

In order to pass from the transposed matrices σ^T and τ^T to the unual ones, let us pass to different wave functions. Let us make on the indices j and β the transformation

$$\varepsilon_{\lambda j} a_j = a'_{\lambda}; \quad \varepsilon_{\nu\beta} a_{\beta} = a'_{\nu}; \quad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (9)$$

Then σ^T_2 will become $\epsilon \sigma^T_2 \epsilon^{-1} = -\sigma_2$ and τ^T_2 will become $(-\tau_2)$. In the exchange term the matrices transform as follows:

$$\delta_{ij}\delta_{j'i'} \to \varepsilon_{jk}\delta_{ik}\delta_{i'm}\varepsilon^{-1}_{mj'} = {}^1\!/_2 (\sigma_1\sigma_2 - 1); \qquad \tau_{\alpha\beta}\tau_{\beta'\alpha'} \to \varepsilon_{\beta n}\tau_{\alpha n}\tau_{m\alpha'}\varepsilon^{-1}_{m\beta'} = {}^1\!/_2 (\tau_1\tau_2 + 3). \quad (10)$$

We then obtain

$$\Delta_2 = \frac{g^2}{8\pi^3} \left\{ \frac{(E_{\mathbf{p}} + M)(E_{\mathbf{p}'} + M)}{4E_{\mathbf{p}}E_{\mathbf{p}'}} \frac{\tau_1\tau_2 (\sigma_1 R)(\sigma_2 R)}{\omega_{\mathbf{p}-\mathbf{p}'} (\omega_{\mathbf{p}-\mathbf{p}'} + E_{\mathbf{p}} + E_{\mathbf{p}'} - W)} \right\} + \frac{g^2 (\sigma_1\sigma_2 - 1)(\tau_1\tau_2 + 3)}{8\pi^3\, 4\mu\, (W - \mu)}. \quad (11)$$

Then, multiplying both sides of Eq. (7) by $(W+2E_p)/(W+2M)$, introducing the binding energy $\mathcal{E}=W-2M$ and passing to the coordinate representation, we obtain the equation

$$\left(\frac{\nabla^2}{M\rho}+\mathcal{E}\right)\Phi^{01}(\mathbf{r})=\int U_2(\mathbf{r}\mathbf{r}'W)\,\Phi^{01}(\mathbf{r}')\,d\mathbf{r}', \qquad (12)$$

where $\rho=1+\mathcal{E}/4M$ and

$$U_2(\mathbf{r}\mathbf{r}'W)=\int\frac{[(2E_{\mathbf{p}}+W)(2E_{\mathbf{p}'}+W)]^{1/2}}{4M\rho}\Delta_2(\mathbf{p}\mathbf{p}'W)\,e^{i(\mathbf{pr}-\mathbf{p}'\mathbf{r}')}d\mathbf{p}\,d\mathbf{p}'. \qquad (13)$$

The quantity M_ρ plays the role of reduced mass. Passing to the nonrelativistic limit $E_\rho\sim M$, $W\sim 2M$, we obtain the equation

$$(\nabla^2/M\rho+\mathcal{E})\,\Phi^{01}(\mathbf{r})=V_2(\mathbf{r})\,\Phi^{01}(\mathbf{r}), \qquad (14)$$

where

$$V_2(\mathbf{r})=-\frac{1}{3}(\tau_1\tau_2)\frac{g^2}{4\pi}\left(\frac{\mu}{2M}\right)^2 \qquad (15)$$

$$\times\left\{\sigma_1\sigma_2+S_{12}\left[1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2}\right]\right\}\frac{e^{-\mu r}}{r}$$

$$-\frac{1}{3}(\sigma_1\sigma_2)(\tau_1\tau_2)(2M)^{-2}\,\delta(r)$$

$$+\frac{(\tau_1\tau_2+3)(\sigma_1\sigma_2-1)}{4\mu(2M-\mu)}\,\delta(r).$$

From this it is seen that the interaction potential between nucleon and antinucleon has a sign opposite to that of the interaction potential between two nucleons. The exchange term does not give a contribution in the nonrelativistic limit. Investigation in the relativistic region shows that the first term of equality (8) gives attraction at small distances, in contradistinction to the case of the two-nucleon system. The exchange term becomes large if $W\sim\mu$. This indicates that the possibility is not excluded of the formation of the bound nucleon-antinucleon system with large binding energy. This problem, which involves the relativistic region, requires a supplementary investigation. It would be interesting to investigate it also using the "new" Tamm method proposed by Dyson.

In conclusion, the author expresses his gratitude to Iu. M. Shirokov for advice and discussion of this problem.

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Angular Distribution of Fission Fragments

V. M. STRUTINSKII

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THE anisotropic angular distribution of fission fragments from fission induced by fast neutrons[1,2], protons[3] and gamma rays[4,5] has been observed experimentally. A number of general features of the angular distribution of the fragments follow from the conservation of angular momentum. Thus the principal qualitative difference of the angular distribution of fragments from nucleon-induced fission from that of photofission, which is the maximum at $\vartheta=0$, as contrasted with the maximum at $\vartheta=\pi/2$ for photofission, results from the different spin orientation of the compound nucleus. When a fast nucleon is captured, the spin of the compound nucleus is oriented predominantly in a direction perpendicular to the beam. The component of the radiation moment along the direction of the beam is ± 1; thus for a dipole radiation the spin of the compound nucleus is oriented predominantly along the beam. This difference in the orientation of the compound nucleus spin results generally in the experimentally observed shape of the angular distribution.*

With the increase of nucleon energy the anisotropy of the angular distribution must increase because of the increased angular momentum transferred to the nucleus. When the target nucleus spin I_0 differs from zero the angular distribution will be more isotropic because of the greater isotropy of spin distribution of the compound nuclei. The angular distribution of photofission fragments is especially dependent on the initial spin when the angular momentum transferred to the nucleus is relatively small. Thus for $I_0\leq 1$ the angular distribution of fission fragments due to