decrease as well. However, the measurements of Kashaev[3] show that the halfwidth increases at first with dilution, reaches a maximum at some value of $f$ and then decreases with further dilution. The increase of the halfwidth at small dilutions may not even contradict an increase in the quantity $X$ at the same time.

Indeed, such case was observed by Kumagi et al[4] who studied the change of the halfwidth of the curve with the change in separation $d$ between two neighborhood paramagnetic ions. In particular, with a decrease in $d$ (as long as the volume forces are small) the halfwidth increases with an increase in the quantity $X$ though the latter is very small. The halfwidth reaches a maximum and then, as the volume forces begin to increase rapidly, it begins to decrease.

A direct comparison of the theoretical moments with the experimental data cannot be carried out not only because the available curves are not measured on sides (only halfwidths were measured) but also because the insufficient sensitivity of the present experimental techniques makes a careful investigation of the absorption curve (particularly its sides) of solid solutions almost impossible since the absorption decreases rapidly with dilution.

In conclusion the author expresses a deep gratitude for the valuable advice and discussions to C. A. Al'tshuler and B. M. Kozyrev.

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## Nonlinear Equations in Quantum Field Theory

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(Submitted to JETP editor, October 25, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 608-609 (March, 1956)

THE purpose of the present note is to show that the equations gotten by Low[1] follow from general relations for the Green's function which were found by Lehman, Symanzik and Zimmerman[2], starting from general requirements of relativistic invariance, causality and boundary conditions. For simplicity, the proof is carried through for the example of the equation for the Green's function of the meson-nucleon system, which is simply related to the matrix element of the $S$-matrix for the scattering of mesons by nucleons.

The Green's function of the meson-nucleon system is defined in the following manner:

$$\tau(xx'; yy') = (\Psi_0, T\{\varphi(x)\varphi(y)\psi(x')\overline{\psi}(y')\}\Psi_0), \tag{1}$$

where $\Psi_0$ is the state vector of the physical vacuum in the Heisenberg representation and $\psi(x)$ and $\varphi(x)$ are the Heisenberg operators corresponding to the nucleon and meson fields. For simplicity the meson is taken to be neutral. Instead of $\tau(x\,x'; yy')$ it is convenient to consider the quantity

$$\langle p'|x, y|p\rangle = \int_{x_0\to+\infty,\, y_0\to-\infty} dx'\, dy\, \overline{u}_{p'}(x')\,\gamma_4\,\tau(xx', yy')\,\gamma_4\, u_p(y') = (\Psi_p, T\{\varphi(x)\varphi(y)\}\Psi_p). \tag{2}$$

where $u_p(x)$ is a solution of the free Dirac equation for particles with momentum p and energy $p_0 = \sqrt{p^2 + M^2}$, and $\Psi_p$ is the state vector of one nucleon in the Heisenberg representation.

If causality and the boundary conditions are employed then, analogously to what is done in Ref. 2, it is possible to obtain relations of the following form for the Green's function:

$$\begin{aligned}\langle p'|x, y|p\rangle = \sum_{n=0}^{\infty}\sum_{\lambda}(-i)^n \Big\{\eta(x-y) &\times \int K_{\xi_1}\ldots K_{\xi_n}\langle p'|x, \xi_1\ldots\xi_n|\lambda\rangle \\ &\times \Delta^{(+)}(\xi_1-\eta_1)\ldots\Delta^{(+)}(\xi_n-\eta_n)K_{\eta_1}\ldots K_{\eta_n} \\ &\langle p|y, \eta_1\ldots\eta_n|\lambda\rangle^* d\xi_1\ldots d\xi_n d\eta_1\ldots d\eta_n \\ &+ \eta(y-x)\int K_{\xi_1}\ldots K_{\xi_n}\langle p'|y, \xi_1\ldots\xi_n|\lambda\rangle\,\Delta^{(+)}(\xi_1-\eta_1)\ldots\Delta^{(+)}(\xi_n-\eta_n) \\ &\times K_{\eta_1}\ldots K_{\eta_n}\langle p|x, \eta_1\ldots\eta_n|\lambda\rangle^* d\xi_1\ldots d\xi_n d\eta_1\ldots d\eta_n\Big\}.\end{aligned} \tag{3}$$

Here $K_x = \Box_x - \mu^2$ where $\mu$ is the renormalized meson mass; * designates the Hermitian conjugate;

$$\Delta^+(x) = -i(2\pi)^{-3}\int dq\,\delta(q^2+\mu^2)\,e^{iqx}\,\eta(q)$$

is a commutator function;

$$\eta(x) = \begin{cases} 1, & x_0 > 0 \\ 0, & x_0 < 0 \end{cases};$$

and the summation in (3) is over the number of mesons $n$ and the number of nucleons and antinucleons $\lambda$.

The matrix element of the $S$-matrix for a transition of a meson from a state of momentum $q$ to a state of momentum $q'$ is related to the Green's function in the following way:

$$\langle p', q' \mid S \mid p, q \rangle \tag{4}$$
$$= -\int f^*_{q'}(x) f_q(y) K_x K_y \langle p' \mid x, y \mid p \rangle \, dx \, dy,$$

where $f_q(x) = (2q_0)^{-1/2} \exp[iqx]$ is a solution of the Klein-Gordon equation;

$$qx = \mathbf{qx} - q_0 x_0, \quad q_0 = \sqrt{\mathbf{q}^2 + \mu^2}.$$

Using this relation, and also the relations

$$\sum_{\mathbf{q}} f_q(\xi) f^*_q(\eta) = i\Delta^{(+)}(\xi - \eta);$$

$$\eta(x) = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{e^{i\alpha x}}{\alpha - i\varepsilon} \, d\alpha,$$

we immediately obtain from (3)

$$\langle p', q' \mid S \mid p, q \rangle \tag{5}$$
$$= \frac{1}{2\pi i} \sum_{\mathbf{q}_1 \ldots \mathbf{q}_n, n, \lambda} \Big\{ \int \frac{d\alpha}{\alpha - i\varepsilon} \langle p', q' + \alpha \mid S \mid \lambda, q_1 \ldots q_n \rangle$$
$$\langle \lambda, q_1 \ldots q_n \mid S \mid p, q + \alpha \rangle \cdot$$
$$+ \int \frac{d\alpha}{\alpha - i\varepsilon} \langle p', -q + \alpha \mid S \mid \lambda, q_1 \ldots q_n \rangle$$
$$\langle \lambda, q_1 \ldots q_n \mid S \mid p_1, -q' + \alpha \rangle \Big\}$$
$$+ \int dx \, dy \, \delta(x_0 - y_0) f^*_{q'}(x) f_q(y) \langle p' \mid [K_x \varphi(x), \dot{\varphi}(y)]_- \mid p \rangle.$$

Here $\langle \lambda q_1 \ldots q_n \mid S \mid p, q + \alpha \rangle$ is the matrix element of the $S$-matrix for a transition of a meson with momentum **q**, energy $q + \alpha$ and a nucleon with energy-momentum $p$ to a state with $n$ mesons and $\lambda$ nucleons and antinucleons. The last term on the right-hand side of (5) is zero if the interaction is linear in the meson field. If the sum in (5) is restricted to the states ($n = 0$, $\lambda = 0$) and ($n = 1$, $\lambda = 1$) and we go over to matrix elements on the energy surface then we get Low's equation.

We emphasize that the relation (5) with the restriction given above becomes an equality only if there is provided the inhomogeneous term ($n = 0$, $\lambda = 1$), which depends on the concrete form of the interaction. The inhomogeneous term can be found exactly only in the limit $q, q' \to 0$, therefore the applicability of the equation is essentially limited to the nonrelativistic domains of nucleon energy and meson energy. For arbitrary $q, q'$ the inhomogeneous term can be expressed in terms of the exact renormalized one-particle Green's function and vertex part. Therefore, attempts to treat the equations found by Low relativistically run into kinds of problems; first, the exact forms of the renormalized one-particle Green's function and vertex part are unknown, and secondly, as shown in the works cited,[3] the renormalized Green's function possesses nonphysical poles, reflecting the fact that the renormalized coupling constant becomes zero.[4]

*Note added in proof.* Analogous questions are considered in recently appearing work.[5]


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Translated by D. Finkelstein
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## Isomeric States of Deformed Nuclei

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(Submitted to JETP editor December 2, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 616-617 (March, 1956)

As is well known, the term "isomer" denotes an excited state of a nucleus characterized by a long lifetime. The small probability of decay of such states may be due to a large difference between their spins and that of the ground state ($\Delta I \geq 3$).[1] Besides this, the decay probability of the excited state depends substantially on the nature of the levels between which $\gamma$-transitions are taking place, and in some cases (the nuclei of $Lu^{177}$, $Ta^{181}$, $Re^{187}$, $Np^{237}$, $Pu^{239}$ and others) the decay probability turns out to be small ( and