

term in Eqs. (76) and (92), and also because of incorrect factors in front of  $V_{\alpha}^R(p)$  and  $V_{\alpha}^{R+}(l)$ .

3. The integral equations for  $g_{\alpha}(p, l)$  and  $V_{\alpha}^R(p, \epsilon)$  have an infinite upper limit of integration. The solution of these equations should be taken as limits of the solutions of these equations with a finite upper limit of integration,  $p_{\max}$ , as  $p_{\max} \rightarrow \infty$ . Then further investigation shows that the equations for  $g_{\alpha}(p, l)$  and  $V_{\alpha}^R(p, \epsilon)$  have finite solutions, behaving for large  $p$  as  $p^{\nu}$  and  $p^{\lambda}$ , respectively, where  $\nu$  and  $\lambda$  are those roots of the equations

$$-Q'g^2/16\pi^2 = F_j(\nu + 1/2), \quad -Q'g^2/16\pi^2 = F_j(\lambda),$$

$$F_{1/2}^{-1}(\lambda) = \pi [\lambda(\lambda + 1) \sin \pi\lambda]^{-1} - \lambda^{-2} \\ - (\lambda + 1)^{-2} + (\lambda^2 - 1)^{-1} + (\lambda^2 + 2\lambda)^{-1},$$

which for  $g^2 \rightarrow 0$  go over into the exponent of the first argument in the asymptotic kernel. For  $T = j = 1/2$  with  $g^2/4\pi > 3\pi(3\pi - 8)^{-1} \approx 6.61$ ,  $\nu$  and  $\lambda$  are complex, and it is possible to show that in this case the equations for  $g_{\alpha}(p, l)$  and  $V_{\alpha}^R(p, \epsilon)$  do not have solutions in the sense that the solutions of the equations with a finite upper limit do not have a well-defined limit for  $p_{\max} \rightarrow \infty$ . Thus  $g^2/4\pi$  should be less than 6.61. This limit on  $g^2/4\pi$  is not connected with the condition of normalization of the full solution, as taken in Ref. 2. In fact, from the integrability of the wave function in  $\mathbf{x}$ -space for  $\mathbf{x} = 0$ , it follows that the wave function in  $\mathbf{p}$ -space for  $p \rightarrow \infty$  should fall off faster than  $p^{-3/2}$ . However, the wave functions in  $\mathbf{p}$ -space are the functions  $\langle b_{-p\alpha} a_{p\alpha} \rangle$ , and not

$$\psi(p) = V_{\omega p}^{-1} \sum_{u^+} u \langle b_{-p\alpha} a_{p\alpha} \rangle,$$

as taken in Ref. 2. Therefore, the functions  $f(p)$  and  $g(p)$  in Ref. 2 should fall off faster than const and not faster than  $p^{-1/2}$ . But this condition does not limit  $g^2$ .

I am very thankful to Academician I.E. Tamm and his colleagues for discussions and advice.

*Note added in proof:* It is necessary to note that the removal of divergences from the "graph with absorption" are connected with a finite charge renormalization not only in the equations of the new Tamm-Dancoff method, but also in covariant equations in the approximation considered here. For the finite renormalization it is necessary that the renormalized vertex function falls off with a negative power of  $p$  for large momenta.

\* This method is essentially the same as that of Ref. 2.

\*\* This equation is obtained from the initial equation for the amplitude "meson, + nucleon" by decomposition of angle variables using spherical harmonics (see Ref. 1).

<sup>1</sup> Silin, Tamm and Fainberg, J. Exptl. Theoret. Phys. (U.S.S.R.) 29, 6 (1955); Soviet Phys. JETP 2, 3 (1956).

<sup>2</sup> R. H. Dalitz and F. J. Dyson, Phys. Rev. 99, 301 (1955).

<sup>3</sup> M. Levy, Phys. Rev. 94, 460 (1954); S. Chiba, Progr. Theor. Phys. 11, 494 (1954).

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## Two $\tau$ Mesons Detected in Photographic Emulsions

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A STACK, consisting of 126 layers of unbacked electron-sensitive emulsions of type R, thickness 450 $\mu$  and diameter 10 cm, were exposed for 7 hours at a height of 27 km. Two  $\tau$ -mesons were found in the scanning; the characteristics of these are given in the table.

One of the  $\tau$ -mesons was observed by following the track of a  $\pi^+$ -meson which had come to rest. All of the prongs of the star in which the  $\tau$ -meson was produced were followed either to their end or to the point where they left the stack. No further heavy particle decay was found among the prongs coming to their end.

Two  $\pi$ -mesons ( $\pi^+$  and  $\pi^-$ ) coming from the decay of the  $\tau$ -meson stopped; the third left the stack. The tracks of these  $\pi$ -mesons lay in a plane; coplanarity was established to an accuracy of 3°. The decay energy was  $Q = (74.2 \pm 2)$  mev, and the mass, determined from the decay scheme,  $m_{\tau} = (965 \pm 4) m_e$ . The second  $\tau$ -meson was

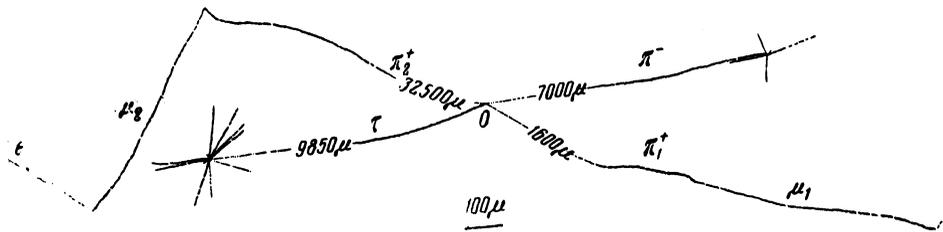
found by area-scanning. A microprojection of it is shown on the picture. All black and grey tracks from the primary star end in the stack; additional heavy unstable particles did not appear among them.

All of the  $\pi$ -mesons produced in the decay also stopped in the stack; two of them were positively

TABLE

No. of the case	Primary star	Track length of $\tau$ -meson in mm	Energy of $\tau$ -meson in mev	Secondary Particles			Decay energy $Q$ in mev	Mass of $\tau$ -meson in units $m_e$	Angles between $\tau$ -mesons in degrees	$\theta$	$\varphi$	$\psi$
				Type	Track length in mm	Energy in mev						
1	12+0n	15±0,5	50±1	$\pi$	—	18±1	74,2±2	965±4	$\alpha_{12} = 89 \pm 2$	56,5	176	59
				$\pi^-$	8,5±0,4	21,0±0,15			$\alpha_{23} = 144 \pm 3$			
				$\pi^+$	21,2±0,8	35,2±1,0			$\alpha_{31} = 126 \pm 3$			
2	14+2n 14+1p	10,5±0,3	40,8±0,8	$\pi^-$	7,7±0,3	19,8±0,4	74,9±1,4	965±3	$\alpha_{12} = 33,5 \pm 1,5$	50	42	33
				$\pi^+$	2,14±0,1	9,5±0,2			$\alpha_{23} = 159 \pm 2$			
				$\pi^+$	33,2±1,5	45,6±1,2			$\alpha_{31} = 167 \pm 2$			

Key:  $\theta$  is the angle between the direction of flight of the  $\tau$ -meson and the normal to its decay plane;  $\varphi$  is the angle between the direction of flight of the  $\tau$ -meson and that of the  $\pi^-$ -meson;  $\psi$  is the angle between the decay plane of the  $\tau$ -meson and the plane of the emulsion. Both  $\tau$ -mesons came off at a large angle with the vertical.



charged, the third, negatively. The tracks of the  $\pi$ -mesons lay in a plane to an accuracy of  $2^\circ$ . The decay energy was\*  $Q = (74.1 \pm 1.4)$  mev and the mass, determined from the decay scheme,  $m_\tau = (964 \pm 3)m_e$ . The first  $\tau$ -meson came out at a very large angle to the plane of the emulsions, which made a direct measurement of its mass impossible. The track of the second  $\tau$ -meson was gently sloping. A direct determination of its mass by multiple scattering and range led to the value  $m_\tau = (1097 \pm 105)m_e$ .

The comparatively small energy of one of the  $\pi^+$ -mesons produced in the decay of the second  $\tau$ -meson stands out. Such a case supports, as is well known (see Ref. 1), the hypothesis that the  $\tau$ -mesons and  $\chi$ -mesons are different particles, and not different modes of decay of a single particle. Such a conclusion possible follows from the analysis of nuclear disintegrations in which heavy unstable particles are produced<sup>2</sup>.

\* In determination of the decay energies of both  $\tau$ -mesons, several differences in the stopping power of emulsions of type R and type G-5 were taken into account. In order to carry out the corresponding corrections, we measured the track lengths of  $110\mu$ -mesons, produced in the decay of  $\pi$ -mesons which stopped. The mean length of track of these  $\mu$ -mesons was  $(584 \pm 2.4)\mu$ , which differs by 2% from the value in G-5 emulsions.

<sup>1</sup> R. H. Dalitz, Phys. Rev. **94**, 1046 (1954).

<sup>2</sup> Gramenitskii, Zamchalova, Podgoretskii, Tret'iakova and Shcherbakova, J. Exptl. Theoret. Phys. (U.S.S.R.) **28**, 616 (1955); Soviet Phys. JETP **1**, 562 (1955).

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## On Mass Renormalization in the Tamm-Dancoff Method

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THE present article is concerned with the consistent application of the counterterm technique for mass renormalization in the Tamm-Dancoff method (abbreviated as the TD method). Because of the mathematical difficulties that arise, we also consider a renormalization method which does not require separation of the counterterms.

Mass renormalization in the TD method is usually accomplished by separation of the counterterms in the Hamiltonian<sup>1</sup>

$$H = H_0(M_1, \mu_1) + H' = H_0(M, \mu) + H' + \delta H',$$

$$H' = i g_1 \bar{\psi}(x) \gamma_5 \tau \varphi(x) \psi(x),$$

$$\delta H' = \delta M \bar{\psi}(x) \psi(x) + \delta \mu^2 \varphi(x) \varphi(x),$$

where  $M_1$  and  $\mu_1$  are bare masses, and  $M$  and  $\mu$  are renormalized masses. The equations of motion for the amplitude  $A$  are then given by the relationship

$$i \delta A / \delta \sigma = [A, H'] + [A, \delta H'].$$

The state vector of the system is resolved into states with different numbers of free particles (but possessing renormalized mass), and all amplitudes with a number of particles smaller than the given number are taken into consideration in each approximation of the method.

I.E. Tamm has noted that there is usually some inconsistency in the manner of application of these amplitude selection rules. These rules are strictly adhered to in relation to terms arising from  $H'$  in