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On the Scattering of Photons by Nucleons

R. I. GURZHI
P. N. Lebedev Physical Institute, Academy of Sciences, USSR

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The crosss section for scattering of photons against nucleons is computed on the basis of a semiphenomenological isobaric theory including absorption.

THE fact that a nucleon is surrounded by a meson field leads to a scattering cross section for photons by nucleons, which differs from the well-known Klein-Nishina formula. By analogy with the scattering and photoproduction of mesons by nucleons, one may expect that in this case, too, the cross section will exhibit a resonance character as a function of energy. Yet, attempts to carry out corresponding calculations meet with the principal difficulties inherent in all present meson theories. But if one includes the meson field in a semiphenomenological way by introducing isobaric states[1,2], then the computations can be carried to completion, as indeed is done in the present note*.

The Lagrangian for the interaction between the nucleon and the photons has the form**:

$$L = \sqrt{4\pi}e\bar{\psi}\tau\hat{A}\psi - \sqrt{4\pi}\,\frac{e}{M}\,\overline{B}_\mu N F_{\mu\nu}\gamma_\nu\gamma_5\psi \qquad (1)$$

$$+ \sqrt{4\pi}\,\frac{e}{M}\,\bar{\psi}N^+F_{\mu\nu}\gamma_\nu\gamma_5 B_\mu;$$

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu};$$

$$A_\mu = \frac{e_\mu}{\sqrt{2\omega}}\,(ce^{ikx} + c^+e^{-ikx}),$$

where $\tau = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ is the nucleonic charge operator,

$$N = \left(\begin{array}{cc|cccc} \multicolumn{2}{c|}{0} & 0 & a & 0 & 0 \\ & & 0 & 0 & b & 0 \\ \hline 0 & 0 & & & & \\ a^+ & 0 & & & & \\ 0 & b^+ & & \multicolumn{2}{c}{0} & \\ 0 & 0 & & & & \end{array}\right)$$

is a Hermitian matrix in isotopic spin space describing transitions from ground to isobaric states, and vice versa.

The incident photon shall be characterized by the polarization vector **e** and the 4-momentum $k(\mathbf{k}, i\omega)$; we shall denote the 4-momentum of the nucleon in the initial state by $p(\mathbf{p}, iE)$. The corresponding quantities in the final state shall be denoted by e', k' and p'. The computations shall be carried out in the center-of-mass system. The secondorder matrix elements U are sums of four terms corresponding to the four possible scattering reactions (two of these are the usual ones, and the other two include an isobar in the intermediate state). We represent the nucleon wave function ψ as a product of u_p, a function of the coordinates and the mechanical spin and χ, a function of the isotopic spin; then:

$$U = \chi'^+\tau^+\tau\chi(A + B) + \chi'^+N^+N\chi(C + D); \qquad (2)$$

$$A = (\sqrt{4\pi}e)^2(2\omega)^{-1}\bar{u}_{p'}\hat{e}'(\hat{p} + \hat{k} + M)^{-1}\hat{e}u_p;$$

$$B = (\sqrt{4\pi}e)^2(2\omega)^{-1}\bar{u}_{p'}\hat{e}(\hat{p} - \hat{k}' + M)^{-1}\hat{e}'u_p;$$

$$C = \left(\sqrt{4\pi}\,\frac{e}{M}\right)^2\frac{1}{2\omega}\,\bar{u}_{p'}\,(\hat{e}'k'_\mu - e'_\mu\hat{k}')$$

$$\times K_{\mu\nu}(p + k)(\hat{e}k_\nu - e_\nu\hat{k})\,u_p;$$

$$D = \left(\sqrt{4\pi}\,\frac{e}{M}\right)^2\frac{1}{2\omega}\,\bar{u}_{p'}\,(\hat{e}k_\mu - e_\mu\hat{k})$$

$$\times K_{\mu\nu}(p - k')(\hat{e}'k'_\nu - e'_\nu\hat{k}')u_p.$$

The isobaric propagator $K_{\mu\nu}$ has the form[5]

$$K_{\mu\nu}(p) = (\hat{p} - \varkappa)^{-1}\Big\{[-i\gamma_\mu(\hat{p} + 3\varkappa)$$

$$+ 2p_\mu]\,\frac{p_l}{6\varkappa^2} + \delta_{\mu l}\Big\}\left(\delta_{l\nu} - \frac{1}{4}\gamma_l\gamma_\nu\right),$$

where $\varkappa$ is the mass of the isobar.

Inasmuch as the expressions for A, B, C and D

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* A similar calculation has already been carried out in Ref. 3. However, the author treats absorption in an approximate fashion by including it only at resonance. In the present article, absorption is treated according to the method of Heitler (see Ref. 4), following the scheme developed in Ref. 2.

** We use here Pauli's notation.

are very cumbersome, let us introduce here coefficients for expanding these quantities into the polynomials $L_{ik}^{\Phi'\Phi}$ of Ref. 6:

$$A = (\sqrt{4\pi}e)^2 \sum_{i,\,k=0}^{5} a_{ik}L_{ik}^{\Phi'\Phi}. \quad (3)$$

$$B = (\sqrt{4\pi}e)^2 \sum_{i,\,k=0}^{5} b_{ik}L_{ik}^{\Phi'\Phi},$$

$$C = \left(\sqrt{4\pi}\frac{e}{M}\right)^2 \frac{\omega}{4(\varepsilon-\varkappa)} \sum_{i,\,k=0}^{5} c_{ik}L_{ik}^{\Phi'\Phi};$$

$$D = \left(\sqrt{4\pi}\frac{e}{M}\right)^2 \frac{E+M}{8E} \sum_{i,k=0}^{5} d_{ik}L_{ik}^{\Phi'\Phi}.$$

where $E = \sqrt{M^2+\omega^2}$ is the energy of the nucleon, and $\epsilon = E + \omega$ is the total energy of the system. The nonzero coefficients of A and C are:

$$a_{00} = -\frac{E+M}{4\omega E(\varepsilon+M)}, \quad a_{11} = -\frac{1}{4E(\varepsilon+M)},$$

$$c_{00} = \frac{M^2(\varepsilon-\varkappa)(\varepsilon+2\varkappa)}{3\varkappa^2\varepsilon E};$$

$$c_{11} = \frac{M^2(\varepsilon-\varkappa)(\varepsilon-2\varkappa)}{3\varkappa^2\varepsilon E};$$

$$c_{22} = \frac{5E+3M+4\omega}{12E}; \quad c_{33} = \frac{\omega(\varepsilon-M)}{4E(\varepsilon+\varkappa)};$$

$$c_{44} = \frac{(\varepsilon-\varkappa)(5E+3M+4\omega)}{12E(\varepsilon+\varkappa)};$$

$$c_{55} = \frac{\omega^2(\varepsilon-\varkappa)}{4E(\varepsilon+\varkappa)(E+M)};$$

$$c_{23} = c_{32} = \frac{\omega(\varepsilon+E+2M)}{4\sqrt{3}E(E+M)};$$

$$c_{45} = c_{54} = -\frac{\omega(\varepsilon-\varkappa)(\varepsilon+E+2M)}{4\sqrt{3}E(\varepsilon+\varkappa)(E+M)}.$$

The coefficients b_{ik} are related as follows to the auxiliary quantities $\beta_1, \beta_2, \ldots, \beta_{10}$:

$$b_{00} = 4f_0(\beta_1 - \alpha\beta_2 - \alpha\beta_3 + \beta_5 + {}^1/_2\beta_7 + {}^1/_2\beta_{10}) + {}^1/_2\ln x(\beta_4 - \beta_3) + f_2(\beta_8 + \beta_9);$$

$$b_{11} = 4f_0(\beta_2 + \beta_4 - \alpha\beta_5 - \alpha\beta_6 + {}^1/_2\beta_8 + {}^1/_2\beta_9) + {}^1/_2\ln x(\beta_1 - \beta_6) + f_2(\beta_7 + \beta_{10});$$

$$b_{22} = 4f_0(\beta_4 - \alpha\beta_5 - 2\alpha\beta_7 + 2\beta_8 + {}^1/_2\beta_9 - \alpha\beta_{10}) - {}^1/_2 f_1(2\beta_6 + \beta_7) + {}^1/_8\ln x\beta_{10} + f_3(-\beta_1 + \alpha\beta_2) + f_5\beta_9;$$

$$b_{33} = 3f_0(-2\beta_3 + \beta_4 - \alpha\beta_5 + {}^1/_2\beta_9) + 3f_1\beta_6 + f_3(-3\beta_1 - 3\alpha\beta_2 - 2\beta_{10}) + f_4\beta_7 + f_5(4\beta_3 + {}^4/_3\beta_8 + {}^1/_3\beta_9);$$

$$b_{44} = f_0(\beta_1 - \alpha\beta_2 - 3\beta_6 - 2\alpha\beta_9 + {}^1/_2\beta_{10}) + f_1(2\beta_3 - {}^1/_2\beta_9) + f_2\beta_8 + f_3(-\beta_4 + \alpha\beta_5) - \alpha f_4\beta_7 + 2f_5\beta_6 + {}^1/_4\beta_7;$$

$$b_{55} = 3f_0(\beta_1 - \alpha\beta_2 - \beta_6 + {}^1/_2\beta_7) + f_3(-3\beta_4 + 3\alpha\beta_5 - 2\beta_8) + f_4\beta_9 + f_5(2\beta_6 + {}^1/_3\beta_7 + {}^4/_3\beta_{10});$$

$$b_{23} = b_{32} = \sqrt{3}f_0(\beta_3 - \beta_4 + \alpha\beta_5 + {}^1/_2\beta_9) + \frac{\sqrt{3}}{2}f_1\beta_7 + \sqrt{3}f_3(\beta_1 - \alpha\beta_2) - \frac{1}{\sqrt{3}}f_5(2\beta_3 + \beta_9);$$

$$b_{45} = b_{54} = \sqrt{3}f_0(\beta_1 - \alpha\beta_2 - 2\beta_6 - {}^1/_2\beta_7) + \frac{\sqrt{3}}{2}f_1(2\beta_3 - \beta_9) + \sqrt{3}f_3(-\beta_4 + \alpha\beta_5) + \frac{1}{\sqrt{3}}f_5(4\beta_6 + \beta_7),$$

where

$$f_0 = {}^1/_4\left(1 - \frac{\alpha}{2}\ln x\right); \quad f_1 = \alpha f_0 + {}^1/_8\ln x; \quad f_2 = -\alpha f_0 + {}^1/_8\ln x;$$

$$f_3 = {}^3/_2\alpha f_0 + {}^1/_{16}\ln x;$$

$$f_4 = -\frac{3}{2}\alpha f_0 + \frac{1}{16}\ln x; \quad f_5 = \frac{3}{2}\alpha^2 f_0 + \frac{1}{8}; \quad x = \frac{\alpha+1}{\alpha-1}.$$

Therefore,

$$\alpha = E/\omega; \qquad \beta_1 = 1 + \beta^{-1}; \qquad \beta_3 = 2;$$

$$\beta_4 = \beta + 1; \qquad \beta_6 = 2;$$

$$\beta_7 = -2\beta_1; \qquad \beta_8 = -2(\beta + 1);$$

$$\beta_2 = \beta_5 = \beta_9 = \beta_{10} = 0; \qquad \beta = (E + M)/\omega.$$

The coefficients d_{ik} are analogously related to the auxiliary quantities $\delta_1, \delta_2, \ldots, \delta_{10}$. Thus,

$$\alpha = (\varkappa^2 - M^2 + 2E\omega)/2\omega^2;$$

$$\delta_1 = \frac{1}{\omega}\left(\frac{\varepsilon + M}{E + M}\right)^2 \frac{M + \varkappa}{6\varkappa^2}(-M^2 + 3\varkappa M + 6\varkappa^2) - \frac{\varepsilon + M}{(E+M)^2}\left(E - \omega - \frac{ME\varepsilon}{3\varkappa^2}\right) - \frac{\omega}{3\varkappa^2(E+M)^2}(M\varepsilon^2 + 6\varkappa^2 M + 2\varkappa M^2 + 3\varkappa^3);$$

$$\delta_2 = \frac{\omega}{3\varkappa^2(E+M)^2}(M^2\varepsilon - 6\varkappa^2\varepsilon + 2\varkappa M^2 + 3\varkappa^3);$$

$$\delta_3 = -\frac{\varepsilon + M}{(E+M)^2}\frac{M + \varkappa}{3\varkappa^2}(-M^2 + 3M\varkappa + 3\varkappa^2) + \frac{2\varepsilon}{E + M};$$

$$\delta_4 = -\frac{1}{\omega}\left(\frac{\varepsilon + M}{E + M}\right)^2 \frac{M + \varkappa}{6\varkappa^2}(-M^2 + 3M\varkappa + 6\varkappa^2) - \frac{\varepsilon + M}{\omega(E+M)}\left(E - \omega + \frac{ME\varepsilon}{3\varkappa^2}\right) + \frac{1}{3\varkappa^2\omega}(M\varepsilon^2 + 6\varkappa^2 M + 2\varkappa M^2 + 3\varkappa^3);$$

$$\delta_5 = -\frac{\varepsilon + M}{E + M}\left(2 + \frac{M\varepsilon}{3\varkappa^2}\right) - \frac{1}{3\varkappa^2\omega}(-M\varepsilon^2 + 6\varkappa^2 M + 2\varkappa M^2 + 3\varkappa^3);$$

$$\delta_6 = \frac{\varepsilon + M}{E + M}\frac{M + \varkappa}{3\varkappa^2\omega}(-M^2 + 3\varkappa M + 3\varkappa^2) + \frac{2\varepsilon}{E + M};$$

$$\delta_7 = -\left(\frac{\varepsilon + M}{E + M}\right)^2 \frac{M + \varkappa}{3\varkappa^2\omega}(-M^2 + 3\varkappa M + 3\varkappa^2) - \frac{2}{3\varkappa^2(E+M)^2}[(\varepsilon + M)(M^2 E + 3\varkappa^2\omega) - \omega(M + \varkappa)(-M^2 + 3\varkappa M)];$$

$$\delta_8 = \left(\frac{\varepsilon + M}{E + M}\right)^2 \frac{M + \varkappa}{3\varkappa^2\omega}(-M^2 + 3\varkappa M + 3\varkappa^2) - \frac{\varepsilon + M}{E + M}\frac{2}{3\varkappa^2\omega}(M^2 E + 3\varkappa^2 M) - 2\frac{M + \varkappa}{3\varkappa^2\omega}(-M^2 + 3\varkappa M);$$

$$\delta_9 = \frac{2\omega}{3\varkappa^2}\frac{(\varepsilon - \varkappa)(-M^2 + 3\varkappa M)}{(E+M)^2}; \qquad \delta_{10} = \frac{2}{3\varkappa^2\omega}(\varepsilon + \varkappa)(-M^2 + 3\varkappa M).$$

In order to determine the scattering cross section including absorption, we apply Heitler's integral equation[4] which yields for the scattering of light by nucleons the following equations:

$$F(p\gamma) = U(p\gamma, p\gamma) - i\eta_q \int U(p\gamma, n^+) F(n^+)\, d\Omega - i\eta_q \int U(p\gamma, p^0) F(p^0)\, d\Omega - i\eta_\gamma \int U(p\gamma, p\gamma) F(p\gamma)\, d\Omega;$$

$$F(n^+) = U(n^+, p\gamma) - i\eta_q \int U(n^+, n^+) F(n^+)\, d\Omega - i\eta_q \int U(n^+, p^0) F(p^0)\, d\Omega - i\eta_\gamma \int U(n^+, p\gamma) F(p\gamma)\, d\Omega;$$

$$F(p^0) = U(p^0, p\gamma) - i\eta_q \int U(p_0, n^+) F(n^+)\, d\Omega - i\eta_q \int U(p^0, p^0) F(p^0)\, d\Omega - i\eta_\gamma \int U(p^0, p\gamma) F(p\gamma)\, d\Omega,$$

where

$$\eta_q = \frac{q\sqrt{q^2 + \mu^2}\sqrt{q^2 + M^2}}{8\pi^2(\sqrt{q^2 + \mu^2} + \sqrt{q^2 + M^2})},$$

$$\eta_\gamma = \frac{\omega^2 E}{8\pi^2(\omega + E)};$$

and where q is the momentum of the meson.

Here, for example, $U(n^+, p\gamma)$ and $F(n^+)$ are the matrix element and amplitude for the photoproduction process $\gamma + p \to n + \pi^+$. These equations can be solved by expanding the F's and the U's in the orthogonal polynomials $L_{ik}^{\Phi'\Phi}$, $L_{ik}^{M\Phi}$ and $L_k^{M'M}$ of Refs. 2 and 6. A simple calculation yields the result

$$F(p\gamma) = \sum_{i,\,k=0}^{5} f_{ik} L_{ik}^{\Phi'\Phi};$$

$$f_{ik} = \frac{u_{ik}}{1 + 4\pi\eta_q\left(u_{(k)}^{1/2} + u_{(k)}^{3/2}\right)},$$

where (k) is an integer equal to either $(k+1)/2$ or $(k+2)/2$, and by means of Eqs. (1)-(3),

$$u_{ik} = (\sqrt{4\pi}e)^2\left[a_{ik} + b_{ik} + \left(\frac{a}{M}\right)^2\left(\frac{\omega}{4(\varepsilon - \varkappa)}c_{ik} + \frac{E+M}{8E}d_{ik}\right)\right];$$

where $u_k^{1/2}$ and $u_k^{3/2}$ are the expansion coefficients of the meson scattering matrix for total isotopic spin $I = 1/2$ and $I = 3/2$ (see Ref. 2). The differential cross section in the center-of-mass system is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2}\frac{\omega^2 E^2}{\varepsilon^2}\frac{1}{4}\sum_{e'e}\mathrm{Sp}\,|F|^2,$$

where

$$\frac{1}{4}\sum_{e'e}\mathrm{Sp}\,|F|^2 = M_0 + M_1\cos\vartheta + M_2\frac{1}{2}(3\cos^2\vartheta - 1) + M_3\cos^3\vartheta,$$

and where ϑ is the angle between **k** and **k**′, and where

$$M_0 = 2(\varphi_{00,00} + \varphi_{11,11}) + 4(\varphi_{22,22} + \varphi_{33,33} + \varphi_{44,44} + \varphi_{55,55}) + 8(\varphi_{23,23} + \varphi_{45,45})$$

$$M_1 = 2(2\varphi_{00,11} + \varphi_{00,22} + 3\varphi_{00,33} + \varphi_{11,44} + 3\varphi_{11,55} + 5\varphi_{22,44} - 3\varphi_{22,55} - 3\varphi_{33,44} - 3\varphi_{33,55}) + 2\sqrt{3}(\varphi_{00,23} - \varphi_{22,45} + \varphi_{11,45} - 3\varphi_{44,54} - \varphi_{44,23} - 3\varphi_{55,23});$$

$$M_2 = \varphi_{22,22} + \varphi_{33,33} + \varphi_{44,44} + \varphi_{55,55} + 4(\varphi_{23,23} + \varphi_{45,45}) + 2(\varphi_{00,44} + 3\varphi_{00,55} + \varphi_{11,22} + 3\varphi_{11,33} + 3\varphi_{22,33} + 3\varphi_{44,55}) + 4\sqrt{3}(\varphi_{00,45} + \varphi_{11,23} - \varphi_{22,23} + \varphi_{33,23} - \varphi_{44,45} + \varphi_{55,45});$$

$$M_3 = 4(3\varphi_{22,55} + 3\varphi_{33,44} + 4\varphi_{33,55} + 6\varphi_{23,45}) + 16\sqrt{3}(\varphi_{33,45} + \varphi_{55,23});$$

$$\varphi_{ik,\,lm} = \frac{1}{2}(f_{ik}^{+}f_{lm} + f_{lm}^{+}f_{ik}).$$

Integrating over angles, we finally obtain the total scattering cross section for the scattering of light against protons.

$$\sigma = (\omega^2 E^2/\pi\varepsilon^2)\,M_0.$$

Numerical computations were carried out with this formula. The values of $u_k^{1/2,\,3/2}$ were taken from Ref. 2, and $a = 1.61$ in accordance with Ref. 6. The following values of σ were obtained as a function of ω (in the laboratory system):

ω (mev)	280	340	400
$\sigma \cdot 10^{-31}$	11.6	21.3	20.3

As could be expected, the cross section has a maximum at $\epsilon = \varkappa$ in the center-of-mass system, corresponding to $\omega = 340$ mev in the laboratory system.

After obtaining the above results, the author found out that an analogous computation had been carried out by V. I. Ritus. The methods of solution differ, however, and so do the numerical results (which may be due to the choice of constants).

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