Energy Dependence of the Fission Width

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At excitation energies of the compound nucleus lying below the fission threshold E_f, the variation of the fission width with energy is determined mainly by the barrier factor. Near threshold the barrier factor has the form $e^{-\Delta E/\epsilon}$, where $\Delta E = E_f - E$. Theoretical estimates[1] give $\epsilon \sim 100$ kev; experimental data agree with this value. Thus, right up to $E = E_f$ the fi ssion width Γ_f is a rapidly increasing function of energy. For $E = E_f$ the tunnel effect largely ceases, and in the region $E > E_f$ the behavior of the fission width is determined by other factors. The statistical theory of the fission width allows one – at least qualitatively– to analyze the dependence of $\Gamma_f(E)$ in this region. We[2] obtained previously the following estimate*:

$$\Gamma_f \sim (\hbar\omega / 2\pi)\, N^*(E - E_f) / N^*(E). \qquad (1)$$

Here ω is the frequency of the vibrations of the nuclear shape which are related to the fission, N^* is the number of levels not connected with the fission degrees of freedom. By $N^*(E)$ the number of levels with excitation energy less than E is indicated.

As shown by Landau[4] the density of levels with a given angular momentum goes basically as e^S, where $S = S(E)$ is the entropy. Therefore, the behavior of the denominator $N^*(N)$ is determined by the factor e^S, that is,it increases monotonically. As concerns the numerator $N^*(E - E_f)$, in the immediate region of the fission threshold it is impossible to apply the exponential factor to it. The formula e^S is valid for the number of levels only if the total number of levels is large and the distance between them small compared with the excitation energy. Near the threshold the number of levels $N^*(E - E_f)$ is the order of unity and the excitation energy $E - E_f$ cannot be considered large in comparison to the distance between levels with the same angular momentum.

Near threshold, the function $N^*(E - E_f)$ has a step-like character. Immediately after the point $E = E_f$ the interval $E - E_f$ contains only one level with the angular momentum considered, i.e., $N^*(E - E_f = 1$. In this region the fission width falls according to the law e^{-S}. Upon further increasing the energy, a second level with the angular momentum considered falls into the interval $E - E_f$, upon which the magnitude of $N^*(E - E_f)$ increases suddently by a factor of 2, and the fission width also increases suddenly by a factor of 2. Then, there follows again a region in which the fission width drops according to the law e^{-S} until a third level occurs in the interval $E - E_f$, etc.

Thus, the dependence of the fission width on energy near to threshold is not monotonic. Of course, the picture described above of the step-like behavior of the fission width is idealized. In fact, the true curve $\Gamma_f(E)$ is found to be smoothed out on account of the tunnel effect and because of participation in the formation of the compound nucleus of neutrons with different angular momentum. It is necessary to take into account the fact that the neutrons used in the experiment are possibly not monochromatic. All the same, there is no reason to think that these factors lead to a complete smoothing out of the non-monotonic behavior of the fission width. For small excitation energies the distance between levels of the same angular momentum can reach ~ 1 mev and the factors enumerated above are not sufficient, generally speaking, to smooth out completely such non-monotonic features of the behavior of the fission width which occur so far apart. As the excitation energy increases these non-monotonic features become more frequent and less pronounced and for $E - E_f$ of the order of several mev they are almost completely smoothed out.

The non-monotonic behavior of the energy-dependence of the fission width near threshold should say something about the behavior of the fission cross section in this region. In fact, for the fission cross section for neutrons in the range $E_n \sim 1$ mev and higher we have

$$\sigma_f \sim \sigma_c \Gamma_f / \Gamma, \qquad (2)$$

where σ_c is the cross section for formation of the compound nucleus and Γ is the total width. In the region of interest the widths Γ_f and $\Gamma - \Gamma_f$ are apparently of the same order of magnitude and, consequently, the non-monotonic behavior of the function $\Gamma_f(E)$ leads to a non-monotonic behavior in the fission cross section $\sigma_f(E)$. For nuclei for which the fission by neutrons has a threshold character this circumstance is confirmed experimentally.[5]

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*The first estimate of the fission width was given by Bohr and Wheeler[3] who, starting from classical considerations, obtained $\Gamma_f \sim (D/2)\, N^* (E - E_f)$. As indicated in the previous note,[2] for $T \gg \hbar\omega$ the expression, Eq. (1), goes over into the formula of Bohr and Wheeler.

[1] S. Frankel and N. Metropolis, Phys. Rev. 72, 914 (1947).

[2] V. Nosov, J. Exptl. Theoret. Phys. (U.S.S.R.) 29, 880 (1955); Sov. Phys. JETP 2, 746 (1956).

[3] N. Bohr and J. Wheeler, Phys. Rev. 56, 426 (1939).

[4] L. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) 7, 819 (1937).

[5] D. Hughes and J. Harvey, *Neutron cross sections*, New York (1955).

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Note on the Theorem of Pauli on the Relation of Spin and Statistics

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THE proof of Pauli[1] is based on a consideration of the irreducible representations of tensor and spinor quantities under transformations of the Lorentz group with determinant equal to unity. Schwinger[2] noted the connection of this theorem with the transformation of quantities under time reversal. We here give proof of the theorem of Pauli which shows that it is sufficient to restrict consideration to the transformation of quantities under inversion of all four coordinate axes (general inversion I) and which emphasizes the close connection of Schwinger's idea with the ideas of Pauli.

Under the transformation of general inversion I, we have for an arbitrary vector A

$$IA = -A, \tag{1}$$

from which it follows that under general inversion, tensors of even rank T_{2n+1} change sign. We say that a quantity belongs to the (+) class if it does not change under inversion I, and belongs to the (−) class if it changes sign. The operation of complex conjugation obviously does not change the class of tensor quantities.

We now consider the transformation of spinors under general inversion. Without restricting the generality, we can consider here and below only spinors of the first rank. As is well-known, upon reflection relative to a two-dimensional plane with normal vector a_k, the spinor U transforms according to the law*

$$U' = \hat{a}U, \quad \overline{U}' = \overline{U}\hat{a}, \qquad \hat{a} = i a_k \gamma_5 \gamma_k, \quad \overline{U} = U^* \gamma_4. \tag{2}$$

We use the notation $\hat{a}^2 = a_k a_k$. We speak about space reflections if $a_k a_k = 1$, and about time reflections if $a_k a_k = -1$. It is easy to show that the bi-linear quantities composed of U and $\overline{U}$, which behave as tensors under spatial reflections, are pseudo-tensors under time reflections. Thus, for example, the scalar $(\overline{U}U)$ goes into $(\overline{U}\hat{a}^2 U)$, and the vector $(\overline{U}\gamma_k U)$ goes into

$$(\overline{U}\hat{a}\gamma_k\hat{a}U) = \hat{a}^2(\overline{U}\gamma_k U) - 2a_k a_i (\overline{U}\gamma_i U).$$

It is possible, however, so to change the definition of the laws of reflection of spinors that the bilinear tensors which are constructed from them behave as tensors not only for space reflections, but also for time reflections. This can be achieved if, in extending the concept of complex conjugation, two quantities, U^* and $-U^*$, are put into correspondence with each spinor U. For this we introduce, in analogy with the theory of functions of a complex variable, a "two sheet" space of spinors, where we arrange that the transformations which do not change the sign of the time, leave the spinor on the same sheet, and the transformations which change the sign of the time carry the spinor to the second sheet. The conjugate spinor U^* is

$$\overset{*}{U} = U^* \quad \text{on the first sheet,} \qquad \overset{*}{U} = -U^* \quad \text{on the second sheet.} \tag{3}$$

(The asterisk on the right indicates the usual complex conjugation.) The dual spinor $\overline{U}$ is defined by the equality