behavior in the fission cross section $\sigma_f(E)$. For nuclei for which the fission by neutrons has a threshold character this circumstance is confirmed experimentally.[5]

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*The first estimate of the fission width was given by Bohr and Wheeler[3] who, starting from classical considerations, obtained $\Gamma_f \sim (D/2)\, N^* (E - E_f)$. As indicated in the previous note,[2] for $T >> \hbar\omega$ the expression, Eq. (1), goes over into the formula of Bohr and Wheeler.

[1] S. Frankel and N. Metropolis, Phys. Rev. 72, 914 (1947).

[2] V. Nosov, J. Exptl. Theoret. Phys. (U.S.S.R.) 29, 880 (1955); Sov. Phys. JETP 2, 746 (1956).

[3] N. Bohr and J. Wheeler, Phys. Rev. 56, 426 (1939).

[4] L. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) 7, 819 (1937).

[5] D. Hughes and J. Harvey, *Neutron cross sections*, New York (1955).

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Note on the Theorem of Pauli on the Relation of Spin and Statistics

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THE proof of Pauli[1] is based on a consideration of the irreducible representations of tensor and spinor quantities under transformations of the Lorentz group with determinant equal to unity. Schwinger[2] noted the connection of this theorem with the transformation of quantities under time reversal. We here give proof of the theorem of Pauli which shows that it is sufficient to restrict consideration to the transformation of quantities under inversion of all four coordinate axes (general inversion I) and which emphasizes the close connection of Schwinger's idea with the ideas of Pauli.

Under the transformation of general inversion I, we have for an arbitrary vector A

$$IA = -A, \tag{1}$$

from which it follows that under general inversion, tensors of even rank T_{2n+1} change sign. We say that a quantity belongs to the (+) class if it does not change under inversion I, and belongs to the (−) class if it changes sign. The operation of complex conjugation obviously does not change the class of tensor quantities.

We now consider the transformation of spinors under general inversion. Without restricting the generality, we can consider here and below only spinors of the first rank. As is well-known, upon reflection relative to a two-dimensional plane with normal vector a_k, the spinor U transforms according to the law*

$$U' = \hat{a}U, \quad \bar{U}' = \bar{U}\hat{a}, \qquad \hat{a} = i a_k \gamma_5 \gamma_k, \quad \bar{U} = U^* \gamma_4. \tag{2}$$

We use the notation $\hat{a}^2 = a_k a_k$. We speak about space reflections if $a_k a_k = 1$, and about time reflections if $a_k a_k = -1$. It is easy to show that the bi-linear quantities composed of U and $\bar{U}$, which behave as tensors under spatial reflections, are pseudo-tensors under time reflections. Thus, for example, the scalar $(\bar{U}U)$ goes into $(\bar{U}\hat{a}^2 U)$, and the vector $(\bar{U}\gamma_k U)$ goes into

$$(\bar{U}\hat{a}\gamma_k\hat{a}U) = \hat{a}^2(\bar{U}\gamma_k U) - 2a_k a_i (\bar{U}\gamma_i U).$$

It is possible, however, so to change the definition of the laws of reflection of spinors that the bilinear tensors which are constructed from them behave as tensors not only for space reflections, but also for time reflections. This can be achieved if, in extending the concept of complex conjugation, two quantities, U^* and $-U^*$, are put into correspondence with each spinor U. For this we introduce, in analogy with the theory of functions of a complex variable, a "two sheet" space of spinors, where we arrange that the transformations which do not change the sign of the time, leave the spinor on the same sheet, and the transformations which change the sign of the time carry the spinor to the second sheet. The conjugate spinor U^* is

$$\overset{*}{U} = U^* \quad \text{on the first sheet,}$$
$$\overset{*}{U} = -U^* \quad \text{on the second sheet.} \tag{3}$$

(The asterisk on the right indicates the usual complex conjugation.) The dual spinor $\bar{U}$ is defined by the equality

$$\overline{U} = \overset{*}{U}\gamma_4. \quad (4)$$

If now, as above, we define the transformation of the spinor U by the equality (2), then the conjugate spinor transforms according to the law:

$$(\overset{*}{U})' = (\overset{*}{\hat{a}U}) = \overset{*}{U}\hat{a}^+ \quad (\text{if } a_k \text{ is a space-like vector}),$$

$$(\overset{*}{U})' = (\overset{*}{\hat{a}U}) = -\overset{*}{U}\hat{a}^+ \quad (\text{if } a_k \text{ is a time-like vector}). \quad (5)$$

With this extension of the concept of conjugate spinor the bi-linear tensors constructed from U and $\overline{U}$ behave in the same way under both space and time reflections.

The transformation of inversion is a reflection of all four coordinate axes. Consequently, the matrix of inversion I in our representation is $i\gamma_5$:

$$IU = i\gamma_5 U. \quad (6)$$

The conjugate spinor transforms according to the law

$$I\overset{*}{U} = (\overset{*}{IU}). \quad (7)$$

Suppose, for definiteness, that the spinor U lies on the first sheet. Then, according to (3) and (3′) the formula (7) becomes

$$IU^* = -(IU)^* = iU^*\gamma_5. \quad (7')$$

Since $(i\gamma_5)^2 = -1$, the eigen spinors of the inversion operator can belong to two classes: to the class (+) if, upon inversion, they are multiplied by $+i$, and to the class (−) if they are multiplied by $-i$. The formulas (6) and (7′) show that the complex conjugate spinors fall into the same class.

We turn now to consideration of the commutators for tensor and spinor quantities. Subjecting the known expression for the commutator of tensor quantities $A(x)$ to the inversion I

$$[A(x'),\ A^*(x'')]_\pm = P_n(\partial/\partial x)\,\Delta(x'-x''), \quad (8)$$

taking into account that $\Delta(-x) = -\Delta(x)$ and that the quantities A and A^* belong to the same class, we obtain the relation

$$P_n(-\partial/\partial x) = P_n(\partial/\partial x). \quad (9)$$

Consequently, $P_n(\partial/\partial x)$ is an even polynomial.

Considering the commutators for spinors, and taking into account the fact that U and U^* belong to the same class, we come to the conclusion

$$[U(x'),\ U^*(x'')]_\pm = P_{2n+1}(\partial/\partial x)\,\Delta(x'-x''), \quad (10)$$

where $P_{2n+1}(\partial/\partial x)$ is an odd polynomial.

We will suppose that A and U are expanded in plane waves. Since the equation connecting the field components should be invariant under the transformation of general inversion, they should have the form

$$\Sigma kA^{(+)} = \Sigma A^{(-)}; \qquad \Sigma kA^{(-)} = \Sigma A^{(+)};$$
$$\Sigma kU^{(+)} = \Sigma U^{(-)}; \qquad \Sigma kU^{(-)} = \Sigma U^{(+)} \quad (11)$$

(we use Pauli's notation).

From these equations it follows that—together with the solutions of Eq. (11) $A^{(+)}, A^{(-)}(U^{(+)}, U^{(-)})$, corresponding to the value k – there exist $A^{(+)}, -A^{(-)}(U^{(+)}, -U^{(-)})$, corresponding to the value $-k$.

We construct the tensors T_{2n} and T_{2n+1} of even and odd rank, respectively, depending on k and depending quadratically on $A^{(\pm)}, A^{*(\pm)}$ or $U^{(\pm)}, U^{*(\pm)}$. We require that the tensors T_{2n} and T_{2n+1} are gauge-invariant and that they behave in the proper way under general inversion. Following Pauli's notation, we have, in the case of spinor quantities,

$$T_{2n} = U^{(+)}U^{*(-)} + U^{*(+)}U^{(-)} + k\,(U^{(+)}U^{*(+)} + U^{(-)}U^{*(-)});$$
$$T_{2n+1} = k\,(U^{(+)}U^{*(-)} + U^{*(+)}U^{(-)}) + U^{(+)}U^{*(+)} + U^{(-)}U^{*(-)}. \quad (12)$$

We show that the quantity $T_{2n}(U)$ cannot be positive. In fact, let it be positive for some $U(k) = U^{(+)}(k) + U^{(-)}(k)$. There exists, however, also a solution of Eq. (11) for which $U(-k) = U^{(+)}(k) - U^{(-)}(k)$. For this solution T_{2n} changes sign with the substitution of $-k$ for k.

Turning now to the quantities $T_{2n}(A)$, $T_{2n+1}(A)$ composed of tensor quantities, we have

$$T_{2n}(A) = A^{(+)}A^{*(+)} + A^{(-)}A^{*(-)} + k\,(A^{(+)}A^{*(-)} + A^{(-)}A^{*(+)});$$
$$T_{2n+1}(A) = k\,(A^{(+)}A^{*(+)} + A^{(-)}A^{*(-)}) + A^{(+)}A^{*(-)} + A^{(-)}A^{*(+)}.$$

Repeating the same considerations, we find that the quantity $T_{2n+1}(A)$ cannot be positive.

Considering only general inversion, we have shown that in the case of a spinor field the energy density cannot be positive definite, and in the case of a tensor field the energy density cannot be positive definite. From these premises and from the expressions, Eqs. (8) and (10), Pauli constructed the proof of his theorem.

[1] W. Pauli, Phys. Rev. 58, 716 (1940) (See *The Relativistic Theory of Elementary Particles* (Russian translation), IIL 1947, appendix)

[2] J. Schwinger, Phys. Rev. 82, 914 (1951) (See *Recent Developments in Quantum Electrodynamics* (Russian translation), IIL 1954, p. 133).

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A Possible Application of Cyclotron Resonance to Mass Spectrometry

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Cyclotron resonance, which was theoretically predicted by Dorfman[1] and has been observed experimentally in semiconductors,[2] consists of absorption of microwave power by an assemblage of electrons or holes upon which a magnetic field H is acting. The absorption takes place at a frequency

$$\nu = eH/2\pi mc, \qquad (1)$$

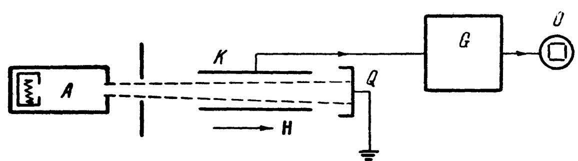
equal to the frequency of rotation of the carriers in the field H and is due to electric dipole transitions[3] whose possibility is controlled by the motion of the charges in quantum orbits. The electric dipole transitions are induced by the electric vector E_ν of the microwave field; the transition probability is a maximum when E_ν is perpendicular to H.

In comparison with paramagnetic resonance, cyclotron resonance absorption is distinguished by its remarkably large intensity. The electric dipole transition probability is approximately 10^{12} times larger than the probability of the magnetic dipole transitions in paramagnetic resonance.[4] Consequently the observation of cyclotron resonance requires a carrier concentration 10^{12} times smaller than is necessary for the observation of paramagnetic resonance. In view of the sensitivity of modern radiospectroscopic techniques it should be possible[4] to observe cyclotron resonance in a gas of free electrons at about 10^4 electrons per cm^3. Cyclotron resonance absorption must occur also in the space charge formed by ions of this or that substance; the relative magnitude of this absorption[3] must be 10^3 to 10^4 times less than for an electron gas under the same conditions of concentration and supplied *rf* power.

The condition that the effect be observable is that $\omega\tau > 1$. It follows from the requirement that the carrier be able to execute at least one complete rotation in the time between successive collisions.

Cyclotron resonance may find application in gas analysis, in studying the properties of plasma (e.g., in measuring ion mobilities and collision times), and in mass spectrometry.

The principle of operation of a possible mass spectrometer is similar to that of the *rf* mass spectrometer propsed by Hipple *et al.*[6] and consists of measuring the cyclotron frequencies of ions of the various masses and subsequently computing m by formula (1). A schematic diagram



Schematic diagram of apparatus.

of the apparatus is shown in Fig. 1. The ion source A sends an unfocused ion beam of low energy into the condenser K which forms part of the resonant circuit of the high-frequency tunable oscillator G. In the field H, parallel to the faces of the condenser, the ions move along helical paths, whose projections upon a plane perpendicular to H are circles of radius $\rho = mcv_\perp / eH$, where $v_\perp$ is the projection of the ion velocity upon this same plane. Even if an ion executes only a single rotation during its flight through the condenser absorption of high-frequency power must be observable at the frequency (1). By modulating the field or the oscillator frequency it is possible by the usual means to display the resonance curve on an oscilloscope or to register it with a recorder. After