[2] G. Askar'ian, J. Exptl. Theoret. Phys. (U.S.S.R.) **28**, 636 (1955); Soviet Phys. JETP **1**, 571 (1955).

[3] P. Argan and A. Gigli, Nuovo Cimento **8**, 5 1171 (1956).

[4] F. F. Wolkenstein, *Breakdown of Liquid Dielectrics*, ONTI, Moscow, 1934.

Translated by R. L. Eisner
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Consequences of Renormalizability of Pseudoscalar Meson Theory with Two Interaction Constants

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THE consequences of renormalizability of the pseudoscalar meson theory with two interaction constants obtained by Shirkov[1] are conveniently formulated as in Ref. 2.

The starting equations are again

$$\alpha(g_0^2, \lambda_0, \xi - L) = \alpha_c(g_c^2, \lambda_c, \xi) / \alpha_c(g_c^2, \lambda_c, L), \quad (1)$$
$$\beta(g_0^2, \lambda_0, \xi - L) = \beta_c(g_c^2, \lambda_c, \xi) / \beta_c(g_c^2, \lambda_c, L),$$
$$d(g_0^2, \lambda_0, \xi - L) = d_c(g_c^2, \lambda_c, \xi) / d_c(g_c^2, \lambda_c, L),$$
$$\mathscr{P}(g_0^2, \lambda_0, \xi - L) = \mathscr{P}_c(g_c^2, \lambda_c, \xi) / \mathscr{P}_c(g_c^2, \lambda_c, L).$$

One then introduces two effective charges $g(\xi)$ and $\lambda(\xi)$:

$$g^2(\xi) = g_0^2\alpha^2(g_0^2, \lambda_0, \xi - L) \quad (2)$$
$$\times \beta^2(g_0^2, \lambda_0, \xi - L)\, d(g_0^2, \lambda_0, \xi - L)$$
$$= g_c^2\alpha_c^2(g_c^2, \lambda_c, \xi)\,\beta_c^2(g_c^2, \lambda_c, \xi)\, d_c(g_c^2, \lambda_c, \xi),$$
$$\lambda(\xi) = \lambda_0 d^2(g_0^2, \lambda_0, \xi - L)\,\mathscr{P}(g_0^2, \lambda_0, \xi - L)$$
$$= \lambda_c d_c^2(g_c^2, \lambda_c, \xi)\,\mathscr{P}_c(g_c^2, \lambda_c, \xi).$$

The notation is the same as in Ref. 2. $\mathscr{P}$ is a quantity whose dependence on the meson-meson scattering amplitude P is given by[3]:

$$\mathscr{P} = (g_0^2 / 4\pi\lambda_0)\, P, \qquad \mathscr{P}_c = (g_c^2 / 4\pi\lambda_c)\, P_c,$$

λ_0 is a constant which appears in front of the Hamiltonian interaction terms*

$$(\lambda_0/4!)\,(\delta_{\tau_1\tau_2}\delta_{\tau_3\tau_4} + \delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} + \delta_{\tau_1\tau_4}\delta_{\tau_2\tau_3})\,\varphi_{\tau_1}\varphi_{\tau_2}\varphi_{\tau_3}\varphi_{\tau_4}.$$

The equivalence of the determination of the effective charges from either nonrenormalized or renormalized quantities follows from Eq. (1) and from the relations which exist between renormalized and nonrenormalized constants

$$g_0^2 = g_c^2\alpha_c^2(g_c^2, \lambda_c, L)\,\beta_c^2(g_c^2, \lambda_c, L)\, d_c(g_c^2, \lambda_c, L), \quad (3)$$
$$\lambda_0 = \lambda_c d_c^2(g_c^2, \lambda_c, L)\,\mathscr{P}_c(g_c^2, \lambda_c, L).$$

This is fundamentally confirmed by the fact that the logarithmic derivatives of α, β, d and $\mathscr{P}$ with respect to ξ are found to be solely dependent on the effective charges** g^2 and λ

$$(4)$$
$$\alpha'/\alpha = \alpha_c'/\alpha_c = F_1(g^2, \lambda), \quad \beta'/\beta = \beta_c'/\beta_c = F_2(g^2, \lambda),$$
$$d'/d = d_c'/d_c = F_3(g^2, \lambda), \quad \mathscr{P}'/\mathscr{P} = \mathscr{P}_c'/\mathscr{P}_c = F_4(g^2, \lambda).$$

Let us derive, for example, the last of Eqs. (4). The quantities g_c^2 and λ_c which appear in $\mathscr{P}_c'$ and $\mathscr{P}_c$ can be expressed in terms of g^2, λ and ξ by means of Eq. (2). Carrying this out, one obtains

$$\mathscr{P}'(g_0^2, \lambda_0, \xi - L) / \mathscr{P}(g_0^2, \lambda_0, \xi - L)$$
$$= F_4[g^2(g_0^2, \lambda_0, \xi - L), \lambda(g_0^2, \lambda_0, \xi - L), \xi].$$

This equation can only be satisfied if F_4 is not an explicit function of ξ; otherwise, if g_0^2 and λ_0 were kept constant, and ξ and L were varied, keeping their difference fixed, then ξ would be the sole variable appearing explicitly in F_4 only.

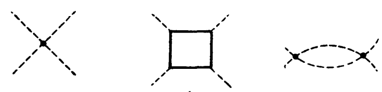
According to Eqs. (2) and (4), the logarithmic derivatives of the effective charges become

$$(g^2)'/g^2 = 2F_1(g^2, \lambda) + 2F_2(g^2, \lambda) \quad (5)$$
$$+ F_3(g^2, \lambda), \quad \lambda'/\lambda = F_4(g^2, \lambda) + 2F_3(g^2, \lambda).$$

If the functions F are known, then Eqs. (5) define a system of differential equations which, together with the boundary conditions $g^2(L) = g_0^2$, $\lambda(L) = \lambda_0$, fully determine the effective charges $g^2(\xi)$ and $\lambda(\xi)$.

The functions F may be obtained from perturbation theory which applies if g_0^2, $\lambda_0 \ll 1$ and ξ is close to L, so that $g_0^2(L - \xi)$ and $\lambda_0(L - \xi)$ are small with respect to unity. We shall consider the most general case, when λ_0 and g_0^2 are of the same order of magnitude. The solution obtained in this case is also valid when one of these quantities is small with respect to the other ($g_0^2 \ll \lambda_0$ or $\lambda_0 \ll g_0^2$), since one may then simply neglect the small quantity with respect to the large one. We shall limit ourselves to the asymptotic solutions in the zero approximation, i.e., we shall consider all quantities as functions of only $g_0^2(\xi - L)$ and $\lambda_0(\xi - L)$ or, what amounts to the same thing, $g_0^2 \times (\xi - L)$ and g_0^2/λ_0. Perturbation theory calculations indicate that α, β, and d do not contain any terms in $\lambda_0(\xi - L)$, and we can therefore apply the results of Ref. 2 which give F_1, F_2, F_3 and the asymptotic forms of α, β and d, and the effective charge $g^2(\xi)$. Perturbation theory yields for $\lambda_0 \mathcal{P}$:

$$\lambda_0 \mathcal{P} = \lambda_0 - (g_0^4/\pi^2)(\xi - L) + {}^{11}/_2 \lambda_0^2 (\xi - L).$$



a b c

The three terms of this formula correspond to the diagrams *a, b, c*. In order to obtain the function F_4 it is necessary to compute $\mathcal{P}'/\mathcal{P}|_{\xi \to L}$ in the last formula and in the resulting expression substitute g_0^2 for g^2 and λ_0 for λ; this yields

$$F_4 = 11\lambda/2 - g^2/\pi^2\lambda. \qquad (4')$$

Applying Eq. (4′) and Eqs. (3′) and (4) of Ref. 2 to Eq. (5) one finds

$$\frac{\lambda'}{\lambda} = \frac{11}{2}\lambda - \frac{g_0^4}{\pi^2 \lambda Q^2} + \frac{2g_0^2}{\pi Q}, \qquad Q = 1 + \frac{5g_0^2}{4\pi}(L - \xi). \qquad (6)$$

Carrying out the substitution

$$\lambda(\xi) = (g_0^2/4\pi)\, \mathcal{P}(x)\, d^2(x), \quad x = Q^{3/5}, \qquad (7)$$

we obtain the equation for the meson-meson scattering amplitude P.

$$dP/dx = {}^{16}/_3 - {}^{11}/_6 (P/x)^2$$

with the boundary condition $P(1) = 4\pi\lambda_0 g_0^2$. The solution of Eq. (7) is

$$P = \frac{16}{11} x \frac{B - x^{-19/3}}{B + ({}^8/_{11}) x^{-19/3}}; \qquad (8)$$

$$B = \left(1 + \frac{1}{2}\frac{4\pi\lambda_0}{g_0^2}\right) \Big/ \left(1 - \frac{11}{16}\frac{4\pi\lambda_0}{g_0^2}\right),$$

which coincides with the results obtained in Ref. 3 for $\lambda_0 = 0$ ($B = 1$).

The present investigation may be used to compute asymptotic solutions to any order in g^2, λ from perturbation theory results to the same order.

In conclusion, the author would like to thank I. Ia. Pomeranchuk and V. B. Berestetskii.

*The symmetric theory is considered here.

**The expressions for the logarithmic derivatives of renormalized quantities are obtained from Eq. (1).

[1] D. V. Shirkov, Dokl. Akad. Nauk SSSR 105, 972(1955).

[2] V. V. Sudakov, J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 729 (1956).

[3] I. T. Diatlov and K. A. Ter-Martirosian, J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 416 (1956); Soviet Phys. JETP 3, 454 (1956).

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Excitation of Nuclear Vibrational Levels by Scattering of Fast Neutrons

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It is shown in Ref. 1 that there exists a large group of even-even nuclei in the interval $36 \leq N \leq 88$, whose first two excited levels apparently have vibrational character; this group property may be explained by the theory of A. Bohr[2] for the case of weak or intermediate coupling between the nuclear surface and the motion of individual nucleons. We shall consider here the excitation of the first vibrational level as it occurs during scattering of fast neutrons by a black nucleus and we shall limit ourselves to the zero coupling case.

The nuclear surface is characterized by the equation

$$r(\vartheta, \varphi) = R\left[1 + \sum_\mu \alpha_\mu Y_{2\mu}(\vartheta, \varphi)\right], \qquad (1)$$

the coordinates A_μ are correlated with creation and