Scattering of K Mesons with Change of Intrinsic Parity

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THE analysis of the experimental data on the decay of K mesons[1] leads, with a high degree of probability, to the conclusion that: 1) the spin of K mesons is equal to zero, and 2) K mesons can occur in states with different intrinsic parities, positive (θ mesons) and negative (τ mesons). In collisions of K mesons with nucleons there can occur changes of the intrinsic parities of the former (conversion of mesons into τ mesons and vice versa). For a consideration of some general features of such a process we shall represent the wave function Ψ of the system K meson + nucleon as a combination of two spinors ψ_θ and ψ_τ transforming differently on reflection,

$$\Psi = \begin{pmatrix} \psi_\theta \\ \psi_\tau \end{pmatrix}, \quad I\psi_\theta = \psi_\theta; \quad I\psi_\tau = -\psi_\tau,$$

where I is the operator of reflection.

In the scattering problem Ψ has, as usual, the following form

$$\Psi = u \exp(ik\,\mathrm{n}_0\mathrm{r}) + F(\mathrm{n})\, e^{ikr}/r,$$

where n_0 and n are unit vectors in the directions of the incident and scattered waves, and u and F are the corresponding amplitudes, which like Ψ, are two-spinor quantities. The amplitude F can be written in the form $F = Ru$, where R is a two-rowed matrix (each of its elements is a two-rowed matrix with respect to the spin variables).

If the properties of θ and τ mesons are the same as regards interaction with nucleons, this last relationship holds also for the "parity-conjugate" amplitudes

$$u' = C_p u; \quad F' = C_p F, \quad C_p = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

where C_p is the operator of parity conjugation introduced by Lee and Yang[2].

Consequently, the matrix R must satisfy the condition $RC_p = C_pR$ and can be written in the form $R = a + bC_p$, where a is a scalar and b is a pseudoscalar (more precisely, corresponding matrices in the spin variables). The amplitude au describes the ordinary scattering (without change of intrinsic parity), and is of a form that is well known from the theory of scattering of spinor waves. Our aim is to find the general form of the amplitude bu that describes the scattering with change of intrinsic parity.

For this purpose we consider the relation between the incident and outgoing waves with definite values of the angular momentum and parity. We write it in the form

$$\Psi^{\mathrm{out}}_{jMg} = S(j,\ g)\,\Psi^{\mathrm{in}}_{jMg},$$

where j, M, and g are the quantum numbers for the angular momentum, one of its components, and the parity, and S is a two-rowed matrix in the same sense as R. We can write this matrix in the form

$$S(j,\ g) = \begin{pmatrix} S_{ll} & S_{ll'} \\ S_{l'l} & S_{l'l'} \end{pmatrix}, \quad g = (-1)^l = (-1)^{l'+1},$$

where $l = j \pm 1/2$ is the orbital angular momentum of the meson and $l' = j \pm 1/2$ is that of the τ meson. By the general symmetry properties of the S-matrix $S_{ll'} = S_{l'l}$.

Furthermore, since

$$\Psi_{jM-g} = C_p \Psi_{jMg},$$

the invariance of the interaction with respect to the parity-conjugation transformation leads to the relation

$$S(j,\ -g) = C_p S(j,\ g)\, C_p^{-1},$$

$$\text{i.e.}\quad S(j,\ -g) = \begin{pmatrix} S_{l'l'} & S_{ll'} \\ S_{l'l} & S_{ll} \end{pmatrix}.$$

We see that the off-diagonal elements $S_{ll'}$ do not depend on the parity g of the state.

We can now construct the expression for the scattering amplitude bu. For this purpose let us consider the amplitude u of an incident wave with components v, 0 (a θ meson). To this there correspond ingoing waves of the form

$$\Psi^{\mathrm{in}}_{jMg} = \begin{pmatrix} c\,\Omega_{jlM}(\mathrm{n}) \\ 0 \end{pmatrix} \frac{e^{-ikr}}{r}, \quad c = \frac{2\pi i^l}{k}\,(\Omega^*_{jlM}(\mathrm{n}_0)\,v),$$

where $g = (-1)^l$ and Ω_{jlM} is a spherical spinor[3].

The outgoing waves will be of the form

$$\Psi^{\mathrm{out}}_{jMg} = \begin{pmatrix} S_{ll}\ c\,\Omega_{jlM}(\mathrm{n}) \\ S_{ll'}\ c\,\Omega_{jl'M}(\mathrm{n}) \end{pmatrix} (-1)^{l+1}\,\frac{e^{ikr}}{r}$$

and consequently we have

$$bv = \frac{2\pi}{ik} \sum_{jlM} (\Omega^*_{jlM}(\mathrm{n}_0)\,v)\,\Omega_{jl'M}(\mathrm{n}).$$

By making use of the following transformations[3]

$\Omega_{jl'M}(\mathbf{n}) = \boldsymbol{\sigma}\mathbf{n}\,\Omega_{jlM}(\mathbf{n}),$

$$\sum_M (\Omega^*_{jlM}(\mathbf{n}_0)\,u)\,\Omega_{jlM}(\mathbf{n}) = \frac{1}{4\pi}\left(\alpha_{jl} + \frac{\beta_{jl}}{i\sin\vartheta}[\mathbf{n}_0\mathbf{n}]\boldsymbol{\sigma}\right),$$

$$\alpha_{jl} = (j + 1/2)\,P_l(\cos\vartheta),\quad \beta_{jl}$$

$$= \mp P_l^1 \text{ for } l = j \mp 1/2,\ \cos\vartheta = \mathbf{n}_0\mathbf{n}$$

(P_l and P_l^1 are Legendre functions), we have

$$\sum_M (\Omega^*_{jlM}(\mathbf{n}_0)\,v)\,\Omega_{jlM}(\mathbf{n}) = \sigma(\mathbf{n}\,r_{jl} + \mathbf{n}_0 q_{jl}),$$

$$r_{jl} = \beta_{jl}/\sin\vartheta;\quad q_{jl} = \alpha_{jl} - \beta_{jl}\,\mathrm{tg}\,\vartheta.$$

Use of well-known relations between Legendre polynomials leads to the equation

$$r_{j,\,j+1/2} + r_{j,\,j-1/2} = q_{j,\,j+1/2}$$

$$+ q_{j,\,j-1/2} = (d/d\cos\vartheta)\,(P_{j+1/2} - P_{j-1/2}).$$

Inserting all of this into the expression for b, we have finally

$$b = B(\vartheta)\,\boldsymbol{\sigma}\,(\mathbf{n}_0 + \mathbf{n}),$$

$$B(\vartheta) = \frac{1}{2ik}\sum_j S_{j+1/2,\,j-1/2}\frac{d}{d\cos\vartheta}(P_{j+1/2} - P_{j-1/2}).$$

For small momenta we can retain in this expression only the term with $j = 1/2$, corresponding to transitions $s_{1/2} \leftrightarrows p_{1/2}$. Then B does not depend on the angles, and the differential cross-section for scattering with change of intrinsic parity takes the form

$$|b|^2 = \sigma_0(1 + \cos\vartheta),$$

where σ_0 is a constant and, according to general properties of the elements of the scattering matrix, $\sigma_0 \sim k^2$. Unfortunately, this dependence of the cross-section on angle and momentum is not sufficient by itself for an experimental singling-out of the process under consideration here, since the differential cross-section for ordinary scattering at small momenta contains an analogous dependence

$$|a|^2 = c_1 + c_2\cos\vartheta,$$

where c_1 and c_2 are constants (the first term corresponds to the s-wave and the second to interference between the s- and p-waves).

From the expression for b it follows that in this type of scattering no polarization of the nucleons occurs. But if the nucleon was polarized before the scattering, then the spin components perpendicular to the vector $n_0 + n$ change sign.

All the preceeding discussion applies also to the scattering of Σ and Λ particles by nuclei of spin 0, if the spin of these particles is equal to ½. The same expressions also describe the processes

$$K + N \to \Sigma + \pi;\quad K + N \to \Lambda + \pi,$$

with the amplitude b in this case referring to the appearance of a hyperon of the same intrinsic parity as the incident K particle (since an odd π meson is produced).

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2 T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956).

3 Cf., e.g., A. Akhiezer and V. Berestetskii, *Quantum Electrodynamics*, Moscow 1953.

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Absorption of γ-Quanta of 500 mev Mean Energy in Lead, Copper and Aluminum

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WE measured the absorption coefficients of γ-quanta of 500 mev energy in Pb, Cu and Al. γ-quanta from the decay of π^0-mesons produced in the internal phasotron target by protons of 660 mev were registered by a 12 channel-pair γ-spectrometer. The spectrometer was placed at the distance of 23 m from the target. A device, periodi-