$$\Theta = 2nA_{dd}\,[A_{sf}]\,/\,1 - \delta\,/\,nI'[nI_1])$$

(the quantities in square brackets are those for Gd, determined, according to our hypothesis, by *s-f* exchange), where $A_{sf} = -\frac{1}{2}(1 - N_f)(I_0 + nI_1)$, $A_{dd} = \frac{1}{2}(1 + N_d^2)(I'_0 + nI')$, $\delta = 0.15 \times (I_0 - 4I_1)^2 n_s \div I_1 n_{d(f)}$, cf. the Table and (22.1) of Ref. 2.

* The same value of m is given by our relation m/M_B

$$m\,/\,M_B = N_{d(f)} - n_s \mp {}^1/_2\,(N_d \mp 1)I.$$

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Quasi-Magnetic Interaction of the Spin of a Nucleon with the Rotation of the Nucleus

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BOHR and Mottelson[1,2] have interpreted the rotational levels of nuclei with spin 1/2 by starting out from the assumption that the Ω-projection of the total angular momentum of nucleons on the axis of symmetry of the nucleus is an integral of the motion. We can conceive of a different interpretation of the rotational levels of the nuclei, however, if we start from the following two assumptions: 1) there are absent from these nuclei the $^2\Sigma$ states which are considered according to a coupling scheme which corresponds to the case of *b* coupling according to Gund.[3] Then in the first approximation, the levels with total momenta $I = K \pm 1/2$ are degenerate (K is the rotational quantum number); 2) this degeneracy is taken into account by the introduction into the Hamiltonian of Bohr and Mottelson of an interaction of the type

$$H_{Rs} = -(\lambda\,/\,mc^2)\,\mathbf{s}[(\nabla U \mathbf{v}_{\text{coll}}],$$

where λ is the dimensionless phenomenological constant which has the same meaning and value as in the usual nuclear spin–orbit coupling (see, for example, Ref. 4); s is the spin vector of the nucleon, U (r) = self-consistent potential of the nucleus, mc^2 = rest energy of the nucleon, $\mathbf{v}_{\text{coll}}$ = velocity with which the nucleon takes part in the collective motion. Introduction of the interaction (1) into the Hamiltonian of the system can be justified if we start out from the model of independent particles which move in a rotating self-consistent field.[5,6]

First, we shall make clear the meaning of $\mathbf{v}_{\text{coll}}$. Consideration of the rotation of the field, as is well known, leads to the appearance in the Hamiltonian of the system (which is written relative to the rotating system of coordinates) of a perturbation of the form $H' = -\hbar\,\omega_x L_x$, where ω_x is the frequency of rotation of the field U(r) relative to the x–axis, perpendicular to the axis of symmetry of the nucleus, L_x is the projection of the orbital momentum of the nucleons on the corresponding axis. The unperturbed Hamiltonian H_0 describes the motion of particles in the non–rotating field. We can write the perturbed wave functions, with accuracy up to first order in ω_x, in the form

$$\psi = \psi_0 + i\omega_x\psi_1, \qquad (2)$$

where ψ_0 is the unperturbed wave function, which is real for the Σ state; ψ_1 is also a real function. Then, with the same accuracy as (2), we get an expression for the current density:

$$\mathbf{j} = \psi_0^2\,\mathbf{v}_{\text{coll}}, \quad \mathbf{v}_{\text{coll}} = (h\omega_x\,/\,m)\,\text{grad}\,(\psi_1\,/\,\psi_0). \qquad (3)$$

We can interpret this expression that for a rotation of the nucleus, each nucleon acquires an additional irrotational velocity $\mathbf{v}_{\text{coll}}$.

We now make clear the meaning and origin of the interaction (1). We represent the wave function (2), with corresponding accuracy, in the form

$$\psi = \psi_0 \exp\,\{i\omega_x\psi_1\,/\,\psi_0\}. \qquad (4)$$

Then the Schrödinger equation in the new representation has the form

$$H_{tr}\psi_0 = E\psi_0, \quad (5)$$

$$H_{tr} = \exp(-i\omega_x\psi_1/\psi_0)\, H \exp(i\omega_x\psi_1/\psi_0),$$

$$H = H_0 + H'.$$

The nuclear spin–orbital interaction, which enters into H_0, has the form

$$H_{ls} = -(\lambda/m^2c^2)\,[\nabla U s]\,p \quad (6)$$

(p is the momentum of the nucleon). Carrying out the transformation, it is easy to see that there is an additional interaction of first order in ω_x in H_{tr}, in addition to the interaction of type (6):

$$H_{Rs} = -(i\lambda/m^2c^2)\,[s\nabla U]\,p\,(\psi_1/\psi_0). \quad (7)$$

Taking into consideration the definition for the collective velocity (3), we bring the interaction (7) to the form (1):

The velocity v_{coll} as a function of the coordinates is for us, strictly speaking, unknown, since it is necessary for its determination to know the wave function of the nucleons in the nucleus. Therefore, as an estimate of the magnitude of the interaction (1), we shall consider that it coincides with the velocity of the irrotational flow of an ideal incompressible fluid which is found in a rotating vessel of spheroidal form.[2] Moreover, as an estinate, we shall consider the interaction (1) in the first non–vanishing approximation in the deformation. In the $^2\Sigma$ states, the integrals of the motion are K and s, where K = L + R, where L is the total orbital momentum of the nucleons, R is the moment of rotation of the nucleus. The rotational frequencies of the nucleus about an axis perpendicular to the axis of symmetry are given in the following form: $\omega_{x,y} = R_{x,y}/I$, where I is the moment of inertia. Averaging (1) over the wave functions of the $^2\Sigma$ states, we obtain

$$\langle H_{Rs}\rangle = \gamma K s, \quad (8)$$

$$\gamma = \frac{3\lambda}{2mc^2}\langle H_{int}\rangle \frac{\hbar^2}{I}, \quad \langle H_{int}\rangle = -\beta\langle r\frac{\partial U}{\partial r}\cdot Y_{20}\rangle,$$

$\langle H_{int}\rangle$ is the energy of interaction of the nucleon with the deformations, β is a parameter of the deformation.

Different estimates[7,8] give for the quantity $\langle H_{int}\rangle$ a value of the order of several mev. Assuming that $\lambda \approx 20$,[4] we get

Nucleus	energy of rotational level (kev)				nuclear term	$3\hbar^2/I$ (keV)	$\gamma/\left(\frac{\hbar^2}{I}\right)$
	1	2	3	4			
Tm^{169} [9]	8.40	118.30	138,10	—	Σ_g^- a Σ_u^+	74.4	0,23
W^{183} [10]	46.80	99.07	207.00	308.94	Σ_u^- a Σ_g^+	78.0	0.81
Pu^{239} [11]	7.85	57.25	75.67	—	Σ_g^- a Σ_u^+	37.5	0.42

$$\gamma \approx 0.1\,\hbar^2/I. \quad (9)$$

We shall make a series of remarks on the character of the $^2\Sigma$ states in the nucleus. If the nucleus has a center and axis of symmetry, then there can exist one of the following states: $\Sigma_g^+, \Sigma_u^+, \Sigma_g^-, \Sigma_u^-$. The sign + or – characterizes the behavior of the wave function for reflection in a plane passing through the axis of symmetry, while the indices g and u characterize the behavior for inversion of the coordinates of the nucleon relative to the origin. In the case of the nucleus, the indices g and u also determine the total parity of the level. Moreover, the states $\Sigma_{g,u}^+$, just as in the shell model, correspond to single nucleon configurations, while the $\Sigma_{g,u}^-$ arise as multi–nucleon configurations. In states Σ_g^+ and Σ_u^-, the quantum number K has the values 0,2,4,. . . , while in the states Σ_u^+ and Σ_u^- it has the values 1,3,5, . . .

Results are shown in the Table of the analysis of the rotational spectra of several nuclei with spin 1/2 from the viewpoint just given. The values found for γ do not contradict, in general, the estimate (9). Therefore, it has not been excluded that for these nuclei there exist $^2\Sigma$ states. The magnetic moment of the nucleus Tm^{149}, computed under the assumption of Σ_u^+ states, agrees with the experimental value if we assume that the gyromagnetic ratio for the collective motion $g_R \approx 1$. Σ states can arise in those cases in which the energy of interaction of the nucleon with deformations is greater than the energy of spin—orbital coupling. Evidently the multiplicity of the radiative transition between components of the doublet $^2\Sigma$ in the rotation spectrum of nuclei with spin 1/2 ought to correspond to a magnetic dipole The available data relative to the multiplicity of the transitions in rotational spectra of the nuclei Tm^{169}, W^{183} and Pu^{239} do not contradict this rule.

In conclusion, we note that the interaction (1) will also play a role in the coupling scheme of Bohr and Mottelson, since in this case the state with given $\Omega = 1/2$ is a combination of Σ and Π states.

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The Threshold of "Creation" and the Threshold of "Generation" of Negative *K*-Particles*

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GELL-MANN and Pais[1] were the first to analyze the peculiar behavior of the θ^0-particles. Starting from the notion of simultaneous creation of *K*-particles and hyperons, which has been confirmed experimentally[2], the small probability for producing two hyperons in a nucleon-nucleon collision[3] leads to the idea that the *K*-particle has isotopic spin ½. This is in agreement with the Gell-Mann classification[4], and implies that the θ^0-particle is not transformed into itself by charge-conjugation, i.e., the θ^0 and $\tilde{\theta}^0$ are distinct particles. Then, when one considers the weak interactions responsible for the θ^0 decay, the transitions $\theta^0 \rightleftarrows \tilde{\theta}^0$ are no longer forbidden. This led Gell-Mann and Pais[1] to the conclusion that the θ^0-particle is a mixture of two particles θ_1^0 and θ_2^0 having different charge-parity and different modes of decay. Pais and Piccioni[5] proposed various versions of an experiment to test the particle-mixture character of the θ^0-particle. We suppose the experimental arrangement of Pais and Piccioni to be known to the reader.

The present paper contains some remarks on the properties of charged *K*-particles, following immediately from the considerations of Pais and Piccioni but nevertheless not yet stated explicitly in the literature. We also propose a version of the Pais-Piccioni experiment which appears to us simpler than the other schemes which have been published.

Consider the process of creation of negative *K*-particles. According to Gell-Mann[4], the threshold for creating these particles (or $\tilde{\theta}^0$-particles) in pion-nucleon or in nucleon-nucleon collisions is much higher than the threshold for creating K^+ (or θ^0) particles. This is because a K^+ or θ^0 can be created together with a hyperon, whereas a K^- or $\tilde{\theta}^0$ cannot be created with a hyperon because of the conservation of strangeness. Thus for example, in nucleon-nucleon collisions the threshold for creation of K^+ or θ^0 is around 1580 mev, while the threshold for creating K^- is around 2500 mev. However, if one looks in detail at the properties of the θ^0-particle predicted by Pais and Piccioni, then it is clear that K^--particles can be obtained from a nucleon or pion beam at an energy below the threshold for the "creation" of K^-, i.e., below the threshold for the production of a pair of *K*-particles. For at an energy below the *K*-particle pair threshold one can produce a θ^0-particle which then undergoes the transition $\theta^0 \to \tilde{\theta}^0 \to K^-$, the first step occurring in the absence of matter and the second step resulting from nuclear scattering with charge exchange. Thus the threshold for the "generation" of K^--particles in thick targets is lower than the threshold for their "creation".

The special feature of our proposed experiment is that the observation of K^--particles below the