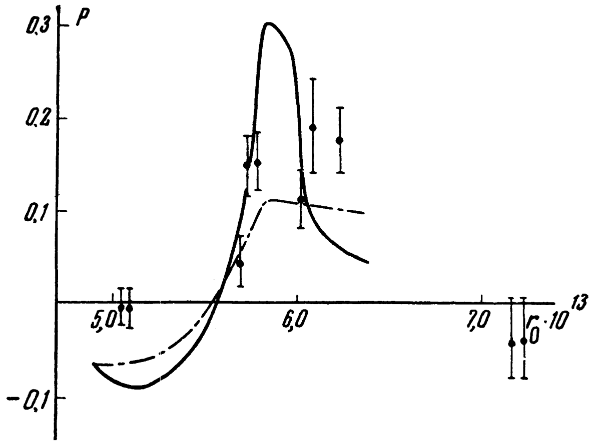
The results are shown in the accompanying figure.




Dependence of the polarization of neutrons on the nuclear radius.

As may be seen from the figure, the experimental points lie in the region of the maximum between the theoretical curves, which would indicate that the closest agreement with experiments is obtained with $0.05 < \zeta < 0.1$. This conclusion is also in good agreement with data on the values of the parameter $\zeta$, obtained in investigating[2] the total cross sections and $\Gamma_n/D$.

Thus it has been shown that using one and the same value of the spin-orbit interaction constant one can describe scattering both from light and from heavy nuclei. This coefficient is somewhat smaller than that given by Ross and coworkers in connection with their investigation of the splitting of the ground level of magic number nuclei.

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## Unitarity Relationships for Elastic Collisions of Particles with Arbitrary Spins

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LET US CONSIDER the collisions of particles without spin. Let the energy of the impinging particles be such that only elastic scattering is possible. Then as Glauber and Schomaker[1] have shown the scattering amplitude $f(\vartheta)$ obeys the following integral relation

$$4\pi \operatorname{Im} f(\vartheta) = k \int f^*(\vartheta'') f(\vartheta')\, d\omega(\mathbf{k}''). \qquad (1)$$

Here $\vartheta$ is the angle between the vectors of the initial $\mathbf{k}$ and final $\mathbf{k}'$ momenta; $\vartheta'$ is the angle between $\mathbf{k}$ and a variable vector $\mathbf{k}''$; $\vartheta''$ is the angle between $\mathbf{k}'$ and $\mathbf{k}''$ ($k = k' = k''$, c. m. s.); and the integration is carried out over all directions of $\mathbf{k}''$. From (1) with $\vartheta \to 0$, we obtain the so-called optical theorem: the relation between the imaginary part of the scattering amplitude forward and the total cross section $\sigma$. Below we shall generalize (1), extending it to the case of elastic collision of particles with arbitrary spins $s_1$ and $s_2$ and shall show that in addition to the optical theorem there can be deduced from (1) a series of other relations connecting scattering matrix elements that do not vanish when $\mathbf{k}' \to \mathbf{k}$ with different spin characteristics.

The process of elastic scattering is fully described by the matrix $M(\mathbf{k}, \mathbf{k}')$ in $(2s_1 + 1)(2s_2 + 1)$-dimensional spin space, determining the amplitude of the scattered wave

$$\Psi_{\mathbf{k}}(r \to \infty) \sim \varphi_{\mathbf{k}'}^{\text{incid}} + \varphi_{\mathbf{k}}^{\text{scat}} = e^{ikr}\chi + M(\mathbf{k}, \mathbf{k}')\chi e^{ikr}/r. \qquad (2)$$

The indices of the total spin of the system and its projection at the spin functions $\chi$ are omitted for simplicity of notation. In view of unitarity of $S$-matrix the wave functions $\Psi_{\mathbf{k}}$ satisfy the same requirements of orthogonality and normalization as the initial functions of the incident wave and form, when $r \to \infty$, a complete system of functions with respect to the angular variables

$$\int \varphi_{\mathbf{k}'}^{+\text{incid}} \varphi_{\mathbf{k}}^{\text{incid}}\, d\omega = \int \Psi_{\mathbf{k}'}^{+} \Psi_{\mathbf{k}}\, d\omega. \qquad (3)$$

The + sign, as usual, denotes Hermitian conjugates.

Utilizing expression (2), the asymptotic representation of plane wave

$$\varphi_{\mathbf{k}}^{\text{incid}} \sim \frac{i}{k}\,\delta\left(1+\frac{\mathbf{kr}}{kr}\right)e^{-ikr}/r-\frac{i}{k}\,\delta\left(1-\frac{\mathbf{kr}}{kr}\right)e^{ikr}/r$$

and the completeness of system of spin functions, one can readily obtain from (3) the sought integral relation for the scattering matrix

$$2\pi\left[M(\mathbf{k},\mathbf{k}')-M^{+}(\mathbf{k}',\mathbf{k})\right] = ik\int M^{+}(\mathbf{k}',\mathbf{k}'')\,M(\mathbf{k},\mathbf{k}'')\,d\omega(\mathbf{k}''), \qquad (4)$$

From (4) there immediately follow the integral relations for the coefficients of expansion of $M(\mathbf{k},\mathbf{k}')$ in terms of the invariant spin matrices given in Ref. 2 for nucleon-nucleon scattering and applied therein to the analysis of the full set of trials aimed at reestablishment of the scattering matrix.

If with $\mathbf{k}'=\mathbf{k}$ we utilize the expansion of $M(\mathbf{k},\mathbf{k}) = \sum_{\mu}\alpha_{\mu}S_{\mu}$ in the orthogonal and normalized Hermitian operators $S_{\mu}$ [the operators are orthogonal and normalized if $\mathrm{Sp}\,S_{\mu}S_{\nu}=(2s_1+1)(2s_2+1)\delta_{\mu\nu}$], then from (4) we can obtain the following relation

$$4\pi\,\mathrm{Im}\,\alpha_{\nu}(2s_1+1)(2s_2+1)=4\pi\,\mathrm{Im}\,\mathrm{Sp}\,S_{\nu}M(\mathbf{k},\mathbf{k}) = k\int \mathrm{Sp}\,S_{\nu}M^{+}(\mathbf{k},\mathbf{k}'')\,M(\mathbf{k},\mathbf{k}'')\,d\omega(\mathbf{k}''). \qquad (5)$$

In particular, assuming $S_{\nu}$ to be unity, we obtain the extension of the optical theory to the case of particles with spins

$$4\pi\,\mathrm{Im}\,\mathrm{Sp}\,M(\mathbf{k},\mathbf{k})=k(2s_1+1)(2s_2+1)\,\sigma.$$

From the last relation there follows the inequality[3] $\sigma(0)\geq(k/2\pi)^2\sigma^2$, limiting the value of the cross section for elastic scattering to 0°. With $S_{\nu}\neq I$ the relations (5) connect $\mathrm{Im}\,\alpha_{\nu}$ with the integral with respect to $\mathrm{Sp}\,S_{\nu}M^{+}M$, determining the addition

$$(2s_1+1)^{-1}(2s_2+1)^{-1}\langle S_{\nu}\rangle_{\text{incid}}\,\mathrm{Sp}\,S_{\nu}M^{+}M$$

to the cross section for scattering of a nonpolarized beam from a nonpolarized target, due to the initial polarization of the colliding particles (in the initial state the mean value of $\langle S_{\nu}\rangle_{\text{incid}}$ of the quantity $S_{\nu}$ differs from zero).

The number of relations (5) is equal to the number of coefficients $\alpha_{\nu}$ that are not zero for $\mathbf{k}'=\mathbf{k}$. Thus in the case of scattering of mesons from nucleons we obtain only the optical theorem [the coefficient at $(\sigma\mathbf{n})$ in the expansion of the scattering amplitude vanishes when $\mathbf{k}'\to\mathbf{k}$]. In the case of nucleon-nucleon scattering we obtain three relations. In this case for $S_{\nu}$ one should select operators

$$I,\ 2^{-1/2}\left[(\sigma_1,\sigma_2)-(\sigma_1\mathbf{l})(\sigma_2\mathbf{l})\right],\ (\sigma_1\mathbf{l})(\sigma_2\mathbf{l})$$

(it is impossible to construct a greater number of scalar expressions from the vectors $\sigma_1$, $\sigma_2$ and $\mathbf{l}=\mathbf{k}/k$). In view of the invariance of the scattering matrix $M(\mathbf{k},\mathbf{k}')$ relative to time reversals, the traces containing the operators

$$2^{-1/2}\left[(\sigma_1\sigma_2)-(\sigma_1\mathbf{l})(\sigma_2\mathbf{l})\right],\ (\sigma_1\mathbf{l})(\sigma_2\mathbf{l})$$

and determining the additions to the cross section, can be expressed[2] in terms of the components of the tensor of correlation of the polarization arising incident to collisions of nonpolarized nucleons.

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## Gadolinium Isotope with Mass 146

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WHEN TANTALUM is bombarded with 660 Mev protons there are formed new isotopes of gadolinium[1] hitherto unreported in the literature. Upon decay, these isotopes in a number of cases form known isotopes of europium, from which it is possible to determine the mass number of the parent substances — the new gadolinium isotopes. In fractions of europium separated from pure fractions of gadolinium (obtained 32 hours after cessation of bombardment) we observed a radioactive isotope that decays with a period of 1.6 days in accord