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ON THE RANGE OF MANY-BODY FORCES BETWEEN A Λ^0-HYPERON AND NUCLEONS

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THE forces between a Λ^0-hyperon and one nucleon [($\Lambda^0 - N$) forces], admissible on grounds of isotopic invariance, have a range several times shorter than the usual nuclear forces.[1,2] Thus, the forces suggested by the experiments on pair production and due to virtual processes with K-meson exchange

$$\Lambda^0 + N \rightarrow N' + \bar{K} + N \rightarrow N' + \Lambda^{0\prime}, \tag{1}$$

have a range $1/m_K$, i.e., a range about three times shorter than the radius of nucleon-nucleon forces, which is of order $1/m_\pi$. However, isotopic invariance allows an interaction in which the Λ^0-hyperon exchanges with the nucleon two (in general, an even number of) π-mesons:

$$\Lambda^0 + N \rightarrow \Lambda^{0\prime} + \pi + \pi + N \rightarrow \Lambda^{0\prime} + N'. \tag{2}$$

One can also conceive of similar 2π-meson forces involving Σ-hyperons:[3]

$$\Lambda^0 + N \rightarrow \Sigma + \pi + N \rightarrow \Sigma + N' \rightarrow \Lambda^{0\prime} + \pi + N' \rightarrow \Lambda^{0\prime} + N'. \tag{3}$$

The forces due to (2) and (3) have a range $1/2m_\pi$.

The existence of Λ^0-hyperfragments indicates that the ($\Lambda^0 - N$) forces are sufficiently strong to lead to binding of a Λ^0-hyperon to nucleons. In this connection it is interesting to determine whether there exist ($\Lambda^0 - N$) forces with the usual range $1/m_\pi$.

Such forces indeed are possible if we consider the interaction of the Λ^0-hyperon with two nucleons instead of one. Thus, the 2π-meson forces between a Λ^0 particle and two nucleons, realizably by virtual processes of the form

$$\Lambda^0 + N_1 + N_2 \rightarrow \Lambda^{0\prime} + \pi + \pi + N_1 + N_2 \rightarrow \Lambda^{0\prime} + N_1' + N_2', \tag{4}$$

have a range $1/m_\pi$, since the Λ^0-hyperon exchanges with each of the nucleons only one π-meson. This conclusion regarding the range of the forces is purely qualitative and follows from the uncertainty relation for the energy and time, and actually involves no approximations.

As an illustration it is easy to calculate the potential of the ($\Lambda^0 - N$) forces of the type (4) in the static approximation. This potential contains the product of the exponentials $\exp(-m_\pi r_{1\Lambda}) \exp(-m_\pi r_{2\Lambda})$, where r_i is the distance between the Λ-hyperon and the i-th nucleon. Thus many-body forces of range $1/m_\pi$ between a Λ^0-hyperon and nucleons are possible, in contrast to two-body ($\Lambda^0 - N$) forces, whose range is less. One can therefore assume that the suggested many-body forces may play an important role in the interaction of the Λ^0-hyperon with nucleons.

This hypothesis could be checked by measuring the cross section for the scattering of a slow Λ^0 particle by a proton and by some simple nucleus such as deuterium or helium. If the present hypothesis is true the cross sections should be considerably larger for scattering by nuclei than for scattering by protons.

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ROTATIONAL BANDS OF EVEN-EVEN AXIALLY SYMMETRIC NUCLEI

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It has been shown by Davydov and Filippov[1] that by using the Hamiltonian obtained by Ford[2] in averaging the interaction between the external nucleons and the nuclear core, one can write the equation for the collective motion of an axially symmetric even-even nucleus with total angular momentum $\hbar J$ in the form

$$d^2U_\nu/d\zeta^2 - 2\zeta dU_\nu/d\zeta + 2\nu U_\nu = 0, \tag{1}$$

where U_ν satisfies the boundary condition

$$U_\nu(-\delta\xi) = 0,\quad U_\nu(\zeta)e^{-\zeta^2/2} \to 0, \quad \text{for } \zeta \to \infty. \tag{2}$$

The eigenvalue ν of Eq. (1) is not in general an integer, and determines the energy $\epsilon_\nu(J)$ of the collective nuclear motion by the equation

$$\varepsilon_\nu(J)/\hbar\omega_0 = (\nu+{}^1\!/_2)\sqrt{1+J(J+1)/\delta^4\xi^4} + J(J+1)/6\delta^2\xi^2 + {}^1\!/_2\,\delta^2(\xi-1)^2, \quad \xi^3(\xi-1) = J(J+1)/3\delta^4. \tag{3}$$

Thus the energy of nuclear collective motion for each value of J = 0, 2, 4, . . . is determined uniquely by just the two parameters ω_0 and δ, which are related to parameters of Bohr and Mottelson's[3] generalized nuclear model by the expressions $\omega_0 = \sqrt{C/B}$, $\delta = \beta(BC/\hbar^2)^{1/4}$.

Davydov and Filippov[1] investigated the solution of Eq. (1) for the case $\delta \leq 1$. In this note we present the results of a solution of this set of equations for the case $\delta > 1$.

The figure gives a graph of $\epsilon_\nu(J)/\hbar\omega_0$ vs. δ; the numbers on the curves give the values of J. It is seen from the figure that when $\delta > 2.5$, the energy spectrum of collective excitations of even-even nuclei breaks up into a set of rotational-vibrational bands. In Table 1 we present a comparison of the theoretical excitation energies of the first and second rotational band for certain nuclei with the experimental data. We also give the values of $\hbar\omega_0$ and δ which have been used to calculate the theoretical excitation energy.

TABLE 1

Nucleus and literature reference	J	Energy level (kev) Theory	Energy level (kev) Experiment	$\hbar\omega_0$ (kev)	δ
W^{182} [4,5]	2	100.09	100.09		
	4	320.3	329.36		
	6	641.6	677.6		
	0	1101	—	1101	3.48
	2	1222	1222		
	4	1481	1488.6		
Th^{232} [6]	2	50	50		
	4	163	165		
	6	332	—		
	0	710	—	710	3.93
	2	770	770		
	4	901	—		
U^{234} [5]	2	43	43		
	4	141	142		
	6	290	295		
	0	803	803	803	4.48
	2	855	—		
	4	966	—		
Pu^{238} [5]	2	44.2	44.2		
	4	147.7	146		
	6	304.8	303		
	0	935	935	935	4.73
	2	986	986		
	4	1100	1073		

In Table 2 we give the δ dependence of the energy ratios of the first and second (1 and 2) rotational-state sublevels in the first and second (I and II) bands of the rotational states of the nucleus.

If the energy of collective oscillations is approximated in the form

$$E_J = n\hbar\omega_0 + AJ(J+1) - BJ^2(J+1)^2, \quad A = \hbar^2/2I, \quad B = a(\hbar\omega_0)^{-2}(\hbar/I)^3, \tag{4}$$