the molecular oscillator; an investigation of this kind, however, is beyond the scope of the present paper.

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CONTRIBUTION TO THE THEORY OF TRANSITION RADIATION

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The transition radiation and the Cerenkov radiation which are produced when a charged particle moves successively through two media which differ in their dielectric and magnetic properties are considered. The cases in which the particle moves from vacuum into the medium and from the medium into vacuum are considered in detail.

THE transition radiation which is produced when a particle moves from a medium characterized by a given dielectric constant into another whose dielectric constant differs from the first was first considered by Ginzburg and Frank[1] (see also Refs. 2—4). In the present paper we consider the radiation fields which are produced in the general case for media which differ in both their dielectric and magnetic properties; certain particular cases are analyzed.

1. GENERAL CASE

We consider the field associated with a particle which has a velocity $\mathbf{v}$ and moves from one medium into another. The first medium will be characterized by the macroscopic constants ϵ_1 and μ_1 (the dielectric constant and magnetic permeability); the second medium is characterized by ϵ_2 and μ_2. We shall assume that the energy lost by the particle per unit length of path is negligibly small compared with its kinetic energy. Under these conditions the field associated with the particle is given by Maxwell's equations

$$\operatorname{curl}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{v}\,e\delta(\mathbf{r}-\mathbf{v}t), \quad \operatorname{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \qquad \operatorname{div}\mathbf{B} = 0, \quad \operatorname{div}\mathbf{D} = 4\pi e\delta(\mathbf{r}-\mathbf{v}t). \tag{1}$$

It will be assumed that the particle moves along the z-axis from $-\infty$ to $+\infty$ and that the interface between the two media is the plane $z = 0$ through which the particle moves at $t = 0$. We resolve the field and currents in triple Fourier integrals:[5]

$$\mathbf{E}(\mathbf{r},t)=\int \mathbf{E}(\mathbf{k})\,e^{i(\mathbf{kr}-\omega t)}\,d\mathbf{k}\quad \text{etc.,} \tag{2}$$

where

$$\omega=\mathbf{kv}=k_z v,\quad \mathbf{D}_{1,2}(\mathbf{k})=\varepsilon_{1,2}(\omega)\,\mathbf{E}_{1,2}(\mathbf{k}),\quad \mathbf{B}_{1,2}(\mathbf{k})=\mu_{1,2}(\omega)\,\mathbf{H}_{1,2}(\mathbf{k})$$

(here and in what follows, quantities which refer to both media will be denoted by the subscripts 1, 2).

The Fourier components of the field have the following form:[6]

$$\mathbf{E}_{1,2}(\mathbf{k})=\frac{ei}{2\pi^2}\frac{1}{\varepsilon_{1,2}}\frac{(\omega/c^2)\chi_{1,2}\mathbf{v}-\mathbf{k}}{k^2-(\omega/c)^2\chi_{1,2}},\quad \mathbf{H}_{1,2}(\mathbf{k})=(\varepsilon_{1,2}/c)\,[\mathbf{v}\times\mathbf{E}_{1,2}(\mathbf{k})], \tag{3}$$

where $\chi_{1,2}=\epsilon_{1,2}\mu_{1,2}$. The fields in (2) with Fourier components (3) do not satisfy the continuity conditions on the tangential components of E and H and on the normal components of D and B at $z = 0$. To satisfy these requirements we must add to the solution of the inhomogeneous Maxwell equations given above the solutions of the homogeneous equations with arbitrary Fourier component, and then determine these from the continuity requirements on the fields at the interface between the two media.

We use the symbols ρ and κ to denote the components of the vectors $\mathbf{r}$ and $\mathbf{k}$ which lie in the xy-plane. The solutions of the homogeneous Maxwell equations are written in the form

$$\mathbf{E}'_{1,2}(\mathbf{r},t)=\int \mathbf{E}'_{1,2}(\mathbf{k})\exp\{i(\varkappa\rho+\lambda_{1,2}z-\omega t)\}\,d\mathbf{k} \tag{4}$$

and similarly for $\mathbf{H}'_{1,2}(\mathbf{r},t)$. In order for the expression given in (4) to be a solution of the homogeneous Maxwell equations, we require that

$$\lambda^2_{1,2}=(\omega/c)^2\chi_{1,2}-\varkappa^2. \tag{5}$$

We use the symbol λ' to denote the real part of λ and λ'' to denote the imaginary part of λ. The first medium is located in the region $z<0$. Hence, to prevent the field given in (4) from diverging at $z\to-\infty$, we require that $\lambda''_1<0$. It is also obvious that the radiation field in the first medium (4) can propagate only in the negative z-direction (reflected waves) whence it follows that $\lambda'_1<0$. From similar considerations we find that $\lambda'_2>0$ and $\lambda''_2>0$. The signs for λ'_1 and λ'_2 which have been indicated refer to positive ω. For negative values of ω these signs must be reversed.

It also follows from the equations for the radiation fields that

$$\mathbf{H}'_{1,2}(\mathbf{k})=(c/\omega\mu_{1,2})[(\varkappa+\mathbf{n}\lambda_{1,2})\times E'_{1,2}(\mathbf{k})],\quad (\varkappa+\mathbf{n}\lambda_{1,2})\,\mathbf{E}'_{1,2}(\mathbf{k})=0 \tag{6}$$

(the unit vector $\mathbf{n}$ is taken in the direction of the positive z-axis). The last condition can be written in another form, resolving $\mathbf{E}'_{1,2}(\mathbf{k})$ into tangential and normal components:

$$\varkappa\mathbf{E}'_{1,2t}(\mathbf{k})+\lambda_{1,2}E'_{1,2n}(\mathbf{k})=0. \tag{7}$$

Equating the field components at $z = 0$, we obtain four conditions for determining the Fourier components of the radiation field. It is easy to show from these conditions that the $\mathbf{E}'_{1,2t}(\mathbf{k})$ vectors are in the same direction as the vector κ. Assuming this to be the case, it turns out that only two of the four conditions are independent; we take the following two conditions:

$$\begin{aligned} -\frac{ei}{2\pi^2}\frac{1}{\varepsilon_1}\frac{\varkappa}{k^2-(\omega/c)^2\chi_1}+E'_{1t}(\mathbf{k}) &= -\frac{ei}{2\pi^2}\frac{\varkappa}{k^2-(\omega/c)^2\chi_2}+E'_{2t}(\mathbf{k}),\\ \frac{ei}{2\pi^2}\frac{(\omega/c^2)\chi_1 v-k_z}{k^2-(\omega/c)^2\chi_1}+\varepsilon_1E'_{1n}(\mathbf{k}) &= \frac{ei}{2\pi^2}\frac{(\omega/c^2)\chi_2 v-k_z}{k^2-(\omega/c)^2\chi_2}+\varepsilon_2E'_{2n}(\mathbf{k}). \end{aligned} \tag{8}$$

From Eq. (8) we obtain the following expressions for the Fourier components of the radiation field:

$$\mathbf{E}'_{1t}(\mathbf{k})=\frac{ei}{2\pi^2}\frac{\varkappa\lambda_1}{\zeta}\eta,\quad E'_{1n}=-\frac{ei}{2\pi^2}\frac{\varkappa^2}{\zeta}\eta,\quad \mathbf{H}_1(\mathbf{k})=-\frac{ei}{2\pi^2c}\frac{k_z\varepsilon_1[\varkappa\mathbf{v}]}{\zeta}\eta. \tag{9}$$

The following notation has been introduced

$$\eta=\left(\frac{\varepsilon_2}{\varepsilon_1}-\frac{v}{\omega}\lambda_2\right)\Big/\left(k^2-\frac{\omega^2}{c^2}\chi_1\right)+\left(-1+\frac{v}{\omega}\lambda_2\right)\Big/\left(k^2-\frac{\omega^2}{c^2}\chi_2\right),\quad \zeta=\varepsilon_2\lambda_1-\varepsilon_1\lambda_2. \tag{10}$$

The radiation fields in the second medium can be obtained from Eqs. (9) and (10) if the subscripts 1 and 2 are interchanged. If in Eq. (9) we take $\mu_1 = \mu_2$ and $\epsilon_1 = \epsilon_2$, all the Fourier components of the radiation field vanish, as is to be expected.

2. VACUUM-TO-MEDIUM CASE

We consider now a case which is of practical interest, i.e., the case in which the particle moves from vacuum into a medium; we set $\epsilon_1 = \mu_1 = 1$, $\mu_2 = 1$ and $\epsilon_2 = \epsilon = \epsilon' + \epsilon''$. We shall be interested in the field which is produced in the vacuum, i.e., the radiation field E_1' and H_1'.

We shall not write expressions for all the vacuum fields, as these can be easily obtained from Eqs. (9) and (10); the expression for E_1' is:

$$E'_{1\rho} = \frac{ei}{2\pi^2}\int \frac{\varkappa\lambda_1 \cos\Phi}{\varepsilon\lambda_1 - \lambda_2}\,\eta_1 \exp\{i(\varkappa\rho\cos\Phi + \lambda_1 z - \omega t)\}\,\varkappa\, d\varkappa\, d\Phi\, \frac{d\omega}{v}; \tag{11}$$

where Φ is the angle between κ and ρ, while

$$\eta_1 = \left(\varepsilon - \frac{v}{\omega}\lambda_2\right)\Big/\left(\varkappa^2 + \frac{\omega^2}{v^2} - \frac{\omega^2}{c^2}\right) + \left(-1 + \frac{v}{\omega}\lambda_2\right)\Big/\left(\varkappa^2 + \frac{\omega^2}{v^2} - \frac{\omega^2}{c^2}\varepsilon\right),\quad \lambda_1^2 = (\omega/c)^2 - \varkappa^2,\quad \lambda_2^2 = (\omega/c)^2\varepsilon - \varkappa^2. \tag{12}$$

The integration over Φ extends from 0 to 2π, that over κ from 0 to ∞, and that over ω from $-\infty$ to $+\infty$. The integral over Φ is easily computed in terms of Bessel functions.

We introduce R, the distance from the point at which the particle enters the medium to the field point being investigated, and the angle θ defined by the expression $\rho = R\sin\theta$ and $z = -R\cos\theta$; it is assumed that R is large. If very small values of θ are not considered the asymptotic expressions for the Bessel functions[7] can be used:

$$J_p(\varkappa R\sin\theta) = \sqrt{\frac{2}{\pi\varkappa R\sin\theta}}\cos\left(\varkappa R\sin\theta - \frac{p\pi}{2} - \frac{\pi}{4}\right). \tag{13}$$

Thus we have

$$E'_{1\rho} = -\frac{e}{\pi v\sqrt{2\pi R\sin\theta}}\int \frac{\varkappa\lambda_1}{\varepsilon\lambda_1 - \lambda_2}\,\eta_1\left(e^{f(\varkappa)R - 3\pi i/4} + e^{\varphi(\varkappa)R + 3\pi i/4}\right)e^{-i\omega t}\sqrt{\varkappa}\,d\varkappa\, d\omega; \tag{14}$$

$$f(\varkappa) = i\varkappa\sin\theta - i\lambda_1\cos\theta;\quad \varphi(\varkappa) = -i\varkappa\sin\theta - i\lambda_1\cos\theta. \tag{15}$$

For very large values of R, an integral of this type is most conveniently computed by the method of steepest descent. In this case the integrand must be an analytic function of κ. However, the presence of λ_1 and λ_2 in the integrand mean that the integrand is double valued. Hence, before deforming the path of integration we must take cuts in the κ-plane to make this function single valued.*

Above we have imposed certain conditions on the signs of the real and imaginary parts $\lambda_{1,2}$ along the real κ axis. We have taken $\kappa = \kappa_1 + i\kappa_2$. We require that $\lambda_1' < 0$ and $\lambda_2' > 0$ over the entire plane of the complex variable κ. It is easy to show that the sign of λ_2'' must correspond to that shown in Fig. 1 so that $\lambda_1'' > 0$ in the 1-st and 3-rd quadrants and $\lambda_1'' < 0$ in the 2-nd and 4-th quadrants. The cuts are taken as shown in Fig. 1 by the heavy lines. The integration over κ from the point 2 to $+\infty$ is taken along the lower edge of the cut.

We now turn to the integral in (14), proceeding with the integration of the first part

$$e^{3\pi i/4}\int_0^\infty \frac{\varkappa\lambda_1}{\varepsilon\lambda_1 - \lambda_2}\,\eta_1\, e^{f(\varkappa)R}\sqrt{\varkappa}\,d\varkappa. \tag{16}$$

It is easy to show that the saddle point is $\kappa_0 = (\omega/c)\sin\theta$. Two lines on which $\mathrm{Re}\, f(\kappa) = 0$ pass through this point. One of these lines is the κ_1-axis while the other is the solid line shown in Fig. 2. In the cross-hatched regions $\mathrm{Re}\, f(\kappa) > 0$; in the non-cross-hatched regions $\mathrm{Re}\, f(\kappa) < 0$. It can be shown that the line of shortest descent in the vicinity of the saddle point is at the angle $\varphi = -\pi/4$ with respect to the κ_1 axis, as shown by the dotted line in Fig. 2. We introduce a new variable b which is defined by the

*The ambiguity due to the presence of $\sqrt{\kappa}$ in Eq. (16) is easily removed by taking the cut along the negative κ-axis.

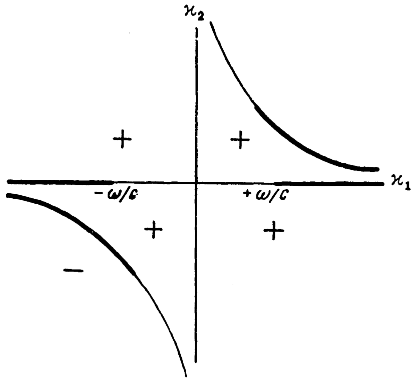

FIG. 1

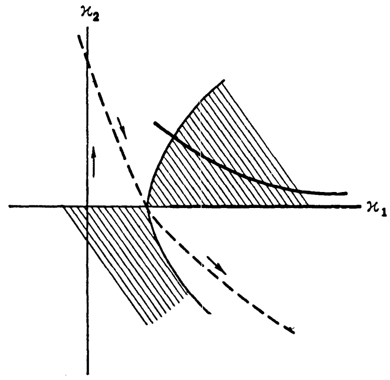

FIG. 2

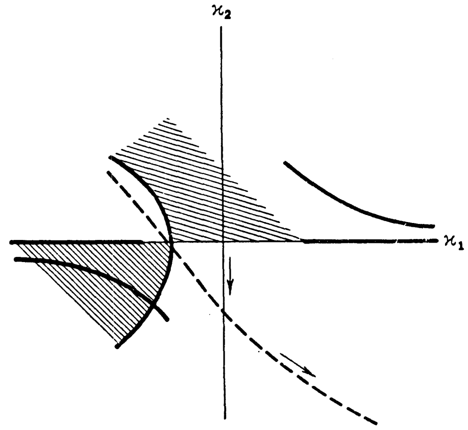

FIG. 3

relation $\kappa - \kappa_0 = b \exp i\varphi$. The integral in (16) along the line of shortest descent is easily computed if we carry out the integration over b from $-\infty$ to $+\infty$. In this case, because of the sharp maximum in the exponential at the saddle point, the expression which stands before the exponential may be taken outside the integral sign since it varies only slightly in this region and we then perform the integration for the exponential function alone.

It is still necessary to integrate the second term in Eq. (14), in the exponential of which we have the function $\varphi(\kappa)$. In a completely analogous way it can be shown that the saddle point is now $\kappa_0 = (\omega/c)\sin\varphi$, while the dotted line in Fig. 3 is the line of shortest descent. It is easy to see, however, that in this case the deformed path of integration does not pass through the saddle point (Fig. 3), so that this integral vanishes in the approximation of large R. It is also easy to show that in this case the poles of the integrand do not affect the calculations.

As a result we have

$$E'_{1\rho} = \frac{e\beta^2}{\pi v R}\int_{-\infty}^{+\infty} \sin\theta\cos^2\theta\,\xi e^{i\omega[(R/c)-t]}\,d\omega, \tag{17}$$

where

$$\xi = \frac{(\varepsilon - \beta\sqrt{\varepsilon-\sin^2\theta})/(1-\beta^2\cos^2\theta) - 1/(1+\beta\sqrt{\varepsilon-\sin^2\theta})}{\varepsilon\cos\theta + \sqrt{\varepsilon-\sin^2\theta}}. \tag{18}$$

Similar calculations lead to the following expression for the normal component of the radiation field:

$$E'_{1n} = \frac{e\beta^2}{\pi v R}\int_{-\infty}^{+\infty} \sin^2\theta\cos\theta\,\xi e^{i\omega[(R/c)-t]}\,d\omega. \tag{19}$$

It is obvious from the formulas which have been obtained that the electric vector of the radiation field E'_1 lies in the plane which passes through the ray to the point of observation and the trajectory of the particle and is perpendicular to the observation ray, i.e., the direction of R. We have

$$E'_1 = E'_{1\rho}\cos\theta + E'_{1n}\sin\theta = \frac{e\beta^2}{\pi v R}\int_{-\infty}^{+\infty} \sin\theta\cos\theta\,\xi e^{i\omega[(R/c)-t]}\,d\omega. \tag{20}$$

Finally, we can also obtain an expression for the magnetic field H'_1 which, as is to be expected, is the same as the expression for E'_1. Thus, at large distances, we have a spherically diverging wave with Poynting vector parallel to R. The Poynting vector flux in the solid angle $d\Omega = \sin\theta\, d\theta\, d\varphi$ during the time of flight of the particle is

$$\frac{dW_{\text{trans.}}}{d\Omega} = \frac{c}{4\pi}R^2\int_{-\infty}^{+\infty} E'_1 H'_{1\varphi}\,dt = \frac{ce^2\sin^2\theta\cos^2\theta}{\pi^2 v^2}\,\frac{\beta^4}{(1-\beta^2\cos^2\theta)^2}\int_0^{\infty}\left|\frac{(\varepsilon-1)(1-\beta^2+\beta\sqrt{\varepsilon-\sin^2\theta})}{(\varepsilon\cos\theta+\sqrt{\varepsilon-\sin^2\theta})(1+\beta\sqrt{\varepsilon-\sin^2\theta})}\right|^2 d\omega. \tag{21}$$

The expression which has been obtained coincides with the corresponding formula in the paper by Ginzburg and Frank,[1] which was obtained in a more approximate manner.*

In the extreme relativistic case the radiation has a sharp maximum in the direction $\theta \sim m/E$. Thus, in integrating Eq. (21) over angle, it is convenient to remove all factors which have a weak angular dependence from under the integral sign, substituting $\theta \sim 0$ in these expressions. The result is

$$W_{\text{trans.}} = \frac{e^2}{\pi c}\int_0^\infty \left(\frac{\sqrt{\varepsilon(\omega)}-1}{\sqrt{\varepsilon(\omega)}+1}\right)^2 \left(\ln\frac{2}{1-\beta}-1\right)d\omega. \tag{22}$$

In conclusion we may note that in integrating over κ certain restrictions have been applied; the results must therefore be qualified. In particular, we have taken the asymtotic expansions in place of the Bessel function and have used method of steepest descent in the integration. It is obvious that this procedure is valid when $(\omega/c)R\sin^2\theta \gg 1$. Thus, in the spatial region close to the trajectory of the particle, bounded by the surface $R \sim \lambda\!\!\!^{-}/\sin^2\theta$, the Poynting vector associated with the transition radiation is not given by Eq. (21). In this region there occurs the "formation" of the transition-radiation wave field which then, at distances $R \gg \lambda\!\!\!^{-}/\sin^2\theta$, is given by Eq. (21). The existence of this region is unimportant for that part of the transition radiation which is not emitted at small angles. However, in the extreme relativistic case, in which the radiation has a sharp maximum in the direction $\theta \sim m/E$, this "formation" region is extremely important and is defined by $R \sim \lambda\!\!\!^{-} E^2/m^2$.

3. MEDIUM-TO-VACUUM CASE

We now consider the case in which the particle moves from medium into vacuum, i.e., $\epsilon_2 = \mu_2 = 1$, $\mu_1 = 1$ and $\epsilon_1 = \epsilon = \epsilon' + i\epsilon''$. It is obvious that in this case the radiation field in the vacuum will consist of both the field due to transition radiation as well as that due to the Cerenkov radiation which is generated in the medium and then propagates into the vacuum.†

Thus, just as in the preceding section we write an expression for $E'_{2\rho}$, integrate over angle Φ and take the values at large distances R ($\rho = R\sin\theta$, $z = R\cos\theta$):

$$E'_{2\rho} = \frac{e}{\pi v\sqrt{2\pi R\sin\theta}}\int \frac{\varkappa\lambda_2}{\varepsilon\lambda_2-\lambda_1}\eta_2\left(e^{f_1(\varkappa)R-3\pi i/4} + e^{\varphi_1(\varkappa)R+3\pi i/4}\right)e^{-i\omega t}\sqrt{\varkappa}\,d\varkappa\,d\omega, \tag{23}$$

where

$$\eta_2 = \left(1-\frac{v}{\omega}\lambda_1\right)\Big/\left(\varkappa^2+\frac{\omega^2}{v^2}-\frac{\omega^2}{c^2}\varepsilon\right)+\left(-\varepsilon+\frac{v}{\omega}\lambda_1\right)\Big/\left(\varkappa^2+\frac{\omega^2}{v^2}-\frac{\omega^2}{c^2}\right),\quad \lambda_1^2=\omega^2\varepsilon/c^2-\varkappa^2,\quad \lambda_2^2=\omega^2/c^2-\varkappa^2,$$
$$f_1(\varkappa) = i\varkappa\sin\theta + i\lambda_2\cos\theta,\quad \varphi_1(\varkappa) = -i\varkappa\sin\theta + i\lambda_2\cos\theta. \tag{24}$$

We set $\lambda_1' < 0$ and $\lambda_2' > 0$ over the entire plane of the complex variable κ. Then the signs of λ_1'' and λ_2'' are the opposite of λ_2'' and λ_1'' respectively, as given in the preceding section.

It is easy to show that the saddle points and lines of steepest descent of the functions $f_1(\kappa)$ and $\varphi_1(\kappa)$ coincide with the saddle points and lines of steepest descent for the functions $f(\kappa)$ and $\varphi(\kappa)$. However, in contrast to the preceding case, the following two factors must be kept in mind in the integration over κ.

First, in deforming the line $0-\infty$ in the line of steepest descent, account must be taken in certain cases of the pole of the function η_2, which adds the residue at this pole to the integral along the saddle-point line. Such a pole of a function η_2 is $\kappa' = (\omega/v)\sqrt{\beta^2\epsilon(\omega)-1}$. Since $\epsilon'' \geq 0$, the pole contributes to the integral in this case if it is located in the κ plane in the region which is cross-hatched in Fig. 4. It will be shown in the following that this term yields the Cerenkov radiation which is generated in the medium and then passes into the vacuum. For small values of ϵ''

$$\varkappa' = \frac{\omega}{v}\left[\sqrt{\beta^2\varepsilon'-1} + i\beta^2\varepsilon''/\sqrt{\beta^2\varepsilon'-1}\right]. \tag{25}$$

*The above-mentioned formula [(32) of Ref. 1] contains an error: the denominator of the first term in circular brackets should read $\omega/v + \sqrt{k_2^2 - k_1^2\sin^2\theta}$ and not $\omega/v - \sqrt{k_2^2 - k_1^2\sin^2\theta}$. This error arises as a result of the wrong sign in the expression for the retarded phase of the refracted wave. The author is indebted to V. L. Ginzburg for this information.

†In this case the expression for the transition radiation cannot be obtained for all media from Eq. (21) by simply reversing the particle velocity. This is immediately obvious from the fact that in the denominator we have the term $1 - \beta\sqrt{\epsilon - \sin^2\theta}$, which vanishes at certain frequencies for transparent media.

Secondly, it can be shown in our case that for certain media and frequencies the above-mentioned pole is close to the saddle point or coincides with it. Then the part of the integrand in front of the exponential is not a slowly-varying function in the region of the saddle point and cannot be taken out from under the integral sign as in the preceding section. It can be shown that such a situation occurs if $\epsilon''(\omega') < \sqrt{c/\omega' R}$ for the frequency ranges $\omega' \pm \Delta\omega'$ where ω' and $\Delta\omega'$ are defined by the relations

$$\beta^2 \varepsilon'(\omega') - 1 = \beta^2 \sin^2\theta \text{ and } \Delta\omega' \sim \sqrt{c/\omega' R}\,(d\varepsilon/d\omega')^{-1}.$$

Since the increment $\Delta\omega'$ is small compared with ω', for the latter case we obtain a formula which refers only to the frequency ω'.

For simplicity let us assume that $\epsilon'' = 0$. Then, from (25) it is obvious that the pole adds to the integral if

$$0 \leqslant \beta^2 \varepsilon' - 1 \leqslant \beta^2 \sin^2\theta. \tag{26}$$

Then we have

$$E'_{2\rho} = \frac{e\beta^2}{\pi v R} \int_{-\infty}^{+\infty} \frac{\sin\theta\cos\theta}{\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta}} \left\{ \frac{\varepsilon + \beta\sqrt{\varepsilon - \sin^2\theta}}{1 - \beta^2\cos^2\theta} - \frac{1}{1 - \beta\sqrt{\varepsilon - \sin^2\theta}} \right\} e^{i\omega(R/c - t)}\, d\omega + \frac{2e}{v\sqrt{2\pi R\sin\theta\cdot v}}$$
$$\times \int_{-\infty}^{+\infty} \frac{\sqrt{1 + \beta^2(1 - \varepsilon')}}{1 + \varepsilon\sqrt{1 + \beta^2(1 - \varepsilon')}} (\beta^2\varepsilon' - 1)^{1/4} \sqrt{-i\omega}\, \exp\left\{ i\left(-\omega t + \frac{\omega}{v}\sqrt{\beta^2\varepsilon' - 1}\, R\sin\theta + \frac{\omega}{v}\sqrt{1 + \beta^2(1 - \varepsilon')}\, R\cos\theta \right) \right\} d\omega. \tag{27}$$

The first term in this expression is to be associated with the transition radiation since the second term appears only if the condition in (26) is satisfied, yielding the Cerenkov radiation. The exponential term in the integrand of this term shows that the Cerenkov field of frequency ω is propagated at an angle $\vartheta(\omega)$ to the direction of motion where $\vartheta(\omega)$ is defined by

$$\sin\vartheta(\omega) = \sqrt{\beta^2\varepsilon'(\omega) - 1}/\beta.$$

This result can be understood on the basis of the following simple observations. Cerenkov radiation of frequency ω moves at an angle given by $\cos\theta' = 1/\beta\sqrt{\epsilon'(\omega)}$ in the first medium. If now we apply the law of refraction, the angle $\vartheta(\omega)$ is given by the above expression. The left-hand part of the double inequality in (26) is the condition for the production of Cerenkov radiation in the first medium. The right-hand part, however, which now can be written in the form $\vartheta(\omega) \le \theta$, indicates that the field at frequency ω moving at an angle $\vartheta(\omega)$ can be seen at angles larger than or equal to $\vartheta(\omega)$.

Using similar arguments, we can obtain expressions for E'_{2n} and $H'_{2\varphi}$. From Eq. (27) it is obvious that the transition radiation and the Cerenkov radiation do not interfere. The total amount of transition radiation emitted during the time of flight of the particle, in the solid angle $d\Omega = \sin\theta\, d\theta\, d\varphi$, is

$$\frac{dW_{\text{trans.}}}{d\Omega} = \frac{ce^2\sin^2\theta\cos^2\theta}{\pi^2 v^2} \frac{\beta}{(1 - \beta^2\cos^2\theta)^2} \int_0^\infty \left| \frac{(\varepsilon - 1)(1 - \beta^2 - \beta\sqrt{\varepsilon - \sin^2\theta})}{(\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta})(1 - \beta\sqrt{\varepsilon - \sin^2\theta})} \right|^2 d\omega. \tag{28}$$

In the case of transparent media, the integration in the last formula does not extend over frequencies which lie in the region $\omega' \pm \Delta\omega'$.

For the Cerenkov radiation, we compute the Poynting vector flux through the annular area ρ, $\rho + d\rho$ during the time of flight of the particle:

$$\frac{dW_{\text{trans.}}}{d\rho} = \frac{c}{4\pi} \int_{-\infty}^{+\infty} dt E'_2 H'_{2\varphi} \cos\vartheta(\omega)\, 2\pi\rho = \frac{4e^2}{v^2} \int_0^\infty \frac{\sqrt{(\beta^2\varepsilon' - 1)(1 + \beta^2(1 - \varepsilon'))}}{(1 + \varepsilon\sqrt{1 + \beta^2(1 - \varepsilon')})^2}\, \omega\, d\omega. \tag{29}$$

The integration in the last integral is performed only over those frequencies for which the condition in (26) is satisfied.

When $\omega = \omega'$ and $\epsilon'' = 0$, the saddle point is simultaneously a pole of the integrand, which is traversed from below. The integral along the line of descent is divided into the integral (in the sense of the principle value) and the half the residue at the pole. As a result, for frequencies close to ω' the following formula applies:

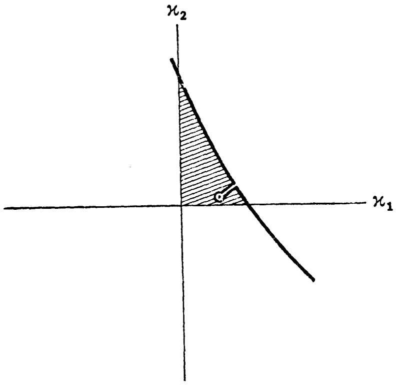

FIG. 4

$$\frac{dW_{\text{trans}}}{d\Omega} = \frac{ce^2 \sin^2\theta \cos^2\theta}{\pi^2 v^2} \frac{\beta^4}{(1-\beta^2\cos^2\theta)^2} \int \left(\frac{\varepsilon+1}{\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta}}\right)^2 d\omega. \quad (30)$$

The Poynting vector associated with the Cerenkov radiation also changes but only at the individual points $\omega = \omega'$.

We may note that here, just as in the preceding case, there is a region in which transition-radiation wave field is formed.

In conclusion the author wishes to express his gratitude to I. I. Gol'dman for a number of interesting discussions.

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EMISSION OF PARTICLES FROM EXCITED NUCLEI

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A formula for the dependence of the cross-section of a nuclear reaction involving the emission of several particles on the energy of incident nuclei has been obtained using the statistical theory of the nucleus and Bohr's concept of nuclear reactions. The formula has been used to compute the cross-sections for nuclear reactions involving the emission of 1, 2, 3, or 4 neutrons from Bi_{83}^{209} and I_{53}^{127} when bombarded with protons, deuterons, or α particles. The dependence of the entropy of a nucleus on the excitation energy and mass number has been determined using the gas model of the nucleus. The results of the calculations agree satisfactorily with the experimental data.

It is known that an excited nucleus emits particles (n, p, α, γ, etc.) in transition to the ground state, the energy distribution of the particles being approximately Maxwellian (as follows, for example, from the evaporation model of the decay[1]). Accordingly, the mean energy carried away by an emitted particle is much smaller than the excitation energy of the nucleus.[2] If the excitation energy is sufficiency large the nucleus cannot, therefore, return to the ground state emitting a single particle.

Both theory[1] and experiment[3,4] show that the probability of emission of a single particle decreases with increasing excitation energy of the nucleus. Consequently, in transition to the ground state such a nucleus