Note added in proof (November 16, 1957). We note that the solution of (1) for both fields of arbitrary intensity is very cumbersome, but does not lead to results new in principle. Because of the transitions through the common level, which is an intermediate one, the absorption maximum is split for the frequency $\omega_1$ as well as for $\omega_2$.

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# *DETERMINATION OF PHASES OF MATRIX ELEMENTS OF THE S-MATRIX*

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For reactions involving two or three channels and reactions with the formation of three low-energy particles, it is shown how the phases of the matrix elements of the S-matrix can be expressed in terms of measurable quantities by using the unitary property of the S-matrix and the invariance of the theory with respect to time reversal. The reactions $\tau^+ \to 2\pi^+ + \pi^-$ and $p + p \to D + \pi^+$ are considered.

In papers by Aizu, Fermi, and Watson[1,2] a relation has been established between the phases of the amplitudes for photoproduction of $\pi$ mesons on a nucleon and the scattering phase shifts for scattering of $\pi$ mesons by a nucleon; this relation follows from the unitary property of the S-matrix and the invariance of the theory with respect to time reversal. Subsequently analogous relations have been obtained in connection with a number of other reactions. But in all these cases the interaction in one of the channels was assumed to be weak. It seems that it may be of use to consider similar relations in the case of strong interactions.

The consequences of the unitary property of the S-matrix and of the invariance of the theory with respect to time reversal are usually discussed by means of the so-called K-matrix. That is, S is written in the form

$$S = (1 - iK)/(1 + iK)$$

and it is shown that in virtue of the properties of the S-matrix the matrix K is real and symmetric in a representation in which spin components are not included in the set of quantum numbers that specify the states of the system. If there are n channels, this means that the matrix elements of the S-matrix are expressed in terms of $n(n+1)/2$ independent real numbers. But the matrix elements of the matrix K are not directly measurable quantities. The quantities directly measured are the squares of the absolute values of matrix elements of the matrix $T = -i(S - 1)$.

The matrix elements of T, however, are complex. Therefore it is of interest to express the phases of the matrix elements of the T-matrix in terms of the squares of their absolute values. In a number of cases this can be done. At the same time, there appear several inequalities connecting the squares of the absolute values of the matrix elements of the T-matrix. The use of the K-matrix is particularly inconvenient for the consideration of states in which there are more than two particles. In these cases it is more convenient to start directly from the conditions for the unitary property of the S-matrix.

In the present paper these conditions are used to determine the phases of elements of the T-matrix in terms of scattering phase shifts and transition probabilities for the case of two or three channels. It is shown that the phases can also be determined in those cases, in which one of the channels contains three particles with small energy of relative motion. The imaginary part of the amplitude for the decay of a $\tau$ meson into three $\pi$ mesons is expressed in terms of the real part and the scattering amplitudes for collisions between $\pi$ mesons. At the same time it is shown that the interaction in the final state changes considerably the angular distribution of the decay products. The corrections caused by the interaction in the final state are determined for the phases of the matrix elements that fix the angular distribution and polarization in the reaction $p + p \rightarrow D + \pi^+$.

## 1. CASE OF TWO OR THREE CHANNELS

From the equation $SS^+ = 1$ it follows that

$$i(T - T^+) = -TT^+.$$

Let us consider the matrix elements of the right and left members between states $|a\rangle$ and $|b\rangle$ of the type indicated. Using the symmetry of S and the rule of matrix multiplication we get

$$\operatorname{Im}\langle a|T|b\rangle = \frac{1}{2}\sum_c \langle a|T|c\rangle\langle b|T|c\rangle^*, \tag{1}$$

$$\langle a|T|b\rangle = \langle a|T'|b\rangle\,\delta(\mathbf{P}_a - \mathbf{P}_b)\,\delta(E_a - E_b), \tag{2}$$

$E_a$, $\mathbf{P}_a$ are the energy and momentum of the system in the state $|a\rangle$, and $\langle a|T'|b\rangle$ is the T matrix on the energy surface. Substituting Eq. (2) into Eq. (1), we get the well known relation

$$\operatorname{Im}\langle a|T'|b\rangle = \frac{1}{2}\sum_c \langle a|T'|c\rangle\langle b|T'|c\rangle^*\,\delta(E_a - E_c)\,\delta(\mathbf{P}_a - \mathbf{P}_c). \tag{3}$$

Carrying out the summation with respect to $\mathbf{P}_c$, $E_c$ on the right side of Eq. (3) we get

$$\operatorname{Im}\langle a|T'|b\rangle = \frac{1}{2}\sum_c \langle a|T'|c\rangle\langle b|T'|c\rangle\,\Gamma_c. \tag{4}$$

Here the summation extends only over states for which

$$\mathbf{P}_c = \mathbf{P}_a = \mathbf{P}_b;\quad E_c = E_a = E_b.$$

$\Gamma_c$ is the phase volume of the states $|c\rangle$ with energy $E_c$. Multiplying the right and left members by $\frac{1}{2}(\Gamma_a\Gamma_b)^{\frac{1}{2}}$ and writing

$$\frac{1}{2}\langle a|T'|b\rangle\sqrt{\Gamma_a\Gamma_b} = f_{ab}, \tag{5}$$

we get

$$\operatorname{Im} f_{ab} = \sum_c f_{ac} f_{bc}^*.$$

In the case a = b

$$\operatorname{Im} f_{ab} = \sum_c |f_{ac}|^2$$

and we get the usual result

$$f_{aa} = (\eta_a e^{2i\delta_a} - 1)/2i,\quad \eta_a = 1 - 4\sum_{c\neq a}|f_{ac}|^2. \tag{7}$$

Let us examine the case in which there are only two channels, as for example in the reaction $\pi^0 + p \to \pi^+ + n$ or in $K + N \to \Sigma + \pi$ if the initial state has isotopic spin $T = 0$ and we can neglect the processes $K + N \to \Sigma + \pi + \bar{n}$, $\Lambda + \bar{n} + \pi$ because of threshold effects for small energies of the K particle. In these cases we have

$$\operatorname{Im} f_{ab} = f_{ab} f_{ab} + f_{ab} f_{bb}^{*}. \tag{8}$$

Substituting Eq. (7) into Eq. (8), we get

$$f_{ab}^{*} \eta_a e^{2i\delta_a} - f_{ab} \eta_b e^{-2i\delta_b} = 0. \tag{9}$$

Since $\eta_a = \eta_b$ by Eq. (7), it follows from Eq. (9) that

$$f_{ab} = X_{ab} e^{i(\delta_a + \delta_b)}, \tag{10}$$

where $X_{ab}$ is real. Equation (10) is a generalization of the relations of Aizu, Fermi, and Watson in the case of strong interactions.

In the case in which there are three channels, as for example in the scattering of K mesons by nucleons in the state with isotopic spin 1, when besides the elastic scattering there are possible the reactions $K + N \to \Lambda + \pi$ and $K + N \to \Sigma + \pi$, we have instead of Eq. (8), if the reactions $K + N \to \Lambda + 2\pi$, $\Sigma + 2\pi$ can be neglected,

$$\operatorname{Im} f_{ab} = f_{aa} f_{ab}^{*} + f_{ab} f_{bb}^{*} + f_{ac} f_{bc}^{*}. \tag{11}$$

Setting $f_{ab} = X_{ab} e^{i\varphi_{ab}}$ and using Eq. (7) we get

$$\frac{1}{2i}\left[\eta_a \exp(2i\delta_b - i\varphi_{ab}) - \eta_b \exp(-2i\delta_a + i\varphi_{ab})\right] X_{ab} + X_{bc} X_{ca} \exp(i(\varphi_{bc} - \varphi_{ca})) = 0.$$

Equating the absolute values of the two terms we have

$$\cos 2(\delta_a + \delta_b - \varphi_{ab}) = \frac{X_{ab}^2 - 4X_{ab}^4 - 2(X_{ac}^2 + X_{bc}^2) X_{ab}^2 - 2X_{ac}^2 X_{bc}^2}{\eta_a \eta_b X_{ab}^2}. \tag{12}$$

Equation (12) solves the problem as stated, but in order for it actually to determine the phase it is necessary that the right member be less than unity in absolute value. This condition gives

$$X_{ab} X_{bc} X_{ac} \geqslant X_{ab}^2 X_{bc}^2 + X_{ac}^2 X_{bc}^2 + X_{ab}^2 X_{ac}^2. \tag{13}$$

Or, solving for $X_{ab}$ for example, we get

$$1/(1 + \eta_c) \leqslant X_{ab} / 2 X_{ac} X_{cb} \leqslant 1/(1 - \eta_c). \tag{14}$$

## 2. DECAY OF THE $\tau$ MESON

For small energy of the relative motion in one of the channels, we can treat also more complicated cases, for example those in which states with three particles are possible. As an example consider the decay of the $\tau$ meson, on the assumption that the decay interaction is invariant with respect to time reversal and that the spin of the $\tau$ meson is zero. We shall treat the $\pi$ mesons as nonrelativistic and regard their wavelengths as large in comparison with the radius of interaction. In this case Eq. (3) gives

$$\operatorname{Im} \langle \tau^+ | T' | \tau_1 \tau_2 \tau_3 \mathbf{p}_3 \mathbf{k}_{12} \rangle = \frac{1}{2} \sum_{\tau_1' \tau_2' \tau_3'} \int d^3p_3' d^3k_{12}' \langle \tau^+ | T' | \tau_1' \tau_2' \tau_3' \mathbf{p}_3' \mathbf{k}_{12}' \rangle \langle \tau_1 \tau_2 \tau_3 \mathbf{p}_3 \mathbf{k}_{12} | T' | \tau_1' \tau_2' \tau_3' \mathbf{p}_3' \mathbf{k}_{12}' \rangle \, \delta(M - E_{\mathbf{p}_3', \mathbf{k}_{12}'}). \tag{15}$$

Here $\tau_1$, $\tau_2$, $\tau_3$, are isotopic spin components for the three mesons, $\mathbf{k}_{12}$ is the momentum of the relative motion of the identical $\pi$ mesons, $\mathbf{p}_3$ is the momentum of the third $\pi$ meson in the rest system of the $\tau$ meson, and M is the mass of the $\tau$ meson.

Equation (15) contains the matrix element of the scattering of the three $\pi$ mesons by each other, which can be represented as the sum of four contributions in the following way:

$$\langle \tau_1\tau_2\tau_3\mathbf{p}_3\mathbf{k}_{12} | T' | \tau_1'\tau_2'\tau_3'\mathbf{p}_3'\mathbf{k}_{12}' \rangle = \delta(\mathbf{p}_3 - \mathbf{p}_3')\,\delta_{\tau_3\tau_3'} \langle \tau_1\tau_2\mathbf{k}_{12} | T' | \tau_1'\tau_2'\mathbf{k}_{12}' \rangle + \delta(\mathbf{p}_2 - \mathbf{p}_2')\,\delta_{\tau_2\tau_2'} \langle \tau_1\tau_3\mathbf{k}_{13}' | T' | \tau_1'\tau_3'\mathbf{k}_{13}' \rangle$$
$$+ \delta(\mathbf{p}_1 - \mathbf{p}_1')\,\delta_{\tau_1\tau_1'} \langle \tau_2\tau_3\mathbf{k}_{23} | T' | \tau_2'\tau_3'\mathbf{k}_{23}' \rangle + \langle \tau_1\tau_2\tau_3\mathbf{p}_3\mathbf{k}_{12} | T' | \tau_1'\tau_2'\tau_3'\mathbf{p}_3''\mathbf{k}_{12}' \rangle_c. \tag{16}$$

In terms of Feynman diagrams the first three terms in Eq. (16) correspond to diagrams in which the line of one of the mesons is not connected with the others, and the last term corresponds to the connected diagrams. If we assume that the interaction between the $\pi$ mesons has a definite radius, then the first three terms correspond to processes in which one of the particles was outside the region of interaction during the scattering of the other two. It is also clear that the first three terms contain scattering amplitudes of pairs of $\pi$ mesons. Such a decomposition has meaning, since it will be shown that for small energy of the relative motion of the $\pi$ mesons the contribution of the last term in Eq. (15) is small. In fact, substituting Eq. (16) into Eq. (15) we get

$$\operatorname{Im} \langle \tau^+ | T' | \tau_1\tau_2\tau_3\mathbf{p}_3\mathbf{k}_{12} \rangle = \frac{1}{4}\mu k_{12} \sum_{\tau_1'\tau_2'} \int d\Omega_{\mathbf{k}_{12}'} \langle \tau^+ | T' | \tau_1'\tau_2'\tau_3\mathbf{p}_3\mathbf{k}_{12}' \rangle \langle \tau_1\tau_2\mathbf{k}_{12} | T' | \tau_1'\tau_2'\mathbf{k}_{12}' \rangle^*$$
$$+ \frac{1}{4}\mu k_{13} \sum_{\tau_1'\tau_3'} \int d\Omega_{\mathbf{k}_{13}'} \langle \tau^+ | T' | \tau_1'\tau_2\tau_3'\mathbf{p}_2\mathbf{k}_{13}' \rangle \langle \tau_1\tau_3\mathbf{k}_{13} | T' | \tau_1'\tau_2'\mathbf{k}_{13}' \rangle^* + \frac{1}{4}\mu \mathbf{k}_{23} \sum_{\tau_2'\tau_3'} \int d\Omega_{\mathbf{k}_{23}'} \langle \tau^+ | T' | \tau_1\tau_2'\tau_3'\mathbf{p}_1\mathbf{k}_{23}' \rangle$$
$$\times \langle \tau_2\tau_3\mathbf{k}_{23} | T' | \tau_2'\tau_3'\mathbf{k}_{23}' \rangle^* + \frac{\mu}{4} \sum_{\tau_1'\tau_2'\tau_3'} \int d^3p_3' \sqrt{\mu\left(M - \frac{3}{4}\frac{p_3'^2}{\mu}\right)}\, d\Omega_{\mathbf{k}_{12}'}$$
$$\times \langle \tau^+ | T' | \tau_1'\tau_2'\tau_3'\mathbf{p}_3'\mathbf{k}_{12}' \rangle \langle \tau_1\tau_2\tau_3\mathbf{p}_3\mathbf{k}_{12} | T' | \tau_1'\tau_2'\tau_3'\mathbf{p}_3'\mathbf{k}_{12}' \rangle^*_c. \tag{17}$$

Here $\mu$ is the mass of the $\pi$ meson.

If we consider the right member as a function of the energy of the relative motion of the $\pi$ mesons, then it can be seen that (because of the tendency of all the matrix elements to a constant limit) the first three terms in Eq. (17) go to zero like $k_{12}$, $k_{13}$, or $k_{23}$, i.e., proportional to $(M - 3\mu)^{\frac{1}{2}}$, while the last term varies as $(M - 3\mu)^2$. If we neglect all powers of $k_{12}$, $k_{13}$, $k_{23}$ except the first, we get

$$\operatorname{Im} \langle \tau^+ | T' | \tau_1\tau_2\tau_3\mathbf{p}_3\mathbf{k}_{12} \rangle = \pi\mu \sum_{\tau_1'\tau_2'\tau_3'} \langle \tau^+ | T' | \tau_1'\tau_2'\tau_3' \rangle \left\{ \delta_{\tau_3\tau_3'} a_{\tau_1\tau_2}^{\tau_1'\tau_2'} k_{12} + \delta_{\tau_2\tau_2'} a_{\tau_1\tau_3}^{\tau_1'\tau_3'} k_{13} + \delta_{\tau_1\tau_1'} a_{\tau_2\tau_3}^{\tau_2'\tau_3'} k_{23} \right\}, \tag{18}$$

Here $<\tau^+|T'|\tau_1'\,\tau_2'\,\tau_3'>$ is the amplitude for decay with neglect of the energy of the $\pi$ mesons (a real quantity) and $a_{\tau_1\tau_2}^{\tau_1'\tau_3'}$ is the scattering amplitude for collisions between $\pi$ mesons of zero energy. If we make use of invariance in isotopic spin space, then we have

$$a_{\tau_1\tau_2}^{\tau_1'\tau_2'} = \sum_{T=0,2} (11\tau_1\tau_2 | 11\,TT')(11\tau_1'\tau_2' | 11\,TT')\, a_T, \tag{19}$$

Here $(1\tau_1 1\tau_2 | 11TT')$ are Clebsch-Gordan coefficients, and $a_0$, $a_2$ are the scattering amplitudes of $\pi$ mesons in the states with isotopic spins $T = 0, 2$.

It must be noted that Eq. (18) is valid only under the condition that the scattering of the $\pi$ meson by the $\pi$ meson does not have resonance character, with $ka \sim 1$ already at very small energies, since in this case the neglect of high powers of $k$ is not permissible. Equation (18) is of interest in the following connection: if we expand the decay amplitude in terms of spherical harmonics, then for example for the decay $\tau^+ \to 2\pi^+ + \pi^-$ we have

$$\langle \tau^+ | T' | 1,1,-1;\, \mathbf{p}_3\mathbf{k}_{12} \rangle = \sum_L P_L(\cos\vartheta) \langle \tau^+ | T' | 1,1 - 1Lp_3,\, Lk_{12} \rangle. \tag{20}$$

Here $L$ is the angular momentum of the relative motion of the two $\pi$ mesons; $L = 0, 2, 4,$ since particles 1 and 2 are identical; $L$ is also equal to the angular momentum of the third particle relative to the center of mass of the first two, because the spin of the $\tau$ meson is taken to be zero; and $\vartheta$ is the angle between $\mathbf{p}_3$ and $\mathbf{k}_{12}$. If we neglect the interaction of the $\pi$ mesons in the final state, then for small $p_3$ and

$k_{12}$ the matrix element in Eq. (20) is proportional to $p_3^L k_{12}^L$, i.e., the coefficient of $\cos^2\vartheta$ in the angular distribution is about $(M - 3\mu)^2$. But according to Eq. (18) the coefficient of $\cos^2\vartheta$ is different from zero already in the first order in $k \sim (M - 3\mu)^{\frac{1}{2}}$, since

$$k_{13} = \sqrt{\frac{9}{16} p_3^2 + \frac{1}{4} k_{12}^3 + \frac{3}{4} p_3 k_{12} \cos\vartheta},$$

and $k_{23}$ has an analogous dependence on the angles.

Accordingly the interaction of the $\pi$ mesons in the final state decidedly changes the energy dependence of the matrix elements that determine the angular distribution. In another paper it will be shown how this fact makes it possible to determine the scattering amplitudes for collisions between $\pi$ mesons from the angular distribution of the decay products. In a similar way one can treat for example the photoproduction of two $\pi$ mesons from a nucleon for energies near the threshold.

## 3. THE REACTION $p + p \rightarrow D + \pi^+$

As another example of the application of Eq. (3) we can take the reaction

$$p + p \rightarrow D + \pi^+.$$

The amplitude for the reaction in the center-of-mass system can be written in the form

$$\langle \mathbf{k} s_z | T' | \sigma_1\sigma_2 \mathbf{p}\rangle = \sum (l\, 1 m s_z | l\, 1 j\mu)(l's'm's_z' | l's'j\mu)(\tfrac{1}{2}\,\tfrac{1}{2}\,\sigma_1\sigma_2 | \tfrac{1}{2}\,\tfrac{1}{2}\, s's_z')\langle jlk | T' | jl's'p\rangle Y_{lm}(\Omega_{\mathbf{k}}) Y_{l'm'}(\Omega_{\mathbf{p}_s}). \tag{21}$$

Here $\mathbf{k}$ is the momentum of the meson and $\mathbf{p}$ that of the proton. The remaining notations are obvious.

Gell-Mann and Watson have shown[3] that if one neglects the scattering of the $\pi$ meson by the deuteron the phase of the quantity $\langle jlk | T' | jl's'p'\rangle$ is equal to the scattering phase shift for proton-proton scattering in the state $|jl's'p'\rangle$. Equation (3) enables us to calculate the correction to the phase caused by the interaction in the final state. Since we are interested in the reaction amplitude for small energy, we confine ourselves to $l = 0, 1$. Under these conditions $l$ and $l'$ are good quantum numbers and it follows from Eq. (3) that:

$$\begin{aligned} \operatorname{Im}\langle jlk | T' | jl's'p\rangle &= \frac{1}{2} pM \langle jlk | T' | jl's'p\rangle \langle jl's'p | T' | jl's'p\rangle^* + \frac{1}{2} k\mu \langle jlk | T' | jlk\rangle \langle jl's'p | T' | jlk\rangle^* \\ &+ \sum_{L\Lambda s''} \int q_1^2 dq_1 p_1^2 dp_1 \delta(E - E_{p_1, q_1}) \langle jlk | T' | jLp_1\Lambda q_1 s''\rangle_0 \langle jl's'p | T' | jLp_1\Lambda q_1 s''\rangle_0^* \\ &+ \sum_{L\Lambda s''} \int q_1^2 dq_1 p_1^2 dp_1 \delta(E - E_{p_1, q_1}) \langle jlk | T' | jLp_1\Lambda q_1 s''\rangle_+ \langle jl's'p | T' | jLp_1\Lambda q_1 s''\rangle_+^*. \end{aligned} \tag{22}$$

$M$ and $\mu$ are the masses of the proton and the $\pi$ meson. The third and fourth terms arise from the respective processes

$$p + p \rightarrow p + p + \pi^0 \rightarrow D + \pi^+, \quad p + p \rightarrow p + n + \pi^+ \rightarrow D + \pi^+,$$

which are possible if the energy of the $\pi^+$ is larger than the binding energy of the deuteron. In the case in question these terms need not be small in comparison with the second, since at low energy the nucleons have a resonance interaction.

Setting

$$\langle jlk | T' | jl's'p\rangle = \rho e^{i\varphi}; \quad \frac{1}{2} pM \langle jl's'p | T' | jl's'p\rangle = (e^{2i\delta_{l'}}\eta_{l'} - 1)/2i; \quad \frac{1}{2} k\mu \langle jlk | T' | jlk\rangle = (e^{2i\delta_l}\eta_l - 1)/2i; \quad \varphi = \delta_{l'} + \varphi'$$

and using equations analogous to Eq. (22) for the amplitudes appearing in the right member, we get the first order correction

$$\begin{aligned} \varphi' &= \delta_l + \frac{1}{\rho} \sum_{L\Lambda s''} \int q_1^2\, dq_1 p_1^2\, dp_1 \delta(E - E_{p_1, q_1}) \operatorname{Re}\langle jlk | T' | jLp_1\Lambda q_1 s''\rangle_0 \\ &\times |\langle jl's'p | T' | jLp_1\Lambda q_1 s''\rangle_0| + \frac{1}{\rho} \sum_{L\Lambda s''} \int q_1^2\, dq_1 p_1^2\, dp_1 \delta(E - E_{p_1, q_1}) \operatorname{Re}\langle jlk | T' | jLp_1\Lambda q_1 s''\rangle_+ |\langle jl's'p | T' | jLp_1\Lambda q_1 s''\rangle_+|. \end{aligned}$$

Since in this approximation

$$\mathrm{Im}\langle jlk|T'|jLp_1\Lambda q_1 s''\rangle = \rho\frac{1}{2}pM|\langle jl's'p|T'|jLp_1\Lambda q_1 s''\rangle|,$$

the right member contains experimentally measurable quantities.

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# *THE FEYNMAN PATH INTEGRAL FOR THE DIRAC EQUATION*

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It is shown that, subject to certain assumptions about the motion of the electron, the Feynman path integral is identical with the propagation function of the Dirac equation.

## 1. INTRODUCTION

The problem considered below was initiated by Feynman and has been discussed in Refs. 1—4. The main result of the first paper[1] is the relation

$$R(x_1t_1|x_2t_2) = \int e^{iS\{x(t)\}/h}\delta x(t). \tag{1}$$

Here $R(x_1t_1|x_2t_2)$ is the solution of the Schrödinger equation which becomes $\delta(x_1 - x_2)$ for $t_1 = t_2$;

$$S = \int_{t_1}^{t_2} L\{\dot{x}(t), x(t)\}\,dt$$

is the increase of the classical action along a path; and $\int\delta x(t)$ denotes integration (summation) over all trajectories $x(t)$ for which $x(t_1) = x_1$ and $x(t_2) = x_2$.

Together with certain other rules, the relation (1) gives a closed formulation of quantum mechanics. This formulation is mathematically equivalent to the usual formalism, but the Feynman approach has a number of advantages. In particular, the formal solution of any problem is obtained in the form of an infinitely multiple integral over the paths. There is a hope that just this approach will give a simple and perspicuous system of concepts and notations which will make it possible to get beyond the framework of perturbation theory in quantum mesodynamics.

Another virtue of the method is the space-time description and the absence of operators. One can follow in thought the motion of the particle—the motion turns out to be a special case of a Markov random process.