$$a_l^{(3)} = \frac{1}{\pi} \sum_{m_j=-j}^{j} \left\{ \frac{(l + {}^1\!/_2 \pm m_j)^2}{(2l+1)^2} \Theta_{l m_j - {}^1\!/_2}(\theta\varphi)\, \Theta_{l m_j - {}^1\!/_2}(\theta'\varphi') \sin\left(m_j - \frac{1}{2}\right)(\varphi - \varphi') \right\},$$

where the plus sign is taken for $j = \ell + \frac{1}{2}$ and the minus sign for $j = \ell - \frac{1}{2}$; $\Theta_{\ell m}(\theta\varphi)$ are normalized associated Legendre polynomials, and the angles θ, φ and θ', φ' fix the direction of the vectors $\mathbf{q}'$ and $\mathbf{q}'' = \mathbf{q}' + \mathbf{q} = \hbar c \mathbf{k}''$. When $q \ll q_n$, q' and q'' are almost independent of the angles and are given by

$$q' \approx q_n, \quad q'' \approx q_n, \quad q'q \approx -q^2/2. \tag{2}$$

In this case Φ_ℓ and Φ'_ℓ are also independent of the angles and the integration in Eq. (1) is greatly simplified. In this approximation the second term in the expression for Q_1 can also be neglected.

We note that when the neutron is ejected from an s state $a_\ell^{(k)} = 0$. Then in the energy region defined by (2) we obtain for the angular distribution of the neutrons the formula

$$d\sigma/d\Omega_n = A\cos^2\theta_n + B\cos\theta_n + C,$$

where θ_n is the angle between $\mathbf{q}_n$ and $\mathbf{q}_i$ and the coefficients A, B and C, after integration with respect to Ω and $k_{n\alpha}$, depend only on the initial energy ϵ_i of the electron.

In conclusion, the author considers it his pleasant duty to thank Professor V. S. Mamasakhlisov for his interest.

[1] M. Ia. Kobiashvili, Труды Тбилисск. ун-та (Trans. Tbilisi Univ.) **62** (1957).
[2] J. M. Kennedy, Phys. Rev. **87**, 953 (1952).
[3] Thie, Mullin, and Guth, Phys. Rev. **87**, 962 (1952).
[4] L. I. Schiff, Quantum Mechanics (McGraw-Hill Book Co., New York 2nd Ed., 1955).

Translated by I. Emin
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LOCALIZATION OF NUCLEONS IN THE O_8^{16} NUCLEUS

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Submitted to JETP editor April 18, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1507-1508 (December, 1957)

CERTAIN problems related to the structure of the nucleus call for a determination of the localization of the nucleons in the nucleus.[1,2] In the present work, an attempt is made to find the localization of the nucleons in the O_8^{16} nucleus by a method developed by Daudel.[3]

Let us consider a system of nucleons in a volume V in a state with a definite value of spin projection; p is the number of protons (neutrons) with spin $\frac{1}{2}$. We break up the space V into p volumes v_i such that one can determine in each of these the probability P_i of meeting a proton (neutron) with spin $\frac{1}{2}$. The volumes v_i are called "boxes" following Ref. 3. The same determination is also applied to spin $-\frac{1}{2}$. The quantity

$$\eta = \left(p - \sum_i P_i\right)/p$$

characterizes the absence of localization of nucleons for two chosen divisions of the space V into boxes. The smaller the value of η for a given volume, the better the division into boxes.

Let $(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$ be the wave function of the nucleus, then

$$P_i = C_p^A\, p \int_{v_i} d\mathbf{r}_1 \int_{V-v_i} d\mathbf{r}_2 \ldots \int_{V-v_i} d\mathbf{r}_p \int_V d\mathbf{r}_{p+1} \ldots \int_V d\mathbf{r}_A \psi^*\psi.$$

According to the oscillator model, the nucleus O^{16} is in an $(s_{1/2})^4\,(1p_{3/2})^8\,(1p_{1/2})^4$ state. The wave function of the nucleus is $\psi = \det \varphi_i \det \varphi_k$; φ_i is the wave function of a single proton in the oscillator model and the subscript k refers to a neutron. Let us break up the volume of the nucleus into four boxes corresponding to the four protons (neutrons) with spin $\frac{1}{2}$ ($-\frac{1}{2}$). From symmetry considerations we examine three divisions of the nuclear volume into boxes: (a) Three concentric spheres; (b) One sphere of radius a with three zones specified in the following manner:

$$0 \leqslant \theta \leqslant x, \quad x \leqslant \theta \leqslant \pi - x, \quad \pi - x \leqslant \theta \leqslant \pi$$

(where $a \le r \le \infty$, $0 \le \varphi \le 2\pi$); (c) The protons (neutrons) with spin $\frac{1}{2}$ ($-\frac{1}{2}$) are situated on the vertices of a tetrahedron. The second division corresponds to a geometrical localization of nucleons over s and p shells. The boundaries of the boxes are chosen to make ΣP_i a maximum for each division. For divisions (a), (b), and (c) the values of η are respectively 0.545, 0.724, and 0.756.

Thus the division of the volume of the nucleus into boxes by concentric spheres is the best of all those considered. The radii of the spheres are equal to 0.768R, 1.023R, and 1.316R, where $R = 3.276 \times 10^{-13}$ cm. is the radius of the nucleus, determined from the maximum slope of the nucleon density distribution curve. For a radius of 0.768R the nucleon density amounts to 88% of its maximum value, so that three boxes are located in the surface layer of the nucleus and one in the center. There is no geometrical localization of nucleons over s and p shells.

By determining the dimensions of a box we establish an upper limit for the diameter of the nucleon. The first spherical layer has the smallest transverse dimension; its thickness is 0.835×10^{-13} cm. Consequently, the radius of a nucleon cannot be greater than 4.18×10^{-14} cm. This is in good agreement with experiments on scattering of electrons by protons which indicate that the radius of a proton is $\sim 4 \times 10^{-14}$ cm (Ref. 4).

[1] W. Hess and B. Moyer, Phys. Rev. **101**, 337 (1956).
[2] W. Swiatecki, Phys. Rev. **98**, 203 (1955).
[3] R. Daudel, J. Phys. et radium **16**, 503 (1955).
[4] Ia. A. Smorodinskii, Usp. Fiz. Nauk, **56**, 425 (1955).

Translated by R. Eisner
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TABULATED MASS DIFFERENCES

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Submitted to JETP editor June 19, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1508-1509 (December, 1957)

RESULTS of high-precision mass spectrometer measurements of atomic masses of stable isotopes from iron to zinc are given in the work of Quisenberry, Scolman and Nier[1] with calculations of atomic masses of the radioactive isotopes. The atomic masses of the radioactive isotopes were calculated from the atomic masses of the stable isotopes with the use of β decay energies from King's tables[2] and reaction energies from the tables of Van Patter and Whaling.[3] A check of these calculations carried out by the author has led to the finding that King's tables are not sufficiently complete and has made possible calculation of the atomic masses of Mn^{55}, Mn^{56} and Fe^{55} with great accuracy and in better agreement with other experimental data. In King's tables, the value 3.65 ± 0.03 Mev given for the total energy of the $Mn^{56} \rightarrow Fe^{56}$ β transition is derived as a weighted mean from the data of three works.[5,6,8] The author knows of seven works in which are published measurements of the limit of the β spectrum and the energy of the γ quanta emitted on β decay of Mn^{56} (they are listed in the table). The weighted mean of all of these values yields 3.710 ± 0.011 Mev for the total energy of the β decay of Mn^{56}. Using this value, one can calculate the mass of Fe^{55} from the mass of Fe^{56} by way of $Fe^{56} \rightarrow Mn^{56} \rightarrow Mn^{55} \rightarrow Fe^{55}$. From the mass difference $Fe^{55} - Fe^{54}$, the binding energy of the neutron in Fe^{55} is calculated from mass spectro-