$$B_{\epsilon\epsilon}^{Ij2} = 1201 g_{\epsilon\epsilon}^4 d, \quad B_{\eta\eta}^{Ij2} = 514 g_{\eta\eta}^4 d, \quad B_{\xi\xi}^{Ij2} = 508 g_{\xi\xi}^4 d,$$

$$B_{\xi\xi}^{Ij} B_{\eta\eta}^{Ij} = 88 g_{\xi\xi}^2 g_{\eta\eta}^2 d, \quad B_{\eta\eta}^{Ij} B_{\epsilon\epsilon}^{Ij} = -603 g_{\eta\eta}^2 g_{\epsilon\epsilon}^2 d,$$

$$B_{\xi\xi}^{Ij} B_{\epsilon\epsilon}^{Ij} = -597 g_{\xi\xi}^2 g_{\epsilon\epsilon}^2 d,$$

where the η axis is directed to the nearest neighbor of the paramagnetic ion in the $\xi\eta$ plane which is perpendicular to ϵ, where $d = \beta^4 a^{-6}$, and where a is the largest direction in the elementary cell.[6] For double nitrates of the rare-earth elements[6] $B_{\epsilon\epsilon}^{Ij2} = 32 g_{\epsilon\epsilon}^4 d$ and for $H_0 \parallel \epsilon$ we have

$$\langle(\Delta\nu)^2\rangle_d = (18 Ph^2)^{-1} \sum_{I(\neq j)} \{e^m (x^4 + 3x^2 + 2) + 0.5 e^{2m} + 1.5 + x^2\} B_{\epsilon\epsilon}^{Ij2}, \quad x = g_\perp / g_\parallel, \tag{6}$$

where $g_\perp$ and $g_\parallel$ are the factors of the spectroscopic splitting.

For ethyl sulfates of the rare-earth elements, the term arising from $\mathcal{H}_d$ has the form

$$\langle\nu^2\rangle_d = 4a^{-6} c \{229 g_\perp^4 + \cos^2\psi (g_\parallel^2 / g^2)(553 g_\perp^4 + 147 g_\perp^2 g_\parallel^2) + \sin^2\psi (g_\perp^2 / g^2)(101 g_\perp^4 + 534 g_\parallel^4 + 871 g_\parallel^2 g_\perp^2)\},$$
$$c = \beta^4 / 16h^2, \quad g^2 = g_\parallel^2 \cos^2\psi + g_\perp^2 \sin^2\psi, \tag{7}$$

where ψ is the angle between H_t and ϵ.

In conclusion, the author expresses his thanks to Professor S. A. Al' tshuler for suggesting this topic and considering this paper.

[1] L. J. F. Broer, Physica, **10**, 801 (1943).
[2] J. H. Van Vleck, Phys. Rev. **74**, 1168 (1948).
[3] K. Kambe and J. F. Ollom, J. Phys. Soc. Japan, **11**, 50 (1956).
[4] J. S. van Wieringen, Disc. Faraday Soc. **19**, 118 (1955).
[5] G. Ia. Glebashev, Уч. зап. Казанского гос. ун-та (Scient. Notes, Kazan' State Univ.) **116**, book 1, 121 (1956).
[6] J. M. Daniels, Proc. Phys. Soc. (London) **A66**, 673 (1953).
[7] K. D. Bowers and J. Owen, Repts. Progr. Phys. **18**, 304 (1955).
[8] B. Bleaney and M. C. M. O'Brien, Proc. Phys. Soc. (London) **B69**, 1216 (1956).

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221

THERMAL CONDUCTION OF SUPERCONDUCTORS

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THANKS to the presence of a gap in the energy spectrum of superconductors,[1] the number of electronic excitations and hence the electronic thermal conductivity is exponentially small $[\sim \exp(-T_k/T)]$ for $T \ll T_k$ (T_k is the temperature of the transition to the non-superconducting state). For $T \ll T_k$ the principal role is therefore played by the lattice thermal conduction, which is connected with the reflection of phonons from boundaries and the scattering of phonons by impurities and lattice defects (the phonon-electron interaction is inappreciable in view of the fact that the number of electronic excitations is very small for $T \ll T_k$), while it is well-known[2] that phonon-phonon interaction does not play a role for $T \ll \Theta_D$, where Θ_D is the Debye temperature).

However, for somewhat higher temperatures, but still appreciably less than T_k, the electronic heat conduction κ_e becomes comparable with the lattice heat conduction and can even exceed it for not very impure specimens. Clearly the largest contribution to κ_e is then given by the scattering of the electrons by impurities. Only for $T \lesssim T_k$ can the interaction of the electrons with the phonons and with one another also play an appreciable role for κ_e.

We consider the scattering of electrons by impurities. Let the Hamiltonian of the interaction of the electrons with the impurity atoms for the normal metal be of the form

$$H' = \sum_{\mathbf{k}} (a_{\mathbf{k},1/2}^+ a_{\mathbf{k}',1/2} + a_{\mathbf{k},-1/2}^+ a_{\mathbf{k}',-1/2}) V_{\mathbf{k},\mathbf{k}'}$$

($\frac{1}{2}$ and $-\frac{1}{2}$ are the spin coordinates, and $a_{\mathbf{k},\pm\frac{1}{2}}$ the amplitude in second quantization). According to Ref. 3 the electronic excitations in superconductors can be described by new amplitudes

$$\alpha_{\mathbf{k}0} = u_k a_{\mathbf{k},1/2} - v_k a_{-\mathbf{k},-1/2}^+; \quad \alpha_{\mathbf{k}1} = u_k a_{-\mathbf{k},-1/2} + v_k a_{\mathbf{k},1/2}^+;$$

$$\left.\begin{matrix} u_k^2 \\ v_k^2 \end{matrix}\right\} = \frac{1}{2}(1 \pm \xi / \sqrt{\Delta^2(T) + \xi^2}); \tag{1}$$

$\xi = (p^2 - p_0^2)/2m \approx v_0(p - p_0)$ is the energy of a normal electron, reckoned from the Fermi surface ($p = p_0$), and $\Delta(T)$ is the value of the gap in the energy spectrum.

Expressing the $a_{\mathbf{k}}$ in terms of the $\alpha_{\mathbf{k}}$ we find

$$H' = \sum_{\mathbf{k}} (u_k u_{k'} - v_k v_{k'}) (\alpha^{+}_{\mathbf{k}0}\alpha_{\mathbf{k}'0} + \alpha^{+}_{\mathbf{k}1}\alpha_{\mathbf{k}'1}) V_{\mathbf{kk}'}.$$

We have omitted here terms of the kind $\alpha^{+}_{\mathbf{k}0}\alpha^{+}_{\mathbf{k}'1}$ and $\alpha_{\mathbf{k}0}\alpha_{\mathbf{k}'1}$, which describe the creation and annihilation of a pair of excitations; these processes are not possible in the case of elastic collisions with impurities. For elastic scattering we have

$$u_k u_{k'} - v_k v_{k'} = u_k^2 - v_k^2 = \xi (\Delta^2 + \xi^2)^{-1/2}.$$

The scattering probability is determined by the equation

$$w_{ab} = \frac{2\pi}{\hbar} |H'|^2_{ab}\rho_E, \quad \rho_E = \frac{p^2\, d\Omega}{h^3}\frac{dp}{d\varepsilon}.$$

The energy of an electron excitation ϵ is of the form[1,3] $\epsilon = \sqrt{\Delta^2(T) + \xi^2}$ $[\Delta(T) = 0$ for $T = T_k]$, so that $\rho_E \approx d\Omega p_0^2 h^{-3}\epsilon/|\xi| v_0$. We see that for electron excitations in a superconductor the probability of scattering by impurities differs from the scattering probability w_0 in normal metals by the factor $(u_k^2 - v_k^2)^2 = \xi^2/\epsilon^2$ which occurs in $|H'|^2_{ab}$, and by the factor $\epsilon/|\xi|$, which occurs in ρ_E. Thus, $w = (|\xi|/\epsilon) w_0$.

Let us assume that there is a temperature gradient $\partial T/\partial x$ in the superconductor. The electrical field in a superconductor is then, in contradistinction to a normal metal, equal to zero. However, in the equilibrium state the current of the normal component j_n is completely cancelled by the opposing superconducting current j_s ($j = j_n + j_s = 0$) (Ref. 4). We write down the transport equation for the distribution function f of the excitations,

$$\frac{\partial f}{\partial x}\frac{\partial \varepsilon}{\partial p_x} - \frac{\partial f}{\partial p_x}\frac{\partial \varepsilon}{\partial x} = \frac{|\xi|}{\varepsilon}\frac{f_0 - f}{\tau_0};$$

so that

$$\frac{\partial f_0}{\partial \varepsilon}\frac{\varepsilon}{T}\frac{\partial \varepsilon}{\partial p_x}\frac{\partial T}{\partial x} = \frac{|\xi|}{\varepsilon}\frac{f - f_0}{\tau_0}, \qquad (2)$$

where τ_0, the relaxation time for the normal electrons, does not depend on the energy; Ref. 5 gives an expression for τ_0; on the left-hand side, we have substituted for f the equilibrium function f_0; $f_0 = [\exp(\epsilon/\Theta) + 1]^{-1}$ (see Ref. 1); $\Theta = kT$.

From Eq. (2) we can find $f_1 = f - f_0$ and we can evaluate the heat flux

$$Q = 2h^{-3}\int v_x \varepsilon f_1\, d\mathbf{p}; \quad \varkappa = -Q \Big/ \frac{\partial T}{\partial x} = \frac{2}{3}\frac{p_0^3\tau_0}{\pi^2\hbar^3 m} F(T);$$

$$F(T) = \Theta^{-1}\int_{\infty}^{\Delta} \varepsilon^2 \frac{\partial f_0}{\partial \varepsilon} d\varepsilon = \frac{\Delta^2(T)}{\Theta}\left(\exp\left(\frac{\Delta}{\Theta}\right) + 1\right)^{-1} \qquad (3)$$

$$+ 2\Theta \sum_{s=1}^{\infty} (-1)^{s+1} e^{-s\Delta/\Theta}/s^2 + 2\Delta \ln(1 + e^{-\Delta/\Theta});$$

Equation (3) describes satisfactorily the experimental data obtained in Ref. 6. The temperature dependence of $\Delta(T)$ can be found in Ref. 1.

We can estimate the magnitude of the convective heat flux $Q_k = TSv_n = TSj_n/\rho_n$, mentioned in Refs. 4 and 7. Using the expressions for S and ρ_n of Ref. 1, we can easily show that the ratio of Q_k to the normal heat flux Q is of the order of magnitude $k(TT_k)^{1/2}/(p_0^2/m)$, i.e., even for $T \approx T_k$ it is of the order 10^{-5} to 10^{-4}.

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[1] Bardeen, Cooper, and Schrieffer, Phys. Rev. **108**, 1175 (1957).

[2] R. Peierls, Ann. Physik. **3**, 1055 (1929); **4**, 121 (1930). R. E. Peierls, Quantum Theory of Solids, Oxford Clarendon Press, 1955.

[3] N. N. Bogoliubov, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 58 (1958), Soviet Phys. JETP **7**, 41 (1958).

[4] V. L. Ginzburg, Сверхпроводимость, (Superconductivity) Acad. Sci. Press, 1946, p. 79; J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 979 (1951).

[5] A. Sommerfeld and H. Bethe, Electron Theory of Metals (Russian translation) ONTI, 1938, p. 227.

[6] N. V. Zavaritskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1085 (1957), Soviet Phys. JETP **6**, 837 (1958).

[7] C. J. Gorter, Can. J. Phys. **34**, 1334 (1956).

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222