mental data. To get agreement with experiment they could essentially still use two parameters. The method proposed by us is free of these shortcomings and enables us to consider the interaction of excitons with light and various centers. One can consider that the result obtained is in satisfactory agreement with the experimental value of ΔE determined from the position of the maximum of the exciton absorption band at $\lambda = 1580$ A (~ 7.85 ev).

In conclusion we note that if we take the translational symmetry of our problem into consideration we can write the wave function in the following form

$$\Psi_k = N^{-1/2}\sum_l \exp(i\mathbf{k}\mathbf{r}_2^l)\,\Psi_B^l. \qquad (2)$$

Expression (2) determines the exciton band, whose width is of the order of

$$\frac{1}{36(1+4A+B)}\sum_{l'(\neq l)}{}' \sum_{l_1, l_1'}^{6} \sum_{x\alpha,\, y\beta}^{6} \int \varphi_1^{*l_1}(\rho)\,\varphi_{2x\alpha}^{*l'}(\rho')$$

$$\times \frac{e^2}{|\rho-\rho'|}\varphi_{2y\beta}^{l}(\rho)\,\varphi_1^{l_1'}(\rho')\,d\tau d\tau' - \text{exch. term.} \qquad (3)$$

The width (3) of the exciton band is, as follows from a numerical calculation, far smaller than ΔE. This is, though, clear from the fact that in (3) functions occur referring to different halide ions and the integrals in (3) are thus much less than the analogous integrals in (1).

Since the width of the exciton band is much smaller than ΔE, the energy of the excitation can be evaluated using the simpler function Ψ_{exc} as was done in the foregoing calculations. In those cases, however, where one is interested in effects which depend essentially on the form and width of the exciton band, it is necessary to use the more exact function (2).

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261

CONSEQUENCES OF THE TWO-COMPONENT BEHAVIOR OF THE ELECTRON IN THE BETA INTERACTION

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THE latest measurements[1-3] of the longitudinal polarization of the electrons emitted in β decay show that the values of the longitudinal polarization $\langle\sigma_\parallel\rangle$ in cases of allowed transitions and first-forbidden transitions in heavy nuclei are to good accuracy equal to v/c. As can be rigorously proved from the formulas[4] for the longitudinal polarization of the electrons from such transitions, a necessary and sufficient condition for the relation $\langle\sigma_\parallel\rangle = v/c$ is the existence of the following relations between the interaction constants conserving parity and violating its conservation:

$$C_S = -C_S', \quad C_T = -C_T', \quad C_A = C_A', \quad C_V = C_V'. \qquad (1)$$

With these conditions the interaction Hamiltonian takes the form

$$H = \sum_\alpha C_\alpha(\overline{\Psi}_p O_\alpha \Psi_n)(\bar{\psi}_e(1-\gamma_5)O_\alpha\psi_\nu) + \text{c. c.}, \qquad (2)$$

and the electronic ψ function is involved in all the types of β interaction through only two components.

Let us examine the consequences of the relations (1), i.e., of the two-component behavior of the electron in the β interaction. When the conditions (1) hold the expressions for the various effects in β decay are decidedly simplified, so that in the case of allowed transitions there remain all told just six independent combinations of the constants and matrix elements:

$$N_0 = (|C_S|^2+|C_V|^2)|M_F|^2 + (|C_T|^2+|C_A|^2)|M_{GT}|^2,$$
$$N_1 = -\lambda_{jj'}(|C_T|^2+|C_A|^2)|M_{GT}|^2 - 2\delta_{jj'}\sqrt{j/(j+1)}\,\mathrm{Re}(C_SC_T^*+C_VC_A^*)M_FM_{GT}^*,$$
$$N_3 = (|C_V|^2-|C_S|^2)|M_F|^2 + {}^1/_3(|C_T|^2-|C_A|^2)|M_{GT}|^2,$$
$$N_4 = 2\delta_{jj'}\sqrt{j/(j+1)}\,\mathrm{Im}(C_VC_A^*-C_SC_T^*)M_FM_{GT}^*,$$
$$N_\nu = -\lambda_{jj'}(|C_T|^2-|C_A|^2)|M_{GT}|^2 + 2\delta_{jj'}\sqrt{j/(j+1)}\,\mathrm{Re}(C_SC_T^*-C_VC_A^*)M_FM_{GT},$$
$$N_5 = 2\delta_{jj'}\sqrt{j/(j+1)}\,\mathrm{Im}(C_VC_A^*+C_SC_T^*)M_FM_{GT}^*. \qquad (3)$$

Here

$$\lambda_{jj'} = [j(j+1)-j'(j'+1)+2]/2(j+1),$$

and $M_F = \left(\int 1\right)$ and $M_{GT} = \left(\int \sigma\right)$ are the nuclear matrix elements. The quantity N_0 determines the total probability of the β transition, N_3 the electron-neutrino angular correlation,

$$W_{e\nu} = 1 + (v/c)(N_3/N_0)(\mathbf{n}_e\boldsymbol{\nu}),$$

and N_1 the angular distribution of the electrons from oriented nuclei:

$$W_{je} = 1 + x(v/c)(N_1/N_0)(\mathbf{n}_e\mathbf{n}_j), \quad x = \langle j_z\rangle / j$$

($\mathbf{n}_e$, $\boldsymbol{\nu}$, and $\mathbf{n}_j$ are unit vectors giving the directions of the momenta of the electron and neutrino and of the spin of the nucleus).

The quantities N_0, N_1, and N_3 have already been determined experimentally. As can be seen from the formulas (3), information new in principle can now be obtained only from experiments in which N_4, N_ν, and N_5 would be measured. The quantity N_ν determines the angular distribution of the neutrinos from oriented nuclei (averaged over directions of emission of the electron)

$$W_{j\nu} = 1 + x(N_\nu/N_0)(\mathbf{n}_j\boldsymbol{\nu}).$$

Therefore the simplest experiment in which the quantity N_ν could be determined is a measurement of the angular distribution of the recoil nuclei from the decay of oriented nuclei.* The coefficient N_4 could be found from a study of the asymmetry of the distribution of recoil nuclei relative to the plane of the electron momentum and the nuclear spin. If, for example, we select electrons with momenta perpendicular to the nuclear spin, the ratio of the numbers of recoil nuclei with directions of motion on opposite sides of this plane will be

$$\left(1 - \frac{x}{2}\frac{v}{c}\frac{N_4}{N_0}\right) \Big/ \left(1 + \frac{x}{2}\frac{v}{c}\frac{N_4}{N_0}\right).$$

The quantity N_5 could be obtained from experiments on the decay of oriented (or aligned) nuclei in which, besides the direction of emission of the electron, one also measured the polarization of the recoil nucleus or the direction of the γ-quantum from a subsequent γ-transition.†

Measurements of the polarization of the electrons (both longitudinal and transverse) from oriented nuclei and in correlation with the neutrinos cannot give anything new as compared with the experiments indicated above. For example, the polarization of the electrons from the decay of oriented nuclei is given by:

$$\langle\sigma\rangle_{je} = \frac{1}{W_{je}}\left\{-\mathbf{n}_e\frac{v}{c}\left[1 + x\frac{c}{v}\frac{N_1}{N_0}(\mathbf{n}_e\mathbf{n}_j)\right] + x\frac{Z}{137\varepsilon}\bar{\eta}_0\frac{N_1}{N_0}[\mathbf{n}_e\times\mathbf{n}_j] + x\frac{\gamma_1\bar{\mu}_0}{\varepsilon}\frac{N_1}{N_0}[\mathbf{n}_e\times(\mathbf{n}_e\times\mathbf{n}_j)]\right\},$$

and the correlation of the polarization with the direction of emission of the neutrino (for unpolarized nuclei) by:

$$\langle\sigma\rangle_{e\nu} = \frac{1}{W_{e\nu}}\left\{-\mathbf{n}_e\frac{v}{c}\left[1 + \frac{c}{v}\frac{N_1}{N_0}(\mathbf{n}_e\boldsymbol{\nu})\right] + \frac{Z}{137\varepsilon}\bar{\eta}_0\frac{N_3}{N_0}[\mathbf{n}_e\times\boldsymbol{\nu}] + \frac{\gamma_1\bar{\mu}_0}{\varepsilon}\frac{N_3}{N_0}[\mathbf{n}_e\times(\mathbf{n}_e\times\boldsymbol{\nu})]\right\}.$$

Here ϵ is the energy of the electron (in units m_ec^2), $\gamma_1 = [1-(Z/137)^2]^{1/2}$, and $\bar{\mu}_0$ and $\bar{\eta}_0$ are coefficients nearly equal to unity which allow for the finite size of the nucleus. Thus measurement of $\langle\sigma\rangle_{je}$ and $\langle\sigma\rangle_{e\nu}$ does not give information that is in principle new as compared with the experiments that have already been carried out, in which the quantities N_0, N_1, and N_3 have been measured.

It is not hard to see that if we write the coefficients C_α in the form $|C_\alpha| e^{i\varphi_\alpha}$, the quantities N_i $(i = 0, \ldots 5, \nu)$ are expressed in terms of only six unknown coefficients: four absolute values $|C_\alpha|$ and the two phase differences $\varphi_T - \varphi_S$ and $\varphi_V - \varphi_A$, since with the two-component behavior of the electron the types A and V do not interfere with S and T. Therefore to obtain complete information about the β interaction in allowed transitions there is in principle no need to measure experimentally all six quantities N_i in Fermi and Gamow-Teller transitions; it is enough if one confines oneself to four of them.

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*An equivalent of this would be an experiment on a $\beta-\gamma$ transition which measured the correlation between the directions of emission of γ-quanta of a given circular polarization and of recoil nuclei.

†The problem of determining the quantity N_5 from experiments on the $\beta-\gamma$ correlation in the decay of oriented (or aligned) nuclei has been considered in detail in Refs. 5 to 7.

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262

PROTON WAVE EQUATIONS

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THE existing techniques of treatment of the electromagnetic field do not allow to handle the interaction of photons with other fields in terms of quantum field theory in a number of cases. These problems include the whole complex of gravitation-electromagnetic interactions: graviton-photon scattering, graviton bremsstrahlung by photons, etc. In order to treat such problems one has to formulate the wave equation of light quanta in matrix form.

The fundamental difficulty in formulating a matrix theory of the photon field lies in the fact that the rest mass of the photon vanishes and further that the wave function contains both electromagnetic potentials and fields. This makes the application of the Kemmer formalism exceedingly difficult.[1,2] However, by applying the Dirac algebra[3] one can remove this difficulty and it is possible to formulate a photon theory analogously to the Lee-Yang theory[4] of rest-mass-zero fermions.

In Ref. 5 it was shown explicitly how to achieve a representation of the 16-row Kemmer algebra by 8- or 4-row representations of the Dirac algebra. These same representations will have to be applied to the photon theory. (A detailed investigation of these algebrae will be published in Nuovo cimento.)

For the photon wave function we shall take the half-undor ψ which includes, besides the fields E and H, two new quantities, a scalar, ψ, and pseudoscalar, $\tilde{\psi}$. Using an 8-row representation of the Dirac algebra one can write the free field wave equation in the form

$$(\boldsymbol{\alpha}, \nabla + \partial / c\,\partial t)\,\psi(\mathrm{x}, t) = 0 \qquad (1)$$

or

$$(\boldsymbol{\alpha}^*, \nabla + \partial / c\,\partial t)\,\psi(\mathrm{x}, t) = 0, \qquad (2)$$

where

$$\tfrac{1}{2}\{\alpha_i\alpha_k\} - \delta_{ik}I = \tfrac{1}{2}\{\alpha_i^*\alpha_k^*\} - \delta_{ik}I = [\alpha_i\alpha_k^*] = 0. \qquad (3)$$

We define a matrix $\alpha_L \neq I$ with the properties

$$[\alpha_L\alpha_i] = [\alpha_L\alpha_i^*] = [r_i\alpha_L] = 0, \quad \alpha_L^2 = I \qquad (4)$$

where $r_i = \alpha_i\alpha_i^*$ are reflection matrices (here one does not sum over the indices i). It leads to the Larmor transformation for ψ: $\psi' = \alpha_L\psi$. The corresponding transformation in the neutrino theory is the Salam transformation[6] $\varphi' = \gamma_5\varphi$.

An explicit expression for α_L is

$$\alpha_L = i\alpha_1\alpha_2\alpha_3 = i\alpha_1^*\alpha_2^*\alpha_3^*, \qquad (5)$$

Besides α_L there exists another pseudoscalar operator, $i\alpha_0 = r\alpha_L$, where r has the properties

$$\{r\alpha_i\} = \{r\alpha_i^*\} = [r_i r] = 0. \qquad (6)$$

Equations (1) and (2) are invariant under Larmor transformations. In order to go over to a 4-row representation one introduces the Larmor-invariant wave function[7] $(I + \alpha_L)\psi$. Then both anticommutative groups G_α and G_{α^*} go over into the group G_γ of the Dirac matrix theory of the electron in the representation where charge conjugation is represented by complex conjugation.[5,8]

It is interesting to note that these matrices are identical with the matrices describing the two internal degrees of freedom of Fock's electron.[9,10] However, they enter linearly the operator of van Wyk's generalized gauge transformation.[11]

The Larmor photons can have different parity and can have a spin of ħ even in the case of longitudinal polarization (longitudinal-magnetic photons). In order to describe Larmor-nonsymmetrical, Maxwell photons one has to go over to a wave function which is a simultaneous solution of (1) and (2), or, of the following system of equations which is equivalent to (1), (2) in this particular case:

$$(\beta^{(+)}, \nabla + \partial / c\,\partial t)\,\psi(\mathrm{x}, t) = 0, \quad \beta^{(-)}, \nabla\,\psi(\mathrm{x}, t) = 0,$$
$$\beta^{(\pm)} = (\boldsymbol{\alpha} \pm \boldsymbol{\alpha}^*)/2. \qquad (7)$$

The wave equations (1) and (2) are derived from the Lagrangian

$$L \sim \bar{\psi}(\boldsymbol{\alpha}, \nabla + \partial / c\,\partial t)\,\psi \qquad (8)$$

(for ordinary photons $\boldsymbol{\alpha}$ here has to be replaced by $\beta^{(+)}$).

The commutation relations are, as usual,

$$[\psi(\mathrm{x}, t), \bar{\psi}(\mathrm{x}', t')] = iS(\mathrm{x} - \mathrm{x}', t - t'), \qquad (9)$$