

of Figs. 1 and 3 and diagrams with a  $\Sigma$  particle and a  $\pi$  meson, possibly with inclusion of additional experimental information.

The equation that has been obtained is the first member of an infinite system of coupled equations for the scattering amplitudes, of the type of relativistic Low equations. In the first approximation, in which one usually solves the Low equations, all the higher amplitudes are omitted, and at sufficiently low energies we arrive at a closed integral equation.\* As follows from our result (28), this equation is simply the corresponding dispersion relation, with the form of the interaction entering through the inhomogeneous term. A special property of the K mesons is the fact that a study of their scattering by means of equations of the type of the Low equations can provide a basis for important conclusions about the structure of the isotopic space. Moreover, qualitative conclusions already offer a possibility, by analogy with the  $\pi$ -meson scattering, of settling in which state the scattering will be largest.

In conclusion the writer expresses his gratitude to Professor D. D. Ivanenko for his interest

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## ONE POSSIBLE MODE OF DEVELOPMENT OF EXTENSIVE AIR SHOWERS

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The development of extensive air showers is studied under the assumption that the fraction of energy lost in interaction of ultra-high energy particles with light nuclei is subject to strong fluctuations. It is shown that the main features of extensive air showers can be explained without recourse to the hypothesis that the nuclear component plays an important role in the development of showers in the depth of the atmosphere.

### 1. INTRODUCTION

It is well known that extensive air showers (EAS) consisting of  $10^4$  to  $10^5$  particles possess a lateral distribution which is independent of the altitude of observation (within the limits of 1000 to 640 g/cm<sup>2</sup>

\*By this approximate equation  $f_{ij}(q_0, e)$  is determined not only in the physical region, but also for  $0 < q_0 < (m_K^2 + p^2)^{1/2}$ .

atmospheric depth) and that the number of such showers varies in the atmosphere exponentially, with an absorption coefficient  $1/\mu = 130$  to  $140$  g/cm<sup>2</sup>. These facts have been explained by several authors<sup>1-3</sup> who have assumed that the development of EAS is determined by the development of nuclear cascade. In that theory, the slow absorption of showers in the atmosphere is explained

by the weak absorption of nuclear active particles. The invariance of lateral distribution is attributed to the fact that high-energy nuclear-active particles carry a large amount of energy into the lower atmosphere and, transferring it to the soft component, counteract the aging of the shower, i.e., the change in the lateral distribution of shower particles.

We have already mentioned<sup>4</sup> that the above features as well as other properties of EAS, can be explained without recourse to the hypothesis that the shower development is influenced materially by the nuclear cascade. It is known that particles of up to  $\sim 10^{12}$  eV have an absorption mean free path in the atmosphere equal to  $\sim 120$  g/cm<sup>2</sup>.<sup>5</sup> We shall assume therefore that the absorption mean free path is  $L_a = 120$  g/cm<sup>2</sup> also for the particles of higher energies ( $E_0 \sim 10^{13}$  to  $10^{14}$  eV) which are responsible for shower productions (it is possible that  $L_p$  decreases with increasing energy). Without contradicting known experimental facts, we shall assume that the interaction of high-energy particles is characterized by larger fluctuations of the fraction of energy lost. In order to simplify the calculations, we shall assume that interactions of ultra-high energy particles can be divided into two groups, one with small energy losses, and another with large energy losses, close to 100%. We shall neglect the contribution of weak interactions to shower development, assuming that a shower is produced when a particle loses a large fraction of its energy, close to 100%.

We shall consider two simplified schemes of large energy loss: (a) the total energy lost is transferred to a single photon with energy  $E_0$  ( $E_0$  is the energy of the primary nuclear-active particle) and the shower develops then as a pure electron-photon cascade, without nuclear-active particles; (b) the collision involving the 100% energy loss corresponds to the process studied by Landau<sup>6</sup> and the shower develops with contribution of nuclear-active particles.

We shall assume, furthermore, that the energy-spectrum of primary nuclear-active particles is

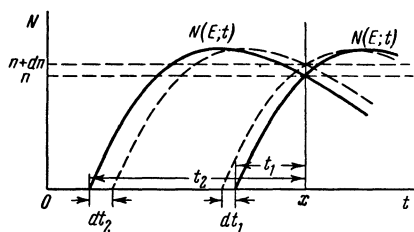


FIG. 1. Illustration for calculation of the number of showers with a given number of particles from  $n$  to  $n + dn$  at the depth  $x$ , produced by primary particles with energy  $E$ .

known and is given by the expression  $F(E)dE = B dE/E^2$ .<sup>7</sup>

The problem consists of calculating those average features of EAS which have been observed in experiments on the basis of the above assumptions (for the two schemes a and b) and to determine if they depend on the presence of high-energy nuclear-active particles in the shower.

## 2. ALTITUDE DEPENDENCE OF THE NUMBER OF SHOWERS WITH A GIVEN NUMBER OF PARTICLES AND THE NUMBER SPECTRUM

Under the above assumptions, the development of EAS is as follows: If a particle with energy  $E$  interacts with an nitrogen or oxygen atom in the atmosphere at the point  $t_0$ , and loses in that interaction its total energy, then a shower will start developing from that point (according to either of the two schemes). Let  $N(E, t)$  be the function giving the number of particles in the shower at a distance  $t$  g/cm<sup>2</sup> from the point of initiation ( $t_0$ ) when the shower is caused by a primary of energy  $E$ . [Obviously, the choice of the scheme of development will determine the function  $N(E, t)$ .] At the point of observation, at atmospheric depth  $x$  g/cm<sup>2</sup>, we shall record a shower consisting of  $n = N(E, x - t_0)$  particles.

If we require that at the observation level  $y$  the shower contain between  $n$  and  $n + dn$  particles and that it be initiated by a nuclear-active primary with energy  $E$ , then it is necessary that that particle interact with large energy loss either in the layer  $dt_1$  at the distance  $t_1$  or in the layer  $dt_2$  at the distance  $t_2$  from observation level (Fig. 1), where  $t_1$  and  $t_2$  are the roots of the equation

$$N(E, t_{1,2}) = n, \tag{2.1}$$

and  $dt_1$  and  $dt_2$  are determined from the equations:

$$dt_1 = dn / \left. \frac{\partial N}{\partial t} \right|_{t=t_1}; \quad dt_2 = dn / \left. - \frac{\partial N}{\partial t} \right|_{t=t_2}. \tag{2.2}$$

If  $F(E)dE$  primary particles are incident on the top of the atmosphere, with energy between  $E$  and  $E + dE$ , then  $F(E) \exp \{ -(x - t_1)/L_a \} dE$  particles with energy  $E$  will arrive at the level  $t_1$  and  $F(E) \exp \{ -(x - t_2)/L_a \} dE$  at the level  $T_2$ . A fraction  $\alpha dt_1/L_1$  of the particles that arrive at the level  $x - t_1$  will interact with a large energy loss in the layer  $dt_1$ . Similarly,  $\alpha dt_2/L_1$  of the primary particles that arrive at the level  $x - t_2$  will interact in the layer  $dt_2$ .  $\alpha$  is the probability of strong interaction with a large ( $\sim 100\%$ ) energy

loss, and  $L_i$  is the interaction mean free path. The number of showers produced by nuclear-active particles with energy between  $E$  and  $E + dE$  and containing between  $n$  and  $n + dn$  particles at observation level will be, therefore, equal to

$$\begin{aligned} & \frac{\alpha}{L_i} dt_1 F(E) \exp\{-(x - t_1)/L_a\} dE \\ & + \frac{\alpha}{L_i} dt_2 F(E) \exp\{-(x - t_2)/L_a\} dE \\ & = \frac{\alpha}{L_i} e^{-x/L_a} dn \cdot F(E) dE \\ & \times \left\{ \frac{\exp\{t_1(E, n)/L_a\}}{(\partial N / \partial t)|_{t_1}} + \frac{\exp\{t_2(E, n)/L_a\}}{-(\partial N / \partial t)|_{t_2}} \right\}. \end{aligned} \quad (2.3)$$

Showers with a given number of particles can, however, be originated by primaries of various energies. The total number of showers  $N_s(n, x)$  with number of particles between  $n$  and  $n + dn$  at the atmospheric depth  $x$  is therefore

$$\begin{aligned} N_s(x, n) dn &= \frac{\alpha dn}{L_i} e^{-x/L_a} \left\{ \int_{E_{\min}}^{\infty} \frac{\exp\{t_1(E, n)/L_a\}}{(\partial N / \partial t)|_{t_1}} F(E) dE \right. \\ & \left. + \int_{E_{\min}}^{E_{\max}} \frac{\exp\{t_2(E, n)/L_a\}}{-(\partial N / \partial t)|_{t_2}} F(E) dE \right\}, \end{aligned} \quad (2.4)$$

where  $E_{\min}$  and  $E_{\max}$  satisfy the following equations:

$$N(E_{\min}; t_{\max}) = n, \quad (2.5)$$

$$N(E_{\max}, x) = n, \quad (2.6)$$

under the additional condition  $t_{\max} < x$  ( $t_{\max}$  is the distance from the origin, which corresponds to the maximum number of particles in the shower).

Cascade curves (near the maximum) can be approximated, with an accuracy sufficient for the calculations, by the expression

$$N(E, \xi) = AE \exp(-a\xi^2 + b\xi^3), \quad \xi = t - t_{\max}. \quad (2.7)$$

The values of  $t_1$  and  $t_2$  are determined from the equations

$$t_1 = t_{\max} + \xi_1, \quad t_2 = t_{\max} + \xi_2,$$

The values  $\xi_1$  and  $\xi_2$  are determined from the equation  $N(E, \xi_{1,2}) = n$ .

If we consider scheme (a), then, for the energy of primaries initiating showers having  $10^4$  to  $10^6$  particles at sea-level or at mountain altitudes, we obtain  $A = 1.2 \times 10^{-9} \text{ ev}^{-1}$  and the coefficients  $a$  and  $b$  in Eq. (2.8) are equal to 0.025 and 0.0008 respectively.  $t_{\max} = \ln(E/\beta)$ , where  $\beta$  is the critical energy in air, equal to  $7.2 \times 10^7 \text{ ev}$ .

If we denote  $\ln(E/E_{\min}) = z$ , then, to a suffi-

cient degree of accuracy, we can put

$$\xi_1 = -\left(\frac{z/a}{1 + (b/a)\sqrt{z/a}}\right)^{1/2}, \quad \xi_2 = \left(\frac{z/a}{1 - (b/a)\sqrt{z/a}}\right)^{1/2}. \quad (2.8)$$

The values of  $\xi_1$ , and  $\xi_2$  obtained from Eq. (2.8) differ from the true values by less than 5% for  $0 \leq z \leq 3.2$ . It should be noted that the primary spectrum is  $F(E) dE = B dE/E^\gamma$ . The number of showers having between  $n$  and  $n + dn$  particles at the observation level is

$$N_s(n, x) dn = \frac{C dn}{n^{\gamma-1/L_a}} e^{-x/L_a} \left[ \int_0^{\infty} \varphi_1(z) dz + \int_0^{z_{\max}} \varphi_2(z) dz \right], \quad (2.9)$$

where

$$\varphi_1(z) = \frac{\exp\{\xi_1/L_a - z(\gamma - 1 - 1/L_a)\}}{-a\xi_1(2 - 3b\xi_1/a)};$$

$$\varphi_2(z) = \frac{\exp\{\xi_2/L_a - z(\gamma - 1 - 1/L_a)\}}{a\xi_2(2 - 3b\xi_2/a)};$$

$$z_{\max} = \ln(E_{\max}/E_{\min}) = z_{\max}(n, x);$$

$$C = (\alpha B / L_i) A^{\gamma-1-1/L_a} \beta^{-1/L_a}.$$

In the case of a nuclear cascade [scheme (b)] the function  $N(E, t)$  has been determined from curves given by Sarycheva,<sup>7</sup> calculated under the assumption that the primary interaction and all secondary interactions of the nuclear-active particles correspond to the Fermi-Landau<sup>6</sup> theory. In that case, too, the function  $N(E, t)$  can be represented in a form analogous to Eq. (2.7):

$$N(E, t) = AE \exp\{-a\xi^2 + b\xi^3 - c\xi^4\}$$

( $a = 3.9 \times 10^{-2}$ ,  $b = 2.8 \times 10^{-3}$  and  $c = 8 \times 10^{-5}$ ). In this scheme,  $A = 10^{-9} \text{ ev}^{-1}$ , and the coefficient  $\beta$  which determines  $t_{\max} = \ln(E/\beta)$  does not represent the critical energy, and is equal to  $7.6 \times 10^9 \text{ ev}$ .

Since  $z_{\max} = z_{\max}(n, x)$ , then, as it can be seen from Eq. (2.9),

$$N_s(n, x) dn = \frac{C dn}{n^{\gamma-1/L_a}} e^{-x/L_a} f(n, x). \quad (2.10)$$

Values of the function  $f(n, x)$  for the two schemes (a) and (b) for  $\gamma = 2.7$  are given in Table I.

In calculation of the altitude dependence of the number of EAS one can use the expression (2.10) for the differential size spectrum at various observation levels. This dependence is shown for the electron-photon cascade [scheme (a)] in Fig. 2 and for the electron-nuclear cascade [scheme (b)] in Fig. 3. The  $x$  axis represents the atmospheric depth in  $t$  units, and the  $y$  axis represents the

TABLE I

Observation level g/cm <sup>2</sup>	Electron-nuclear cascade			Electron-photon cascade		
	n = 10 <sup>4</sup>	n = 10 <sup>5</sup>	n = 10 <sup>6</sup>	n = 10 <sup>4</sup>	n = 10 <sup>5</sup>	n = 10 <sup>6</sup>
640	15.5	13.4	10.1	15.2	12.1	8.8
815	19.1	17.8	16.0	22.2	18.2	14.1
1000	20.6	19.7	18.7	25.3	21.9	18.7

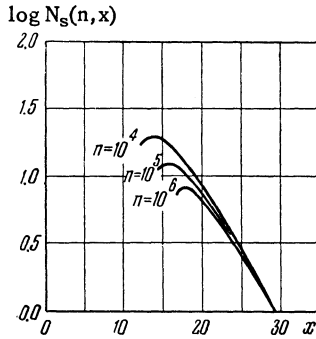


FIG. 2. Dependence of the number of showers N<sub>s</sub>(n, x) with a given number of particles n on the atmospheric depth x (in t units). Calculated according to scheme (a).

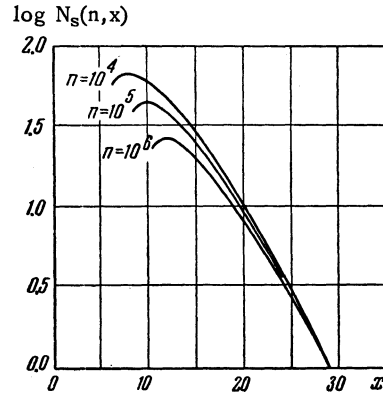


FIG. 3. Dependence of the number of showers N<sub>s</sub>(n, x) with a given number of particles n on the atmospheric depth x (in t units). Calculated according to scheme (b).

relative intensity of EAS with a given number of particles.

Information on the variation of the number of showers of given n with the atmospheric depth, for depths greater than 1000 g/cm<sup>2</sup>, can be obtained from the zenith-angle dependence of showers at sea-level. Experimental data of Bassi et al.<sup>8</sup> and results of calculations of the zenith angle distribution of showers at sea-level for scheme a are given in Fig. 4.

In the altitude interval from mountain elevations to sea-level, the dependence of the number of EAS on x, expressed in g/cm<sup>2</sup>, can be written in the form

$$N_s(n, x) = N_s(n, 1000) e^{\mu(1000-x)}. \quad (2.11)$$

Values of 1/μ for altitudes from 640 to 1000 g/cm<sup>2</sup> are given in Table II for the cases of elec-

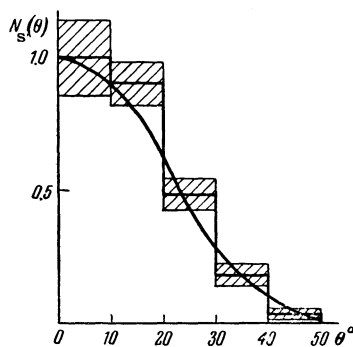


FIG. 4. Zenith angle dependence of the number of EAS N<sub>s</sub>(θ). The curve is calculated according to the scheme a. The histogram represents experimental data.<sup>8</sup>

tron-photon and electron-nuclear cascades. Let us mention that the calculated values of the shower absorption coefficient do not contradict the experimental value 1/μ = 130 to 150 g/cm<sup>2</sup> for showers with n = 10<sup>4</sup> to 10<sup>5</sup>.

It should be noted that the fluctuation mechanism of development of EAS yields, for showers of a given n, an absorption mean free path much larger than the absorption mean free path of the shower-producing primaries. It follows that at a depth of 700 to 1000 g/cm<sup>2</sup>, no equilibrium is reached yet between the electronic component and the nuclear-active particles.

For a given x and a small region of n, we can represent the differential size spectrum by a power law

$$N_s(n, x) dn = kdn/n^{\kappa+1}. \quad (2.12)$$

comparing the right hand sides of Eqs. (2.10) and (2.12) we obtain

$$\kappa + 1 = \gamma - 1/L_a - \partial \ln f(n, x) / \partial \ln n. \quad (2.13)$$

Values of κ + 1, calculated according to Eq. (2.13)

TABLE II

n	1/μ, g/cm <sup>2</sup>	
	Electron-nuclear cascade	Electron-photon cascade
10 <sup>4</sup>	131	143
10 <sup>5</sup>	137	150
10 <sup>6</sup>	150	157

TABLE III

Observation level g/cm <sup>2</sup>	x + 1					
	Electron-nuclear cascade			Electro-photon cascade		
	n = 10 <sup>4</sup>	n = 10 <sup>5</sup>	n = 10 <sup>6</sup>	n = 10 <sup>4</sup>	n = 10 <sup>5</sup>	n = 10 <sup>6</sup>
640	2.47	2.49	2.52	2.58	2.61	2.64
1000	2.43	2.44	2.45	2.46	2.47	2.48

for  $\gamma = 2.70$  for two observation levels ( $x = 640 \text{ g/cm}^2$  and  $x = 1000 \text{ g/cm}^2$ ), are given in Table III.

Table III shows that the fact that  $\kappa$  is independent of the altitude of observation level cannot serve as an argument for the decisive role of the nuclear-active particles in the development of showers in the depth of the atmosphere.

### 3. BAROMETRIC EFFECT OF EXTENSIVE AIR SHOWERS

If fluctuations of the energy lost by the primary particle in interaction with air atoms represent the main factor in the development of EAS, then the value of the barometric coefficient for showers with a given  $n$  is determined, essentially, by the absorption mean free path of ultra-high-energy particles.

In fact, the barometric coefficient  $b$  is defined

$$b = -\partial N_s(n, x)/\partial x = 1/L_a - \partial \ln f(n, x)/\partial x. \quad (3.1)$$

Values of  $\partial \ln f(n, x)/\partial x$  at sea-level for showers with various  $n$  are given in Table IV for both development schemes.

Calculations show that the increase in  $\gamma$  with  $n$  causes a slow decrease of  $\partial \ln f(n, x)/\partial x$  which contributes only about 10% to the barometric coefficient.

Experimental data<sup>9</sup> indicate that the barometric coefficient increases with  $n$ . It follows from Eq. (3.1) that the increase in  $b$  is due, mainly, to the

TABLE IV

Electron-nuclear cascade			Electron-photon cascade		
n = 10 <sup>4</sup>	n = 10 <sup>5</sup>	n = 10 <sup>6</sup>	n = 10 <sup>4</sup>	n = 10 <sup>5</sup>	n = 10 <sup>6</sup>
0.016	0.020	0.030	0.026	0.037	0.056

as [we make use of Eq. (2.10)]

TABLE V

Observation level g/cm <sup>2</sup>	$\bar{E}(n, x), \text{ev}$					
	Electron-nuclear cascade			Electron-photon cascade		
	n = 10 <sup>4</sup>	n = 10 <sup>5</sup>	n = 10 <sup>6</sup>	n = 10 <sup>4</sup>	n = 10 <sup>5</sup>	n = 10 <sup>6</sup>
640	$1.3 \cdot 10^{13}$	$1.2 \cdot 10^{14}$	$1.2 \cdot 10^{15}$	$4.2 \cdot 10^{13}$	$3.4 \cdot 10^{14}$	$2.5 \cdot 10^{15}$
1000	$3.9 \cdot 10^{13}$	$2.7 \cdot 10^{14}$	$2.2 \cdot 10^{15}$	$1.2 \cdot 10^{14}$	$9.2 \cdot 10^{14}$	$7.2 \cdot 10^{15}$

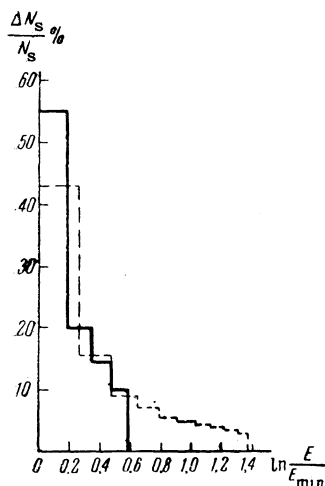


FIG. 5. Energy distribution of primary particles producing showers with 10<sup>5</sup> particles at the atmospheric depth  $x = 640 \text{ g/cm}^2$ . Solid line calculated according to scheme (a), the dashed according to scheme (b)

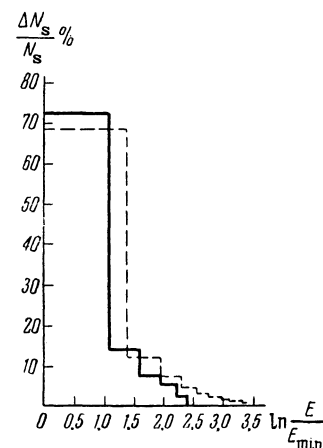


FIG. 6. Energy distribution of primary particles producing showers with 10<sup>5</sup> particles at the atmospheric depth  $x = 1000 \text{ g/cm}^2$ . Solid line - scheme (a), dashed - scheme b.

decrease in  $L_a$ . The absorption mean free path  $L_a$  decreases, therefore monotonously with increasing energy of the primary particle. The decrease may be due either to an increase in the value of the interaction cross-section, or to increased inelasticity, or to both effects.

#### 4. ENERGY SPECTRUM OF PRIMARY PARTICLES PRODUCING EAS WITH A GIVEN NUMBER OF PARTICLES

The number of EAS containing between  $n$  and  $n + dn$  particles at the observation level and produced by primary particles with energies between  $E_1$  and  $E_2$  is given by the expression

$$\Delta N_s(n, x) dn = \frac{Cdn}{n^{\gamma-1}L_a} e^{-x/L_a} \int_{z_1}^{z_2} [\varphi_1(z) + \varphi_2(z)] dz, \quad (4.1)$$

$$z_{1,2} = \ln(E_{1,2}/E_{\min}).$$

Since the total number of showers with  $n$  to  $n + dn$  particles at the observation level is given by Eq. (2.9), then the relative number of cases when a shower with a given  $n$  is produced by a nuclear-active particle with energy between  $E_1$  and  $E_2$  will be

$$\frac{\Delta N_s(n, x)}{N_s(n, x)} = \frac{\int_{z_1}^{z_2} [\varphi_1(z) + \varphi_2(z)] dz}{\int_0^{\infty} \varphi_1(z) dz + \int_0^{z_{\max}} \varphi_2(z) dz}. \quad (4.2)$$

Using Eq. (4.2), one can compute the energy spectrum of particles producing showers containing  $n$  particles at a given level.

Energy distributions of the primaries producing showers, containing  $10^5$  particles at two observation levels, are shown in Figs. 5 and 6 for electron-photon (solid line) and electron-nuclear (dashed) cascades. The  $x$  axis represents the logarithm of the energy of nuclear-active particles in units of  $E_{\min}$ , and the  $y$  axis the relative number of showers produced by particles of the corresponding energy interval.

The mean energy of primary particles that produce showers containing  $n$  to  $n + dn$  particles at the observation level is given by

$$\frac{\bar{E}(n, x)}{E_{\min}} = \left\{ \int_0^{\infty} e^z \varphi_1(z) dz + \int_0^{z_{\max}} e^z \varphi_2(z) dz \right\} / \left\{ \int_0^{\infty} \varphi_1(z) dz + \int_0^{z_{\max}} \varphi_2(z) dz \right\}. \quad (4.3)$$

The values of  $\bar{E}(n, x)$  calculated according to Eq. (4.3) are given in Table V.

It can be seen from Figs. 5 and 6 that if the fluctuation mechanism of development of EAS

takes place in the reality, then EAS observed at sea-level with  $n = 10^4$  to  $10^6$  are produced by primaries of a wide energy range. In connection with this, a study of the intensity distribution of the Cerenkov radiation of EAS with a given number of particles  $n$  can give information about the role of energy-loss fluctuations in the formation of EAS,<sup>10</sup> and a measurement of the absolute pulse size may indicate which of the considered development schemes (a) and (b) is closer to reality.

#### 5. LATERAL DISTRIBUTION OF PARTICLES IN EAS

If we assume that EAS are produced by a single high-energy electron (photon) [scheme (a)] then the lateral distribution of shower particles at the observation level will be determined in a unique way by the value of the cascade parameter  $s$ . In scheme (a), the mean value  $\bar{s}$  can be calculated as a function of  $n$  and  $x$ .

TABLE VI

Observation level g/cm <sup>2</sup>	$\bar{s}$	
	$n = 10^4$	$n = 10^6$
640	1.11	1.04
1000	1.27	1.14

It is evident that, within the framework of the development scheme used, a shower produced by a particle with energy  $E$ , at an atmospheric depth  $x - t$ , is characterized at the observation level by the value  $s = s(E, t)$ . The function  $s(E, t)$  is approximated by an empirical formula which is in a good agreement (within a few percent), for primary energies of  $\sim 10^{13}$  to  $10^{16}$  ev and for  $0.6 \leq s \leq 1.4$ , with the values of  $s$  obtained from cascade curves:

$$s = 1 + \frac{0.63}{t_{\max}} \xi - 10^{-(t_{\max}/13.7 + \xi)} \xi^2, \quad \xi = t - t_{\max}, \quad (5.1)$$

and  $t_{\max} = \ln(E/\beta)$ .

The mean parameter  $\bar{s}(n, x)$ , which characterizes showers containing  $n$  to  $n + dn$  particles at the observation level  $x$ , has been calculated using Eq. (5.1). The calculations were carried out for sea-level ( $x = 1000$  g/cm<sup>2</sup>) and mountain altitudes ( $x = 640$  g/cm<sup>2</sup>), for showers consisting of  $10^4$  to  $10^5$  particles. The results are given in Table VI. It can be seen that in the presence of large fluctuations in the primary interaction process, even a purely electromagnetic development of the shower, without participation of nuclear-active particles, can cause showers with a given number of particles, observed at two different altitudes, to have a prac-

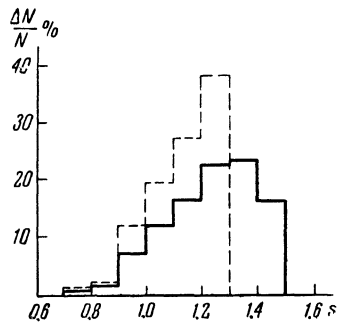


FIG. 7. Age distribution of EAS with  $10^4$  particles, solid line – at observation level  $x = 1000 \text{ g/cm}^2$ , dashed – at  $x = 640 \text{ g/cm}^2$ .

tically identical “age,” i.e. the same mean  $\bar{s}$ .

The distribution of showers with  $10^4$  particles is given in Fig. 7 for two observation levels,  $x = 640 \text{ g/cm}^2$  (dashed line) and  $x = 1000 \text{ g/cm}^2$  (solid line). The  $x$  axis represents the age parameter  $s$ , and the  $y$  axis the relative number of showers with  $s$  in the corresponding interval.

Experimental study of the age distribution of EAS with given  $n$  might also throw some light upon the role of fluctuations in the development of EAS.

## 6. CONCLUSION

As can be seen from the above calculations, in the presence of large fluctuations in the energy loss of ultra-high-energy particles interacting with atomic nuclei, such parameters of EAS as the absorption mean free path  $1/\mu$ , the mean age  $\bar{s}$ , and the shower spectrum exponent  $\kappa$  are practically independent of the fact whether high-energy nuclear-active particles are present in the EAS, or whether the shower develops as an electron-photon cascade without a marked “feeding” of the shower in the depth of the atmosphere by nuclear-active particles.

We think therefore that it is highly probable that interactions of primary particles corresponding to large energy losses, and which possibly represent the main contribution to EAS production, may belong to several types. Moreover, properties of the interactions may vary within wide limits, from the case of energy transfer to one or several  $\gamma$ -quanta to the case of production of a large number of secondary particles with a large degree of degradation of the primary-particle energy.

We should like to stress again that the purpose of the present calculation is not to choose the most probable mode of development of EAS (according

to our view, the present experimental data are not sufficient for such a decision) but to show that such experimental characteristics as the shower absorption coefficient and the independence of  $\bar{s}$  and  $\kappa$  from the altitude of the observation level are by far insufficient to conclude about the large role of nuclear-active particles in the development of showers in the depth of the atmosphere and, even less so, to determine more precisely the number of these particles determining this development, as it has been done by several authors.<sup>11-12</sup> It seems to us that so long as the role of energy loss fluctuations in the production of EAS is not explained, conclusions concerning the mechanism of interactions of ultra-high-energy particles based upon such average characteristics of EAS as  $1/\mu$ ,  $\bar{s}$ , and  $\kappa$  must be regarded as tentative.

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