ON THE THEORY OF PARAMAGNETIC RESONANCE IN METALS

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On the basis of the general theory,[1] we have investigated the dependence of the surface impedance of metals in paramagnetic resonance on the dispersion of conduction electrons and on the angle of inclination of the constant magnetic field to the surface of the metal. The case of resonance saturation is examined.

1. INTRODUCTION

PARAMAGNETIC resonance occurs in a metal the electrons of which possess paramagnetic susceptibility when the metal is placed in both a constant magnetic field H_0 and an alternating electromagnetic field H_1 with the frequency $\omega = \Omega_0 \equiv 2\mu H_0/\hbar$. Much experimental work[2] has been done in recent years on paramagnetic resonant absorption of electromagnetic waves impinging on metals.

Dyson[3] was the first to include electron diffusion from the skin layer in a theoretical examination of this effect. His theory is based on the free-electron model, that is, the conduction electrons were regarded as free particles obeying the dispersion law $\epsilon(\mathbf{p}) = p^2/2m^*$. A formula was derived for the surface impedance when the constant magnetic field H_0 is perpendicular to the metal surface and the strength of the alternating field $H_\perp$ is such that resonance saturation is far from being reached.

In reference 1 the present authors developed a theory of paramagnetic resonance, based on a solution of the equation for the density operator of electrons regarded as a gas of noninteracting quasi-particles with an arbitrary dispersion law $\epsilon(\mathbf{p})$. This theory is free of limitations on the direction of the constant field H_0 and the strength of the alternating field H_1. However, in the surface impedance of the metal was not obtained in that article. In addition to the case considered by Dyson, it is of interest to consider the dependence of surface impedance on the angle of inclination of the constant magnetic field to the metal surface, to determine the influence of dispersion upon impedance, and to study the case of alternating fields strong enough to produce resonance saturation. The present article is devoted to these questions.

We shall make a few preliminary remarks on the relation of impedance to dispersion and to the angle of inclination ψ of H_0 to the metal surface. First, it is clear that in specimens of thickness $d \ll \delta$ (δ is the skin thickness) the electron dispersion is quite unimportant. This is due to the fact that an electron is always in a field the amplitude and phase of which are identical at all points of the specimen, so that the probability of spin reversal per unit time does not have to depend on the character of electron motion. We shall not consider the question of the influence of dispersion on the spin relaxation time T_{sp}, which is always included in the theory as a parameter. It is also evident that impedance will not depend on the angle of inclination of H_0 to the surface.

The situation changes when we pass to bulk metal, where $\delta_{eff} \ll d$ (δ_{eff} is the electron diffusion depth; it will be shown below that for metals we almost always have $\delta \ll \delta_{eff}$). In a strong field H_0, when the radius r of the electron orbit is much shorter than the mean free path l, we can expect the impedance to depend on the angle ψ. Indeed, in a strong field ($r \ll l$) an electron can perform l/r revolutions between two successive collisions with the lattice. It is also easily seen that for a strong field the resonance frequency $\omega = \Omega_0$ is in the region of the anomalous skin effect. For all actually obtainable fields $r \gg \delta$. Therefore when the field H_0 is almost parallel to the metal surface ($\psi \ll r/l$) electrons which are close to the surface in a layer with thickness of the order of r will return l/r times to the skin layer, that is, they will remain in the skin layer l/r times longer than in the case of a perpendicular field. Correspondingly, the resonance amplitude will be enhanced by a factor of l/r compared with the effect when the field H_0 is perpendicular to the metal surface. Stronger resonance results from the fact that the probability of spin reversal increases with the time that an electron spends in the skin layer. In a weak magnetic field ($r \gg l$) the resonance is not enhanced because collisions with the lattice will prevent an electron from continually returning to the skin layer.

It must be noted that the picture of resonance which has just been given becomes somewhat complicated for the following reasons. We know that electron energy levels ϵ are isoenergetic surfaces $\epsilon(\mathbf{p}) = \epsilon$ in momentum space possessing a definite orientation with respect to the axes of the reciprocal lattice. Two kinds of surfaces can be distinguished—closed and opened surfaces. In the first case for any orientation of the constant magnetic field H_0 with respect to the crystal axes the electron trajectory in the momentum space (the cross section of the surface $\epsilon(p) = \epsilon$ with the plane $p_z = \text{const}$, where p_z is the conserved projection of the quasi-momentum on the direction of H_0) will be closed, that is, the electron will perform a finite (periodic) motion in a plane perpendicular to the field. For open surfaces both closed and open trajectories in momentum space are possible depending on the orientation of the magnetic field; in the latter case the electron will perform an infinite motion in a plane perpendicular to the field. We now note that because of the Fermi-Dirac distribution a contribution to resonance will not be given by all conduction electrons but only by those with energy in the range $\Delta\epsilon \sim kT$ close to the Fermi energy limit ϵ_0, where $kT \gg \mu H_0$ (or in the range $\Delta\epsilon \sim \mu H_0$, where $kT \lesssim \mu H_0$). Therefore the following cases are possible:

(a) In the interval $\Delta\epsilon$ there are no open surfaces. Then in a strong field H_0 parallel to the metal surface we can expect intensified resonance compared with the case of a field perpendicular to the surface.

(b) In the interval $\Delta\epsilon$ we have open and closed isoenergetic surfaces. Here also resonance will be intensified in a strong field H_0 parallel to the metal surface because of the electrons with closed trajectories in momentum space.

(c) In the interval $\Delta\epsilon$ there are only open isoenergetic surfaces. If the orientation of the metal surface plane is then such that in a field parallel to the surface the electron trajectories in momentum space are open, in any field H_0 the electrons cannot repeatedly return to the skin layer because of infinite motion in a plane normal to the metal surface; therefore the resonance cannot be intensified. This is to some extent a special case because of the strict conditions under which it occurs.

Thus we can almost always expect that in a strong magnetic field H_0 the surface impedance will depend essentially on the inclination of the field to the surface of the metal.

2. A GENERAL EXPRESSION FOR THE IMPEDANCE

Let the test specimen be a flat plate of thickness d which is infinite in the two other dimensions. We shall use two coordinate systems, $(\xi y \zeta)$ and (xyz), with the same origin. The ζ axis is directed into the metal normal to its surface; z is in the direction of H_0. The slope of the magnetic field H_0 with respect to the metal surface is given by the angle ψ between the ξ and z axes. The incident plane electromagnetic wave is normal to the metal surface.

In reference 1 it was shown that the field H_1 inside the metal can be represented in the form $H_1 \approx H_1^0 - 4\pi M$, that is, $B = H_1 + 4\pi M \approx H_1^0$. Here H_1^0 is the field in the absence of paramagnetic resonance (attenuated at the depth δ); M is the resonance magnetization, which is attenuated (near resonance) at the depth $\delta_{\text{eff}} \sim l(T_{\text{sp}}/t_0)^{1/2}$; t_0 is the mean free electron time. The relation $B \approx H_1^0$ holds true when the following inequality is satisfied:*

$$|\varkappa|\,\delta^2 \ll \delta_{\text{eff}}^2, \quad \varkappa = 1 - iT_{\text{sp}}(\omega - \Omega_0),$$

i.e., the frequency difference $\Delta\omega = \omega - \Omega_0$ is subject to the limitation $|\Delta\omega|\,T_{\text{sp}} \ll \delta_{\text{eff}}^2\ \delta^2$. However this limitation is not important since the resonance width is determined mainly by the spin relaxation time: $|\Delta\omega|\,T_{\text{sp}} \sim 1$, and the inequality $\delta \ll \delta_{\text{eff}}$ is almost always satisfied for metals. Indeed, in the anomalous skin effect $\delta \ll l$, while $\delta_{\text{eff}} \gg l$ for all actually attainable fields† H_0, so that obviously $\delta \ll \delta_{\text{eff}}$. In the normal skin effect $\delta/\delta_{\text{eff}} \lesssim 10^{-14}/t_0$ (for $\omega T_{\text{sp}} \gtrsim 10$), so that $\delta \ll \delta_{\text{eff}}$ fails to hold only at temperatures which are higher or of the order of room temperature, in which case $t_0 \lesssim 10^{-14}$ sec.

We shall now use the relation $B \approx H_1^0$ to determine the surface impedance of a metal under the given limiting conditions. The impedance when $\delta \gtrsim \delta_{\text{eff}}$ is found in Appendix (a). We define the

*In reference 1 the interaction Hamiltonian $\hat{H} = \mu\boldsymbol{\sigma}\hat{\mathbf{H}}$ ($\mu = -|e|\hbar/2mc$) did not take the sign of μ into account explicitly. When this is done and by μ we mean $|\mu|$, then in the final formulas for the magnetization we must make the substitutions $\Omega_0 \equiv 2\mu H_0/h \to -\Omega_0 \equiv -\frac{2|\mu|H}{\hbar}$ and $\omega \to -\omega$. These substitutions do not affect the results for nuclear polarization and the selective transparency of films. This comment will be taken into account in the present article.

†In a strong field H_0 the part of l is played by r, the radius of the electron orbit in a magnetic field.

surface impedance tensor z_{ik} by means of

$$E_i = (c/4\pi)\, Z_{ik} H_k \ (i, k = \xi, y).$$

(We perform a summation over repeated subscripts.) In the absence of resonance $E_i^0 = (c/4\pi)\, Z_{ik}^0 H_k^0$. Since $B \approx H_i^0$ and $E = E^0 + E'$, where $E' \sim cZ^0 (\delta/\delta_{\text{eff}}) M$, neglecting terms of the order of E', we have

$$E_i = \frac{c}{4\pi} Z_{ik}^0 B_k = \frac{c}{4\pi} Z_{ik}^0 (H_{1k} + 4\pi M_k). \tag{1}$$

Using the results obtained in reference 1, we can write

$$4\pi M_k = \alpha_{kn} E_n(0) = \alpha_{kn} Z_{nl}^0 H_{1l}^0.$$

Substituting this expression into (1), we obtain accurately to within linear terms in α:

$$E_i = \frac{c}{4\pi}\left(Z_{ik}^0 + \frac{c}{4\pi} Z_{in}^0 \alpha_{nl} Z_{lk}^0\right) H_{1k}.$$

Hence we obtain the surface impedance tensor in paramagnetic resonance (for $\delta \ll \delta_{\text{eff}}$):

$$Z_{ik} = Z_{ik}^0 + \frac{c}{4\pi} Z_{in}^0 \alpha_{nl} Z_{lk}^0, \tag{2}$$

where Z^0 is the impedance in the absence of resonance. In the present case

$$\alpha_{\xi\xi} = \alpha_{yy} = ia \sin\psi, \quad \alpha_{\xi y} = -a \sin^2\psi, \quad \alpha_{y\xi} = a.$$

For a specimen of thickness $d \gg \delta$ (far from resonance saturation)

$$a = -A 2\pi\chi \frac{\Omega_0 c T_{\text{sp}}}{\omega \delta_{\text{eff}} \sqrt{\varkappa}} \coth \frac{d}{\delta_{\text{eff}}} \sqrt{\varkappa}, \tag{3}$$

where A is a numerical coefficient of the order of unity which is associated with the conditions for electron reflection from the metal boundary (see Appendix b); $\delta_{\text{eff}} = \lambda (T_{\text{sp}}/t_0)^{1/2}$ is the distance traversed by an electron during diffusion in the time T_{sp}. The magnitude of λ depends essentially on the strength and inclination to the metal surface of the field H_0 and the electron dispersion law.

For closed Fermi surfaces

$$\lambda^2 = \frac{r_0^2\gamma}{2\int \theta\, dp_z} \int \frac{dp_z}{(e^{\gamma\theta} - 1)^3} \int_0^\theta d\tau \int_0^\theta d\tau' e^{\gamma\tau'} \{\widetilde{V}_\zeta^2 (e^{\gamma\theta} - 1)^2 - 2\widetilde{V}_\zeta \overline{V}_\zeta e^{\gamma\theta} (e^{\gamma\theta} - 1) + \overline{V}_\zeta^2 e^{\gamma\theta} (e^{\gamma\theta} + 1)\},$$

where

$$V_\zeta = \frac{v_\zeta}{v_0}, \quad v_\zeta = \frac{\partial\varepsilon(\mathbf{p})}{\partial p_\zeta}, \quad \widetilde{V} = \int_0^{\tau+\tau'} V\, d\tau'', \quad \overline{V} = \int_0^\theta V\, d\tau;$$

$$\gamma = T_0/t_0; \quad r_0 = v_0 T_0;$$

$$\theta = T(p_z)/T_0; \quad T(p_z) = \frac{c}{eH_0} \frac{\partial S(p_z)}{\partial \varepsilon} \equiv \frac{2\pi c m^*(p_z)}{eH_0};$$

$\tau = t/T_0$ is the dimensionless time which denotes the position of the electron in an orbit in momentum space; $T(p_z)$ is the orbital period of electrons with given p_z in the magnetic field H_0; $S(p_z)$ is the area of the intersection of the surface $\epsilon(\mathbf{p}) = \epsilon$ with the plane $p_z = \text{const}$; T_0 is the characteristic time of revolution of an electron in the magnetic field, say on the order of $\overline{T(p_z)}$; v_0 is the characteristic electron velocity on the Fermi surface. Specifically, for quadratic dispersion $\epsilon = m^* v^2/2$, $(2\pi)^{-1} \partial s/\partial\epsilon = m^*$, i.e., $m^*(p_z)$ is independent of p_z; therefore it is convenient to set $T_0 = T = 2\pi m^* c/eH_0$ and $v_0 = v$. It is easily seen that in actuality T_0 and v_0 do not enter into λ and are introduced only for convenience in obtaining values.

In the selected coordinate systems $V_\zeta = V_z \sin\psi - V_x \cos\psi$. Therefore, remembering that $\overline{V}_x = 0$ in virtue of the periodic motion of electrons in the xy plane, we obtain

$$\lambda^2 = l^2 \sin^2\psi + s^2 \sin\psi \cos\psi + r^2 \cos^2\psi, \tag{4}$$

where

$$l^2 = \frac{r_0^2\gamma}{2\int \theta\, dp_z} \int \frac{dp_z}{(e^{\gamma\theta} - 1)^3} \int_0^\theta d\tau \int_0^\theta d\tau' e^{\gamma\tau'} \times \{[(e^{\gamma\theta} - 1)\widetilde{V}_z - e^{\gamma\theta}\overline{V}_z]^2 + e^{\gamma\theta}\overline{V}_z^2\};$$

$$s^2 = \frac{r_0^2\gamma}{\int \theta\, dp_z} \int \frac{dp_z}{(e^{\gamma\theta} - 1)^2} \int_0^\theta d\tau \int_0^\theta d\tau' e^{\gamma\tau'} \widetilde{V}_x \{e^{\gamma\theta}\overline{V}_z - (e^{\gamma\theta} - 1)\widetilde{V}_z\};$$

$$r^2 = \frac{r_0^2\gamma}{2\int \theta\, dp_z} \int \frac{dp_z}{e^{\gamma\theta} - 1} \int_0^\theta d\tau \int_1^\theta d\tau' e^{\gamma\tau'} \widetilde{V}_x^2.$$

For weak magnetic fields H_0 ($\gamma \gg 1$):

$$l \sim s \sim r \sim r_0/\gamma = v_0 t_0,$$

so that for any angle ψ the magnitude of λ is of the order of the mean free electron path.

For strong fields H_0 ($\gamma \ll 1$):

$$l \sim r_0/\gamma, \quad s \sim r_0/\sqrt{\gamma}, \quad r \sim r_0, \tag{5}$$

so that for angles $\psi \gg \gamma$ the magnitude of λ is of the order of the mean free electron path and for $\psi \ll \gamma\lambda$ it is of the order of the radius of the electron orbit in the magnetic field.

For quadratic dispersion in any field H_0 (assuming $T_0 = T$ and $v_0 = v$) we have

$$l = \frac{r_0}{\gamma\sqrt{3}}, \quad s = 0, \quad r = \frac{r_0}{\sqrt{3(1+\gamma^2)}}$$

$$\text{or} \quad \lambda^2 = \frac{v^2 t_0^2}{3} \frac{\gamma^2 + \sin^2\psi}{\gamma^2 + 1}.$$

In the case of an open Fermi surface when the field H_0 is oriented with respect to this surface in such a way that the electron performs periodic motion in the xy plane, that is, $\overline{V}_x = 0$, a is also given by (3). When $\overline{V}_x \neq 0$, a is given by (3) with the following substitution for λ:

$$\lambda^2 = r_0^2 \frac{\gamma}{2} \lim_{\theta\to\infty} \frac{1}{\int \theta dp_z} \int dp_z \int_0^\theta d\tau \int_{-\infty}^{\tau} d\tau' e^{\gamma(\tau'-\tau)} \left\{ \int_{\tau}^{\tau'} V_\zeta d\tau'' \right\}^2 .$$

This expression is obtained for λ when we seek a solution of Eq. (30) in reference 1 which is aperiodic and bounded with respect to τ. Similarly to (4),

$$\lambda^2 = l^2 \sin^2\psi + s^2 \sin\psi\cos\psi + r^2\cos^2\psi,$$

where, as is easily seen, for any field H_0 (with arbitrary γ)

$$l \sim s \sim r \sim r_0/\gamma = v_0 t_0.$$

If in $\Delta\epsilon \sim kT$ (or $\Delta\epsilon \sim \mu H_0$, with $kT \ll \mu H_0$) there are both closed and open surfaces $\epsilon(\mathbf{p}) = \epsilon$, then generally the magnetization can be represented in the form

$$M(\zeta) = M_1(\zeta) + M_2(\zeta), \tag{6}$$

where M_1 and M_2 are the contributions to the magnetization from the groups of electrons the energy levels of which are represented in momentum space by closed and open isoenergetic surfaces, respectively. Keeping in mind all that has been said above regarding resonance, we can infer that for a weak field H_0 ($\gamma \gg 1$), independently of the angle ψ_1, M_1 and M_2 attenuate at the depth $\delta^0_{\text{eff}} \sim v_0 t_0 (T_{sp}/t_0)^{1/2}$, with $M_1 \sim M_2$. For a strong field H_0 ($\gamma \ll 1$) and $\psi \gg \gamma$ we again have $M_1 \sim M_2$ and the attenuation depth of both expressions is of the order of δ^0_{eff}. For angles $\psi \ll \gamma$ when the field orientation with respect to open surfaces is such that $\overline{V}_x = 0$, the attenuation depth of M_1 and M_2 of the order of $\gamma\delta^0_{\text{eff}}$; if $\overline{V}_x \neq 0$, M_1 attenuates at the depth $\gamma\delta^0_{\text{eff}}$ and M_2 at the depth δ^0_{eff}. Then $M_1(0) \sim M_2(0)/\gamma \gg M_2(0)$.

When $\Delta\epsilon$ has only open surfaces $M(\zeta) = M_2(\zeta)$ and $M_2(\zeta)$ has the previous properties. It was stated above that the resonance part of the impedance is given by $M(0)$. We can therefore affirm that the specific form of the dispersion law does not essentially affect the dependence of the impedance on the angle ψ (with the exception of the "special" case). The presence of both closed and open surfaces in the interval $\Delta\epsilon$ when the magnetization is given by (6) could, for example, affect the selective transparency of films.[1]

We now note that for films with $d \ll \delta_{\text{eff}}$, as can easily be seen from (3), impedance is independent of ψ. Therefore the most interesting case is that of bulk metal ($d \gg \delta_{\text{eff}}$), which we shall consider hereafter.

3. SURFACE IMPEDANCE IN AN OBLIQUE FIELD

In the general case the formulas for surface impedance are very complicated. Therefore for simplicity we shall consider only the case in which the incident wave in linearly polarized (with non-vanishing components E_ξ and H_{1y}). Then the absorption of electromagnetic energy is given by the component $Z_{\xi y}$ of the impedance tensor:

$$P = (c/4\pi)^2 |H_1|^2 \operatorname{Re} Z_{\xi y}.$$

From (2) and (3) we have (omitting the subscripts ξ, y)

$$Z = Z^0 \left\{ 1 - Z^0 \frac{\chi\Omega_0 T_{sp} c^2}{\omega\lambda(\varkappa T_{sp}/t_0)^{1/2}} \right\}. \tag{7}$$

It is easily seen that Z will depend on ψ only in the anomalous skin effect. Equation (5) shows clearly that this requires a strong magnetic field H_0. For $\gamma \ll 1$ we have $H_0 \gg (3m^*/5m_0)10^{-7}/t_0$ oersteds, where m^*/m_0 is the ratio of the affective mass to the free electron mass; $\omega \gg 2\pi m^*/5m_0 t_0 \ \text{sec}^{-1}$. Thus for $t_0 \sim 2\times 10^{-11}$ sec and $m^*/m \sim 1$ we must have fields $H_0 \gg 3000$ Oe and resonance frequencies $\omega \gg 2\pi\times 10^{10} \ \text{sec}^{-1}$.

Two cases are conveniently distinguished: (a) An oblique field and (b) a parallel field.

(a) As shown in reference 5, the impedance Z^0 in a field H_0 inclined to the metal surface differs from the impedance for $H_0 = 0$ only in a very narrow range of angles $\psi < \psi_1 \sim (r_0/l)(\delta/r_0)^{2/3}$. Therefore when $\psi \gg \psi_1$ we use for Z^0 the familiar expression

$$Z^0 = \frac{C_1}{\sqrt{2}} \frac{(\delta^2 l)^{1/3}\omega}{c^2}(1 + i\sqrt{3}), \quad C_1 = \frac{8}{9}(4\pi^2\sqrt{6})^{1/3}. \tag{8}$$

From (7) and (8) we obtain

$$\operatorname{Re} Z = \frac{C_1^2}{\sqrt{2}} \frac{\omega(\delta^2 l)^{1/3}}{c^2} \left\{ \frac{1}{C_1} + \frac{\chi\Omega_0 T_{sp}(\delta^2 l)^{1/3}}{\lambda(T_{sp}/t_0)^{1/2}} \frac{\alpha + \sqrt{3}\beta}{\sqrt{1 + T_{sp}^2\Delta\omega^2}} \right\},$$

$$\operatorname{Im} Z = \frac{C_1^2}{\sqrt{2}} \frac{\omega(\delta^2 l)^{1/3}}{c^2} \left\{ \frac{\sqrt{3}}{C_1} - \frac{\chi\Omega_0 T_{sp}(\delta^2 l)^{1/3}}{\lambda(T_{sp}/t_0)^{1/2}} \frac{\sqrt{3}\alpha - \beta}{\sqrt{1 + T_{sp}^2\Delta\omega^2}} \right\}, \tag{9}$$

where

$$\alpha = (\sqrt{1 + T_{sp}^2\Delta\omega^2} + 1)^{1/2},$$
$$\beta = \operatorname{sign}\Delta\omega\,(\sqrt{1 + T_{sp}^2\Delta\omega^2} - 1)^{1/2}. \tag{10}$$

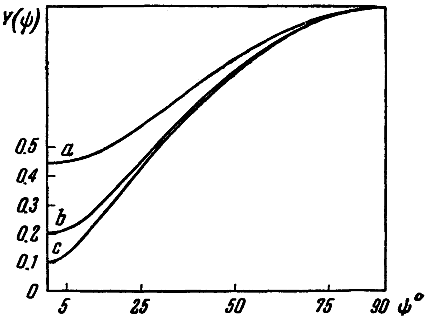
Equation (9) shows that the width of the resonance curve is independent of ψ while the resonance intensity changes considerably as ψ is varied from

0 to $\pi/2$ (for closed electron trajectories in momentum space). The ratio of the ordinate of the resonance curve at $\psi = \pi/2$ to the ordinate for arbitrary ψ will be

$$Y(\psi) = \frac{\operatorname{Re} Z_{res}(\pi/2)}{\operatorname{Re} Z_{res}(\psi)} = \frac{\operatorname{Im} Z_{res}(\pi/2)}{\operatorname{Im} Z_{res}(\psi)}$$

$$= \frac{1}{l}\left(l^2 \sin^2\psi + s^2 \sin\psi\cos\psi + r^2\cos^2\psi\right)^{1/2},$$

where Y varies between r/l and 1 (see the figure).



$Y(\psi)$ for the quadratic dispersion law: a) $-\gamma = 0.5$; b) $\gamma = 0.2$; c) $\gamma = 0.1$.

(b) In a strong field H_0 $(\gamma \ll 1)$ which is almost parallel $(\psi \ll \psi_1)$ to the metal surface the impedance Z^0 depends on the magnetic field. We note that in such a field cyclotron resonance occurs at frequencies ω which are multiples of $2\pi/T_0$ and that this resonance depends essentially on the law of electron dispersion in the metal. Paramagnetic resonance occurs at frequencies $\omega = \Omega_0$ which generally do not coincide with the frequencies $2\pi/T_0$ since T_0 involves the effective electron mass while Ω_0 clearly involves the free electron mass. Therefore for $Z^0(H_0)$ we can use the expression for nonresonant impedance in a parallel field with quadratic dispersion (dispersion is not important far from resonance[6]):

$$Z^0(H_0) = (1+\omega^2 t_0^2)^{1/6}\left(\frac{2\pi H}{H_0}\right)^{1/3}\frac{16}{9}\left(\frac{\sqrt{3}\pi\omega l}{c^4\sigma}\right)^{1/3}\exp\left\{i\,\frac{\pi + \tan^{-1}\omega t_0}{3}\right\}.$$

Here

$$H = \frac{mc}{et_0}, \quad v_0\sqrt{2\pi mN}\left|1 + \frac{i}{\omega t_0}\right|^{-1/2} \gg H_0 \gg H\,|1 + i\omega t_0|.$$

When $\omega t_0 \ll 1$ the corresponding formulas for Re Z and Im Z are obtained from (9) after the substitution $l \to l2\pi H/H_0$ with $\lambda = r_0 = v_0 T_0$.

4. RESONANCE SATURATION

In the calculation of energy absorption in quite strong fields H_1 we must take resonance absorption into account. Here again we shall consider for simplicity the incident wave to be linearly polarized. For a specimen of thickness $d < \delta$ the expression for the resonance part of the absorption is analogous to the corresponding expression in the theory of nuclear resonance:

$$P_{res} = \frac{\omega H_1^2}{4}\,\frac{\Omega_0 \chi T_{sp}}{|\varkappa|^2 + \Omega_1^2 T_{sp}^2}, \quad \Omega_1 = \frac{2\mu H_1}{\hbar}.$$

Resonance absorption occurs when $H_1 \gg 5 \times 10^{-8} \times T_{sp}^{-1}$ oersteds.

For bulk metal $(d \gg \delta_{eff})$ in the limiting case $\delta \gg \delta_{eff}$ the absorption is given by $P = (c/4\pi)^2 H_1^2 R$, where

$$Z \equiv R + iX = \frac{4\pi}{c}\frac{E(0)}{H_1(0)} \approx Z^0\left\{1 + \frac{4\pi M(0)}{H_1^0(0)}\right\}.$$

The magnetization $M(0)$ with resonance saturation taken into account will be[1]

$$M(0) = -\frac{\chi H_0 u}{1 + \operatorname{Re}(uu_r)}, \quad u = \frac{c\Omega_0 T_{sp} E^0(0)}{\omega H_0 \delta_{eff}\sqrt{\varkappa}}, \quad u_r = u^*\Big|_{\omega=\Omega_0}$$

We shall now give the formulas for the resonance part of the impedance. For the normal skin effect

$$R_{res} = -4\pi^2\sqrt{2}\,\frac{\beta}{|\varkappa|}\,\frac{\chi\omega\Omega_0 T_{sp}\delta^2}{c^2\delta_{eff}}\left(1 + \frac{\alpha}{|\varkappa|}\,\frac{\sqrt{2}T_{sp}^2\omega\delta^2}{\Omega_0\delta_{eff}^2}\,\Omega_{1i}^2\right)^{-1},$$

$$X_{res} = -(\alpha/\beta)R_{res},$$

where α and β are given by (10); $\Omega_{1i} = 2\mu H_{1i}/\hbar$, H_{1i} is the strength of the wave impinging on the metal. We note that both far from resonance and at resonance $R_{res}/X_{res} = -\beta/\alpha$.

For the anomalous skin effect

$$R_{res} = \frac{C_1^2}{\sqrt{2}}\,\frac{\chi\Omega_0 T_{sp}(\delta^2 l)^{2/3}\omega}{c^2\delta_{eff}}\,\frac{\alpha + \sqrt{3}\beta}{|\varkappa|}$$

$$\times\left\{1 + \frac{C_1^2\sqrt{2}T_{sp}\,\omega\,(\delta^2 l)^{2/3}}{4\pi^2\Omega_0\delta_{eff}^2}\,\frac{\alpha}{|\varkappa|}\,\Omega_{1i}^2\right\}^{-1}, \quad X_{res} = -\frac{\sqrt{3}\alpha - \beta}{\alpha + \sqrt{3}\beta}R_{res}.$$

The ratio R_{res}/X_{res} is independent of H_{1i} just as for the normal skin effect.

We note that for bulk metal resonance saturation requires considerably higher incident field strength than in the homogeneous case. Thus for the normal skin effect with $\omega \sim 10^8\ \text{sec}^{-1}$, $t_0 \sim 10^{-13}$ sec and $T_{sp} \sim 10^{-7}$ sec we have $H_{1i} \gg 5$ Oe, whereas in the homogeneous case under the same conditions we have $H_{1i} \gg 0.4$ oersteds.

APPENDIX

(a) The region where (2) can be applied is bounded by the inequality $\delta \ll \delta_{eff}$. $\delta \gtrsim \delta_{eff}$ refers entirely to the region of the normal skin effect $(j = \sigma E)$, where without any special difficulty we can obtain an exact expression for the impedance through simultaneous solution of the equations[1]

$$\frac{\partial E}{\partial z} = \frac{\omega}{c}B_1; \quad \frac{\partial H_1}{\partial z} = -\frac{4\pi i\sigma}{c}E; \qquad (1a)$$

$$v_z\frac{\partial u}{\partial z} + \frac{u}{t_0^*} = \frac{\bar{u}}{t_0} + \frac{2\mu B_1}{\hbar};$$

$$u(v_z, 0) = qu(-v_z, 0) + (1-q)\bar{u}(0), \quad (2a)$$

where

$$B_1 = H_1 + 4\pi M, \quad M = i\chi H_0 \bar{u},$$
$$1/t_0^* = 1/t_0 + 1/T_{sp} - i(\omega - \Omega_0).$$

Equation (2a) applies to a field H_0 prependicular to the metal surface with quadratic dispersion. For simplicity we also assume $d = \infty$. Electron reflection from the metal boundary is characterized by the reflection coefficient q, with $q = 1$ for specular reflection and $q = 0$ for diffuse reflections.

Solving (2a) and averaging the solution over the Fermi surface, for the magnetization M with specular reflection we obtain the integral equation

$$M(z) = \frac{t_0^*}{t_0} \times \int_{-\infty}^{\infty} R(|z-\zeta|)\{(1 + 4\pi i\chi t_0\Omega_0) M(\zeta) + i\chi t_0\Omega_0 H_1(\zeta)\} d\zeta, \quad (3a)$$

where

$$R(z) = \frac{1}{2l_0}\int_1^{\infty} \exp\left(-\frac{zx}{l_0^*}\right)\frac{dx}{x}, \quad l_0 = vt_0, \quad l_0^* = vt_0^*.$$

Performing a Fourier transformation of (3a) and of the equation for H_1, we obtain

$$\frac{t_0}{t_0^*} M(k) = R(k)\{(1 + 4\pi i\chi t_0\Omega_0) M(k) + i\chi t_0\Omega_0 H_1(k)\},$$
$$2H_1'(0) + k^2 H_1(k) - 2i\delta^{-2} H_1(k) = 8\pi i\delta^{-2} M(k), \quad (4a)$$

where

$$R(k) = \int_{-\infty}^{\infty} e^{ikz} R(|z|)\, dz = \frac{\tan^{-1} kl_0^*}{kl_0^*}, \quad (5a)$$
$$\delta = \frac{c}{\sqrt{2\pi\omega\sigma}}.$$

Hence

$$H_1(k) = \frac{2\delta^2 H_1'(0)}{k^2\delta^2 - 2i} \times \left\{-1 + \frac{8\pi\chi t_0\Omega_0 R(k)/(\delta^2 k^2 - 2i)}{t_0/t_0^* - R(k)[1 + 4\pi i\chi t_0\Omega_0\delta^2 k^2/(\delta^2 k^2 - 2i)]}\right\}. \quad (6a)$$

The second term within square brackets in the denominator can always be neglected compared with unity since usually $t_0\Omega_0 \lesssim 1$, whereas $\chi \sim 10^{-6}$. Performing the inverse Fourier transformation of (6a) and remembering that $H_1^1(0) = -(4\pi i\sigma/c) \times E(0)$, for the impedance $Z = (4\pi/c)E(0)/H_1(0)$ we obtain

$$Z = Z^0\left\{1 - iZ^0\frac{2\chi t_0\Omega_0 c^2}{\pi\omega} \times \int_0^{\infty}\frac{R(k)\,dk}{[1 + t_0/T_{sp} - it_0(\omega - \Omega_0) - R(k)][1 - \delta^2 k^2/2i]^2}\right\}, \quad (7a)$$

where $Z^0 = 4\pi\omega\delta/ic^2\sqrt{2i}$. In deriving this expression we have assumed the second term in curly brackets to be small compared with unity. Equation (7a) agrees, as was to be expected, with (71) in Dyson's article[3] for $d = \infty$.

The integration in (7a) is easily performed if we note that for $t_0|\Delta\omega| \ll 1$ we can use the resolution $R(k) \approx 1 - k^2 l_0^{*2}/3$, since under this condition only small values of kl_0^* are significant in the integrand. The integral then becomes

$$\frac{3\delta_{eff}^2}{l_0^{*2}}\int_0^{\infty}\frac{dk}{(\varkappa + \delta_{eff}^2 k^2)(1 - \delta^2 k^2/2i)^2}, \quad \delta_{eff} = l_0\sqrt{\frac{T_{sp}}{3t_0}}.$$

For $\delta \ll \delta_{eff}$, subject to the condition $|\kappa| \ll (\delta_{eff}/\delta)^2$, we obtain for the impedance:

$$Z = Z^0\left\{1 - iZ^0\frac{\chi\Omega_0 T_{sp} c^2}{\omega\delta_{eff}\sqrt{\varkappa}}\right\},$$

which agrees exactly with (7). (The appearance of the factor i in this and subsequent equations of the Appendix is associated with circular polarization of the incident wave.)

We note that for the magnetization M we obtain from (4a)

$$M(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ikz} M(k)\,dk \approx -i\frac{\chi T_{sp}\Omega_0 cE(0)}{\omega\delta_{eff}\sqrt{\varkappa}}\exp\left\{-\frac{z\sqrt{\varkappa}}{\delta_{eff}}\right\}. \quad (8a)$$

(b) We shall now show that the conditions for electron reflection from the metal boundary affect only the numerical coefficient in the magnetization (assuming that the electron spin does not change in collisions with the surface). For this purpose it is sufficient to obtain a solution of (2a) when $q = 0$ and to compare this solution with (8a). Solving (2a) and averaging the solution over the Fermi surface, we obtain for M the integral equation

$$M(z) = K(z) M(0) + \int_0^{\infty} R(z-\zeta)\{M(\zeta) + i\chi t_0\Omega_0 B_1(\zeta)\} d\zeta,$$

where

$$K(z) = \frac{1}{2}\int_1^{\infty}\exp\left\{-\frac{zx}{l_0^*}\right\}\frac{dx}{x^2}, \quad R(z) = \frac{1}{2l_0}\int_1^{\infty}\exp\left\{-\frac{zx}{l_0^*}\right\}\frac{dx}{x}.$$

Following Fock,[7] the solution of this equation can be written as

$$M(z) = M(0) J_2(z) + J_1(z), \quad (1b)$$

$$J_1(z) = \frac{1}{(2\pi)^2}\int_{b-i\infty}^{b+i\infty} e^{kz}\psi_1(k)\,dk\int_{-i\infty}^{i\infty}\psi_2(x) G(x) L(x)\frac{dx}{x-k}, \quad (2b)$$

$$J_2(z) = \frac{1}{(2\pi)^2}\int_{b-i\infty}^{b+i\infty} e^{kz}\psi_1(k)\,dk\int_{-i\infty}^{i\infty}\psi_2(x) N(x)\frac{dx}{x-k}, \quad \mathrm{Re}\, b > 0. \quad (3b)$$

Here

$$\ln\psi_1(k) = -\frac{k}{\pi}\int_0^\infty \frac{dx}{x^2+k^2}\ln\left(1-\frac{t_0^*}{t_0}\frac{\tan^{-1} l_0^* x}{l_0^* x}\right),$$

$$\psi_1(k) = \psi_2(-k),$$

$$L(k) = \int_{-\infty}^{\infty} e^{-kz}R(z)\,dz = \frac{t_0^*}{t_0}\frac{1}{2l_0^* k}\ln\frac{1+l_0^* k}{1-l_0^* k},$$

$$G(k) = i\chi t_0\Omega_0\int_0^\infty e^{-kz}B_1(z)\,dz,$$

$$N(k) = \int_0^\infty e^{-kz}K(z)\,dz = \frac{l_0^*}{2}\frac{l_0^* k + \ln(1+l_0^* k)}{l_0^{*2}k^2}.$$

The functions $\psi_2(k)$ and $\psi_1(k)$ have simple poles at the points

$$k = k_0 \equiv (1/l_0^*)\sqrt{3t_0\varkappa/T_{sp}} \equiv \sqrt{\varkappa}/\delta_{eff} \text{ and } k = -k_0$$

respectively. We shall now obtain the magnetization M. For calculation of the integrals in (1b) it is convenient to expand $\psi_1(k)$ in a series with respect to $\epsilon = (t_0/T_{sp})(1 + iT_{sp}\Delta\omega)$ ($\epsilon \ll 1$, when $t_0|\Delta\omega| \ll 1$). We now have

$$\ln\psi_1(k_0) = -\frac{\sqrt{3\varepsilon}}{\pi}\int_0^\infty \frac{dx}{x^2+3\varepsilon}\ln\left(1+\varepsilon-\frac{\tan^{-1} x}{x}\right)$$

$$-\frac{\sqrt{3\varepsilon}}{\pi}\ln\frac{t_0}{t_0^*}\int_0^\infty\frac{dx}{x^2+3\varepsilon}.$$

By adding and subtracting the integral

$$\frac{\sqrt{3\varepsilon}}{\pi}\int_0^\infty \frac{dx}{x^2+3\varepsilon}\ln\left(\varepsilon+\frac{x^2}{3}\right) = \ln 2\sqrt{\varepsilon},$$

we obtain

$$\ln\psi_1(k_0) = -\ln 2\sqrt{\varepsilon}$$

$$+\frac{\sqrt{3\varepsilon}}{\pi}\int_0^\infty\frac{dx}{x^2+3\varepsilon}\ln\frac{\varepsilon+x^2/3}{1+\varepsilon-x^{-1}\tan^{-1}x}+\frac{1}{2}\ln(1+\varepsilon).$$

It can be shown that

$$\int_0^\infty \frac{dx}{x^2+3\varepsilon}\ln\frac{\varepsilon+x^2/3}{1+\varepsilon-x^{-1}\tan^{-1}x}$$

$$=\int_0^\infty\frac{dx}{x^2}\ln\frac{x^2/3}{1-x^{-1}\tan^{-1}x}-(3\varepsilon)^{1/2}\frac{9\pi}{20}$$

$$+3\varepsilon\int_0^\infty\frac{dx}{x^2}\left(\frac{5}{6}-\frac{1}{x^2}-\frac{1/3}{1-x^{-1}\tan^{-1}x}-\frac{1}{x^2}\ln\frac{x^2/3}{1-x^{-1}\tan^{-1}x}\right)$$

$$-(3\varepsilon)^{3/2}\frac{159\pi}{1120}+\ldots$$

By numerical integration we find that

$$\int_0^\infty\frac{dx}{x^2}\ln\frac{x^2/3}{1-x^{-1}\tan^{-1}x} = \frac{2}{3}\pi\pm\eta$$

($\eta < 10^{-4}$; the requirement for resonance leads to $\eta = 0$). Thus, confining ourselves to terms of the expansion which are of the order of ϵ,

$$\ln\psi_1(k_0) = -\ln 2\sqrt{\varepsilon}+\frac{2}{\sqrt{3}}\sqrt{\varepsilon}-\frac{17}{20}\varepsilon$$

$$\approx\ln\left(\frac{1}{2\sqrt{\varepsilon}}+\frac{1}{\sqrt{3}}-\frac{17\sqrt{\varepsilon}}{40}\right). \quad (5b)$$

It can be shown similarly that for small k

$$\ln\psi_1(k) = -\ln\left[\sqrt{\varepsilon}\left(1+\frac{k}{k_0}\right)\right]$$

$$+k\left(\frac{2}{3}-\frac{9}{20}\sqrt{3\varepsilon}\right)+\frac{\varepsilon}{2}\operatorname{sign}k$$

$$\approx\ln\left\{\frac{1}{\sqrt{\varepsilon}}\left[1+k\left(\frac{2}{3}-\frac{9}{20}\sqrt{3\varepsilon}\right)+\frac{\varepsilon}{2}\operatorname{sign}k\right]\left(1+\frac{k}{k_0}\right)^{-1}\right\}. \quad (6b)$$

We shall now calculate the integrals $J_1(z)$ and $J_2(z)$. We apply the residue theorem to the inner integral in (2b). The integrand possesses poles in the right half plane at the points $x = k$ and $x = k_0$ because of the functions $1/(x-k)$ and $\psi_2(x)$. The function $G(x)$ has poles only in the left half plane. The "distant" pole $x = l/l_0^*$ of $L(x)$ can be neglected since $1/l_0^* \gg k$. Then

$$J_1(z) = \frac{i}{2\pi}\int_{b-i\infty}^{b+i\infty} e^{kz}\psi_1(k)\,dk$$

$$\times\left\{\psi_2(k)\,G(k)\,L(k)+\frac{G(k_0)\,L(k_0)}{k_0-k}\operatorname{Res}\psi_2(k_0)\right\}.$$

The residue theorem can also be applied to this integral. $G(k)$ has a pole at $k = -1/\delta$ and $\psi_1(k)$ has a pole at $k = -k_0$. If $\delta \ll 1/|k_0| = \delta_{eff}/|\sqrt{\kappa}|$ the pole at $G(k)$ like that of $L(k)$ can be neglected. Noting that $L(k_0) = L(-k_0) \approx 1$, we obtain

$$J_1(z) = e^{-k_0 z}\left\{\psi_2(-k_0)\,G(-k_0)\right.$$

$$\left.+\frac{1}{2k_0}G(k_0)\operatorname{Res}\psi_2(k_0)\right\}\operatorname{Res}\psi_1(-k_0).$$

Hence, using (5b) and (6b) and retaining only zero order terms in (5), we obtain

$$J_1(z) = 2G(0)\,e^{-k_0 z},\quad G(0) = -i\chi t_0\Omega_0 cE(0)/\omega l_0. \quad (7b)$$

Similarly

$$J_2(z) = \left(1-\frac{211}{240}\sqrt{3\varepsilon}\right)e^{-k_0 z}. \quad (8b)$$

From (1b), (7b) and (8b) we have

$$M(z) = -\frac{160}{211}\frac{i\chi T_{sp}\,\Omega_0 cE(0)}{\omega\delta_{eff}\sqrt{\varkappa}}\exp\left\{-\frac{z\sqrt{\varkappa}}{\delta_{eff}}\right\},$$

which agrees with (8a) except for the coefficient. Thus the coefficient A in (3) equals unity for specular reflection of electrons from a metal boundary, while A = 160/211 for diffuse reflection.

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