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ABSORPTION OF POLARIZED μ^--MESONS BY NUCLEI ANGULAR DISTRIBUTION OF THE NEUTRONS

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The angular distribution of neutrons produced in the absorption of polarized μ^- mesons by nuclei is calculated. Numerical computations are performed for O^{16} and Ca^{40}.

1. INTRODUCTION

THE investigation of the nuclear absorption of μ^- mesons is a possible means to determine the character of the non-electrodynamic interaction of the μ^- mesons with nuclei. The nuclear absorption process of the μ^- mesons goes through an intermediate stage under formation of a mesic atom and continues via the reaction

$$\mu^- + P \rightarrow N + \nu + Q, \qquad (1)$$

in which the proton (P) absorbing the μ^- meson transforms into a neutron (N) and a neutrino (ν). The difficulties involved in the discussion of process (1) in nuclei are not merely of computational nature. The trouble is that, owing to the large amount of released energy ($Q \sim 100$ Mev), the neutrino and the neutron emitted by the nucleus in the capture of the μ^- meson carry rather large angular momenta, and the remaining nucleus can acquire a high excitation energy. One must therefore consider transitions involving comparatively large changes in the angular momentum and highly excited nuclear states, which obviously is a difficult task, because of our insufficient knowledge of the nuclear wave functions.

The nuclei in the nuclear μ^--meson absorption processes were described in most calculations by the nuclear shell model with jj coupling. In this case the ratio of the probabilities for μ^- capture in different nuclei depends on the form of the μ meson-nucleon interaction, so that the Fermi interaction (scalar, vector) can be distinguished from the Gamow-Teller interaction (tensor, axial vector).[1-3]

The experimental ratios of the μ^--capture probabilities for six pairs of nuclei were compared with the computed values of Tolhoek and Luyten[3] in reference 4. In the opinion of the authors, the results point to the possible predominance of the Gamow-Teller interaction. This conclusion is, however, not sufficiently reliable, since the above-mentioned calculations have at best semi-quantitative character.

The discovery of parity nonconservation in weak interactions increases the number of effects which may be utilized to decide the question of the type of the μ meson-nucleon interaction. A number of papers[5-9] propose to investigate the asymmetry in the angular distribution of the neutrons from reaction (1) during the absorption of polarized μ^- mesons. In references 5, 9, and 10 it is shown that the nucleus remaining after the μ^- capture is polarized, where the degree of polarization depends on the type of the μ meson-nucleon interaction (the μ^- mesons can be unpolarized). The investigation of the circular polarization and the angular distribution[8] of the γ quanta in the radiative μ^- capture (where, in the second case, the μ^- mesons are assumed to be polarized) also permits the determination of the form of the interaction.

We point out that there are practically no experiments on the absorption of polarized μ^- mesons in μ mesic hydrogen, since the hyperfine structure in the mesic hydrogen atom and the effect of the jumping of the μ^- meson from one proton to another lead to the complete depolarization of the μ^- mesons.[11,12] Furthermore, the probability for absorption of the μ^- mesons in mesic hydrogen is extremely small. It is therefore of greatest interest to investigate theoretically the effects connected with the μ^- capture in complex nuclei. In the present paper we calculate the angular distribution of the neutrons produced according to reaction (1) during the absorption of polarized μ^- mesons by complex nuclei. Preliminary results have been published earlier.[13]

2. ANGULAR DISTRIBUTION OF THE NEUTRONS

The μ^- mesons produced in the decay of π^- mesons are polarized in the direction of their motion. During the slowing down in the medium and the following formation of the mesic atom a depolarization of the μ^- mesons is possible, which can, however, be incomplete.[14] A part of the neutrons produced in process (1) leaves the nucleus immediately after the capture of the μ^- meson, i.e., reaction (1) is a direct process. Under the assumption that, as in other weak interaction processes, parity is not conserved in the μ^- capture, the angular distribution of the neutrons from the direct process has the form

$$1 + P_\mu \alpha \cos\theta, \tag{2}$$

where P_μ is the degree of polarization of the μ^- mesons at the moment of capture, α is asymmetry coefficient depending on the type of the four-fermion interaction of the μ meson-nucleon system and on the degree to which parity is not conserved, and θ is the angle between the direction of polarization of the μ^- meson and the direction of emission of the neutron.

It should be noted that, in addition to the angular distribution (2) of the neutrons emitted by the nucleus in the direct process, there is an isotropic background of neutrons from the decay of the compound nucleus formed in reaction (1). The spectrum of the neutrons of the isotropic background has a maximum at the energy $E_N \sim 1$ to 1.5 Mev, while the maximum for the neutrons of the direct process lies at $E_N \sim 5$ Mev (see Sec. 3). One can thus decrease the background considerably by selecting the neutrons with energies $E_N \gtrsim 3$ Mev. It is to be expected that, for the lighter nuclei, the total number of neutrons consists essentially of neutrons from the direct process.

Assuming parity nonconservation, we can write the Hamiltonian for the four-fermion interaction of the μ mesons with the nucleons in the form

$$H = \sum_k (\overline{\Psi}_N O_k \Psi_P)(\overline{\Psi}_\nu [g_k - g'_k \gamma_5] O\, \Psi_\mu) + \text{herm. conj.}, \tag{3}$$

where k = s, v, p, t, a denotes, respectively the scalar, vector, pseudoscalar, tensor, and axial vector variant, and

$$O_s = 1,\ O_v^\alpha = \gamma_\alpha,\ O_p = \gamma_5,\ O_t^{\alpha\beta} = -i2^{-1/2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha),\quad O_a^\alpha = -i\gamma_\alpha\gamma_5, \quad (\alpha,\ \beta = 1,\ 2,\ 3,\ 4). \tag{4}$$

If $g'_k = -g_k$ we have the theory of the longitudinal neutrino,[15] in which the neutrino produced in process (1) is, with our Hamiltonian (3), polarized in the direction of its motion. If, during the absorption of the μ^- meson, a neutrino is created which is polarized opposite to its direction of motion, then we must change the sign in front of the asymmetry coefficient α.

Since the neutrino carries away most of the energy released in the μ^- capture, while the nucleons in the nucleus acquire only energies of a few Mev, we can treat the nucleons nonrelativistically in the s, v, t, and a variants of the interaction. For the p variant (which vanishes in the nonrelativistic approximation) we use the first approximation for the nucleons (in terms of v/c), in which the small components of the wave function (φ) are expressed in terms of the large ones (ψ) according to the well-known formula $\varphi = [(\sigma\times\mathbf{p})/2Mc]\,\psi$, where $\mathbf{p} = -i\hbar\nabla$. Furthermore, we can neglect terms of order $\lambda\!\!\!^{-}_\nu/a_\mu$ in the p variant ($\lambda\!\!\!^{-}_\nu \sim 2\times10^{-13}$ cm is the de Broglie wavelength of the neutrino, and $a_\mu \sim 2\times10^{-1}Z^{-1}$ cm is the radius of the K orbit of the μ^- mesic atom).

Since the binding energy of the μ^- meson in the K orbit of the mesic atom is much less than its rest energy, we may restrict ourselves to the nonrelativistic approximation of the μ^- meson wave function. The nucleus is described by the shell model with jj coupling. The recoil energy of the nucleus is neglected. For the wave function of the emitted neutron we take the complex potential solution, which takes into account the interaction of the neutron with the remaining nucleus. The potential of the shell model and the potential describing the interaction of the neutron with the nucleus are taken to be spherically symmetric.

With these assumptions the wave functions of the proton in the (n, j, l) subshell, of the neutron, of the μ^- meson, and of the neutrino can be written in the form

$$\psi_P = R_{njl}(r)\,\Omega_{jll_z}(\mathbf{r}/r), \quad \int_0^\infty R_{njl}^2(r)\,r^2dr = 1, \tag{5}$$

$$\psi_N = \psi_N(\mathbf{r})\,\chi_{s_N}, \quad \Psi_N(\mathbf{r}) = \sum_L i^L(2L+1)\,a_L(r)\,P_L(\mathbf{k}_N\mathbf{r}/k_N r), \tag{6}$$

$$\psi_N(\mathbf{r}) \sim (2\pi)^{-3/2}(e^{i\mathbf{k}_N\mathbf{r}} + (f/r)\,e^{-ik_N r}) \text{ for } r\to\infty,^*$$

$$\psi_\mu = (4\pi)^{-1/2}R_\mu(r)\,\chi_{s_\mu}, \quad \int_0^\infty R_\mu^2(r)\,r^2\,dr = 1, \tag{7}$$

$$\psi_\nu = (2\pi)^{-3/2}u_\nu e^{i\mathbf{k}_\nu\mathbf{r}}. \tag{8}$$

Here n, j, l are the quantum numbers specifying the nuclear subshell in the shell model with jj coupling (n is the number of nodes of $R_{njl}(r)$ plus one, l is the orbital angular momentum, and

*This asymptotic form takes account of the creation of the neutron.[16]

$j = l \pm \frac{1}{2}$ is the total momentum of the proton), χ are normalized spinors, u_ν is a normalized Dirac bispinor, and $\Omega_{jlj_z}(\mathbf{r}/r)$ is a spherical spinor.[17] Disregarding the finite dimensions of the nucleus (which is admissible for $Z < 30$), we have

$$R_\mu(r) = 2a_\mu^{-3/2}\exp(-r/a_\mu),\quad a_\mu = \hbar^2/m_\mu e^2 Z.$$

The computational results listed below apply, strictly speaking, only to nuclei with completely filled proton subshells. In this case, according to the Pauli principle, a proton state in the nucleus is described by a wave function in form of a determinant made up of one-particle wave functions of form (5). It is easily shown, however, that the use of wave function (5) directly for the single proton in the calculations, with the subsequent summation over the projections of the total angular momentum j_z and over all proton subshells (n, j, l) in the nucleus, leads to the same result as the calculation using the determinant. If some subshell (n, j, l) in the nucleus is not completely filled, we must, generally speaking, include in the calculations the correlation between the angular momenta of the separate protons in this subshell.* We further note that, if the nuclear spin is different from zero, with the protons contributing to the spin, it is necessary to include the hyperfine structure of the mesic atom in the calculations. Since the latter causes a strong additional depolarization of the μ^- mesons, such cases are not of great interest among the experiments under discussion, and we shall therefore leave them out of our considerations.

With the usual formulas of first order perturbation theory we obtain from Eqs. (3) to (8) for the probability of emission of the neutron from the nucleus with an energy in the interval E_N to $E_N + dE_N$ $(E_N = \hbar^2 k_N^2/2M)$ into the solid angle $d\Omega_N$ under the given angle θ with respect to the direction of polarization of the μ^- meson:†

$$\begin{aligned} dW(E_N, \theta) = \{[((f_{ss} + 2\mathrm{Re} f_{sv} + f_{vv}) \\ + 3(f_{tt} + 2\mathrm{Re} f_{ta} + f_{aa}))A_0(E_N) \\ - 2\mathrm{Re}(f_{pt} + f_{pa})A_1(E_N) + f_{pp}A_2(E_N)] \\ + P_\mu[(-(h_{ss} + 2\mathrm{Re}\,h_{sv} + h_{vv}) + (h_{tt} + 2\mathrm{Re}\,h_{ta} + h_{aa})) \\ \times B_0(E_N) + 2\mathrm{Re}(h_{pt} + h_{pa})B_1(E_N) - h_{pp}B_2(E_N) \\ + 2\mathrm{Im}(h_{st} + h_{sa} + h_{vt} + h_{va})G_0(E_N) \\ + 2\mathrm{Im}(h_{pt} + h_{pa})G_1(E_N)]\cos\theta\}\, dE_N\, d\Omega_N/4\pi, \end{aligned} \tag{9}$$

where $A_k(E_N)$, $B_k(E_N)$, and $G_k(E_N)$ $(k = 0, 1, 2)$ are functions of the energy of the emitted neutron depending on the properties of the nucleus under consideration.*

With the help of (9) we can write the angular distribution of the neutrons emitted from the nucleus in the form

$$q(E_N, \theta) = 1 + P_\mu\alpha(E_N)\cos\theta, \tag{10}$$

where

$$\begin{aligned} \alpha(E_N) = \{[(h_{ss} + 2\mathrm{Re}\,h_{sv} + h_{vv}) \\ - (h_{tt} + 2\mathrm{Re}\,h_{ta} + h_{aa})]\beta_0(E_N) \\ - 2\mathrm{Re}(h_{pt} + h_{pa})\beta_1(E_N)\gamma_1(E_N) + h_{pp}\beta_2(E_N)\gamma_2(E_N) \\ + 2\mathrm{Im}(h_{st} + h_{sa} + h_{vt} + h_{va})\delta_0(E_N) \\ + 2\mathrm{Im}(h_{pt} + h_{pa})\delta_1(E_N)\}\{(f_{ss} + 2\mathrm{Re} f_{sv} + f_{vv}) \\ + 3(f_{tt} + 2\mathrm{Re} f_{ta} + f_{aa}) - 2\mathrm{Re}(f_{pt} + f_{pa})\gamma_1(E_N) \\ + f_{pp}\gamma_2(E_N)\}^{-1}. \end{aligned} \tag{11}$$

We remember that for μ^- capture in mesic hydrogen[5] (neglecting the fine structure)†

$$\beta_k = 1,\quad \delta_k = 0,\quad \gamma_1 = 0.53\cdot 10^{-1},\quad \gamma_2 = 0.28\cdot 10^{-2}. \tag{12}$$

Our case is therefore different from that of the mesic hydrogen in that the quantities $\beta_k(E_N)$ and $\gamma_k(E_N)$ are here functions of the neutron energy depending on concrete properties of the nucleus, and also in that additional terms involving $\delta_k(E_N)$ appear. The order of magnitude of the functions $\gamma_k(E_N)$ for the nucleus agrees with the values in formula (12).

In the theory of the longitudinally polarized neutrino $(g_k' = -g_k)$ formula (11) takes the form

$$\begin{aligned} \alpha^{\mathrm{long.}}(E_N) = \{(-|g_s + g_v|^2 + |g_t + g_a|^2)\beta_0(E_N) \\ + 2\mathrm{Re}[g_p^*(g_t + g_a)]\beta_1(E_N)\gamma_1(E_N) - |g_p|^2\beta_2(E_N)\gamma_2(E_N) \\ - 2\mathrm{Im}[(g_s^* + g_v^*)(g_t + g_a)]\delta_0(E_N) \\ - 2\mathrm{Im}[g_p^*(g_t + g_a)]\delta_1(E_N)\}\{|g_s + g_v|^2 + 3|g_t + g_a|^2 \\ - 2\mathrm{Re}[g_p^*(g_t + g_a)]\gamma_1(E_N) + |g_p|^2\gamma_2(E_N)\}^{-1}. \end{aligned} \tag{13}$$

In the absence of pseudoscalar coupling we obtain from (13)

$$\begin{aligned} \alpha_{svta}^{\mathrm{long.}}(E_N) = \{(-|g_s + g_v|^2 + |g_t + g_a|^2)\beta_0(E_N) \\ - 2\mathrm{Im}[(g_s^* + g_v^*)(g_t + g_a)]\delta_0(E_N)\} \\ \times \{|g_s + g_v|^2 + 3|g_t + g_a|^2\}^{-1}. \end{aligned} \tag{13a}$$

*If the incompletely filled nuclear subshell (n, j, l) contains two protons with total spin zero, the formulas below remain valid provided we change $(2j + 1)$ to 2 for this subshell in all formulas of the Appendix.

†The notations used in (9) and in the following formulas of this section are explained in the Appendix at the end of this article.

*Formulas (2) and (3) of reference 13 do not include the terms describing the interference between the Fermi and Gamow-Teller interactions. The interference term missing in (3) is given by formula (A 12) of the Appendix. Furthermore, (2) and (3) should be multiplied by the function $\rho_{njl}(E_N)$, which is also given in the Appendix.

†In reference 5 the tensor and axial vector operators are defined as $O_t^{\alpha\beta} = (1/2)(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)$ and $O_a^\alpha = \gamma_\alpha\gamma_5$, respectively. Here we use the usual definition [see (4)].

For pure p coupling we have

$$\alpha_p^{\text{long.}}(E_N) = -\beta_2(E_N). \tag{13b}$$

Integrating over the energy of the emitted neutron E_N, we obtain, instead of (9), (10), (11), and (13), formulas of completely analogous form, in which the functions $A_k(E_N)$, $B_k(E_N)$, $G_k(E_N)$, $\beta_k(E_N)$, $\gamma_k(E_N)$, and $\delta_k(E_N)$ are replaced by the corresponding constants $\tilde{A}_k$, $\tilde{B}_k$, $\tilde{G}_k$, $\tilde{\beta}_k$, $\tilde{\gamma}_k$, and $\tilde{\delta}_k$, which depend, of course, on the properties of the given nucleus.

We note that in the special case of emission of the neutron and the neutrino in opposite directions ($\mathbf{n}_N + \mathbf{n}_\nu = 0$) the angular distribution of the neutrons has the form (10) with α given by (11) and (13), $\beta_k = 1$, $\delta_k = 0$, and γ_k very close to the γ_k of formula (12).

We add a few remarks concerning the formulas obtained.

1. If it turns out that the interaction of the μ mesons with the nucleons is that of the Gell-Mann — Feynman theory,[18] which assumes the presence of the a and v variants with one common coupling constant, then, according to (13), the asymmetry effect in the angular distribution of the neutrons disappears.

2. The pseudoscalar coupling is multiplied by a factor $\sim 1/20$ in all formulas.

3. The spin-orbit interaction of the emitted neutron with the nucleus has been neglected in the derivation of our formulas. The inclusion of this interaction leads to the expressions given in reference 13. Estimates made for some special cases lead us to assume that the spin-orbit interaction has little effect on the angular distribution of the neutrons. It may, however, become quite important in the computation of the polarization of the neutrons in process (1).

3. NUMERICAL CALCULATIONS FOR O^{16} AND Ca^{40}

The quantities entering into the formulas (9) and (11) were calculated for the μ^- capture in the nuclei O^{16} and Ca^{40}. The calculations were made under the following assumptions.

(A) For the radial proton wave functions in the nucleus we used the wave functions corresponding to a square-well potential. The dependence of the radius of the well R on the mass number A was assumed to have the form $R = r_0 A^{1/3}$. We neglected the dependence of the radial wave function on the quantum number j, so that the binding energies of the protons in the nucleus are completely determined by the values of the quantum numbers n and l. The order in which the proton shells are filled was, hence, taken to be: $1s^2$, $1p^6$, $1d^{10}$, $2s^2$, etc. From the binding energy of the proton in the nucleus, ϵ_p, for a given radius of the potential well, R, we found the well depth U_P with the help of well-known formulas. Then the binding energies of the remaining protons in the nucleus were determined. We note that $G_k(E_N) \equiv 0$ for nuclei with closed proton shells, if we neglect the dependence of the radial wave function of the proton in the nucleus on j. This is easily seen by immediately carrying out the summation over $j = l \pm \frac{1}{2}$ in formula (A4) of the Appendix.

(B) The interaction of the neutron with the nucleus, $V_N(r)$, is described by a complex square well ($V_N(r) = -U_N(1 + i\xi)$ for $0 \le r < R$, and $V_N(r) = 0$ for $r > R$) with the same radius R as in the case of the proton.

The use of the complex potential allows us to consider a definite absorption probability for the neutrons in the nucleus. In contrast to the scattering problem where $\xi > 0$, the choice of the asymptotic neutron wave function (6) presupposes in our case that $\xi < 0$.

(C) The coordinate dependence of the μ^--meson wave function in the K orbit is neglected.

The parameters of the proton and neutron potentials and of the proton states in the nucleus are listed in Table I. The values of r_0 and U_N are chosen in agreement with the paper of Feshbach, Porter, and Weisskopf.[19]

TABLE I

Nucleus	$r_0 \cdot 10^{13}$(cm)	U_N (Mev)	U_P (Mev)	Proton energy levels in the nucleus (in Mev)			
				1s	1p	1d	2s
O^{16}	1.45	42	32.3	22.1	12.1*	—	—
Ca^{40}	1.45	42	31.7	25.6	19.3	11.6	8.35*

*The data on the proton binding energy ε_p are taken from Ref. 21.

The calculations were carried out on the "Strela" computer of the Moscow State University. In order to guarantee a sufficient accuracy ($\sim 5\%$), we had to include all orbital angular momenta L of the emitted neutron up to L = 3, inclusively, in the case of oxygen, and up to L = 4 for the case of calcium. For the following it is convenient to introduce the dimensionless coupling constants c instead of the coupling constants g_k:

$$\text{for } g_k = c_k (10^{-49} \text{ erg cm}^3)$$

(similarly for g'_k), and to measure the energy

TABLE II

Nucleus	ξ	$\tilde{W}_0$ (sec^{-1})	$\tilde{W}_1$ (sec^{-1})	$\tilde{W}_2$ (sec^{-1})	$\tilde{\beta}_0$	$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\tilde{\gamma}_1$	$\tilde{\gamma}_2$	Ratio of the probabilities for μ^- capture in different shells*	Ratio of specific probabilities (per proton) for μ^- capture in different shells
1	2	3	4	5	6	7	8	9	10	11	12
O^{16}	0	$1.62\cdot10^3$	$0.742\cdot10^3$	$0.341\cdot10$	0.034	0.037	0.040	$0.458\cdot10^{-1}$	$0.210\cdot10^{-2}$	1 : 65**	1 : 21.6**
	−0.15	$0.718\cdot10^3$	$0.324\cdot10^2$	$0.146\cdot10$	0.476	0.482	0.489	$0.453\cdot10^{-1}$	$0.203\cdot10^{-2}$	1 : 38**	1 : 12.6**
Ca^{40}	0	$1.33\cdot10^5$	$0.618\cdot10^4$	$2.96\cdot10^2$	0.145	0.146	0.144	$0.415\cdot10^{-1}$	$0.222\cdot10^{-2}$	1 : 7.8 : 290 : 190***	1 : 2.6 : 58 : 190***
	−0.15	$0.327\cdot10^5$	$0.153\cdot10^4$	$0.712\cdot10^2$	0.528	0.528	0.530	$0.418\cdot10^{-1}$	$0.218\cdot10^{-2}$	1 : 11.4 : 148 : 149***	1 : 3.8 : 29.6 : 149***

* These numbers refer to the values of $(\tilde{W}_{nl})_0$; analogous results are obtained for $(\tilde{W}_{nl})_{1,2}$.
** 1s : 1p.
*** 1s : 1p : 1d : 2s.

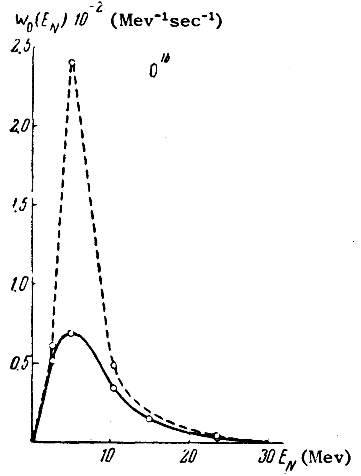

FIG. 1

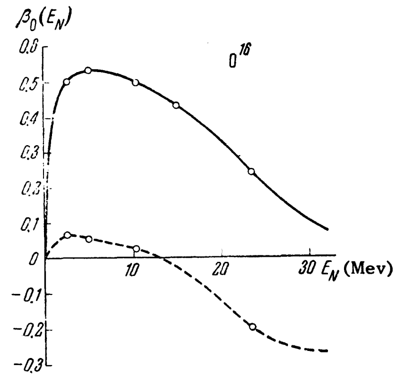

FIG. 2

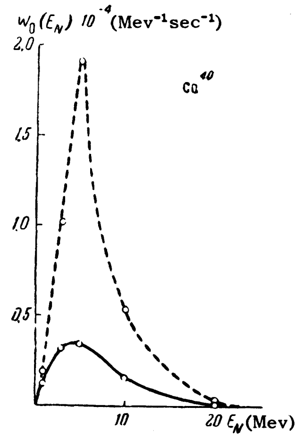

FIG. 3

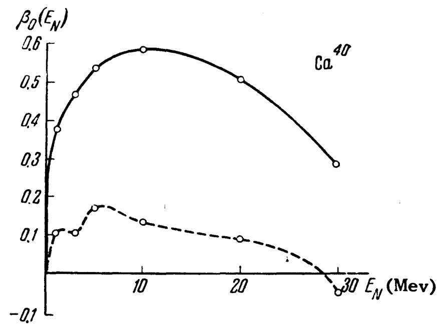

FIG. 4

E_N in Mev. Formula (9) then yields for the probability of emission of neutrons with energies in the interval E_N to $E_N + dE_N$ as a result of the direct process:

$$dW(E_N) = \{[(\varphi_{ss} + 2\mathrm{Re}\,\varphi_{sv} + \varphi_{vv}) + 3(\varphi_{tt} + 2\mathrm{Re}\,\varphi_{ta} + \varphi_{aa})]W_0(E_N) - 2\mathrm{Re}(\varphi_{pt} + \varphi_{pa})W_1(E_N) + \varphi_{pp}W_2(E_N)\}\,dE_N, \qquad (14)$$

where $\varphi_{ik} = c_i^*c_k + c_i'^*c_k'$, and the quantities $W_k(E_N)$ are related to the quantities $A_k(E_N)$ in an obvious manner. For the total probability $\tilde{W} = \int dW(E_N)$ we obtain a completely analogous formula with $W_k(E_N)$ replaced by $\tilde{W}_k = \int W_k(E_N)\,dE_N$.

The results of the calculations are given in Table II and in Figs. 1 to 4.* The dotted curves in the figures refer to calculations done assuming no absorption of the neutrons in the nucleus ($\xi = 0$), the solid curves refer to calculations including the absorption effect ($\xi = -0.15$).

Figure 1 (O^{16}) and Fig. 2 (Ca^{20}) show that the probability of emission of the neutrons on account of the direct process has a clearly expressed

*Figures 1 to 4 show the dependence of the quantities $W_0(E_N)$ and $\beta_0(E_N)$ on the energy E_N. The form of the curves for $W_1(E_N)$ and $W_2(E_N)$ is practically the same as that of the curve for $W_0(E_N)$. The deviation of $\beta_1(E_N)$ and $\beta_2(E_N)$ from $\beta_0(E_N)$ does not exceed a few percent.

maximum at neutron energies of $E_N \sim 5$ Mev. The inclusion of absorption in the nucleus levels out the maximum without changing its position and lowers, of course, the magnitude of the probability.

In columns 3, 4, and 5 of Table II we list the values of W_0, W_1, and W_2 integrated over the energy of the neutron E_N. Columns 11 and 12 of Table II give the ratios of the μ^--capture probabilities integrated over E_N for the different shells. It follows from these data that, at least for the light nuclei, the μ^- mesons are mainly absorbed by protons belonging to the outer shells of the nucleus. Of the eight protons in O^{16} only the six protons in the outer 1p shell are effective in the μ^- capture, and of the 20 protons in Ca^{20}, only the 12 protons of the two outer shells (1d and 2s). Column 12 shows clearly that our calculations do not support the assumption that the protons in the nucleus absorb the μ^- mesons with approximately equal probabilities (see, e.g., reference 20).

Figures 2 and 4 show the dependence of $\beta_0(E_N)$ on E_N. The effect of the absorption of the neutrons in the nucleus is quite large. It greatly enlarges the value of $\beta_0(E_N)$ while, at the same time, smoothing out its dependence on E_N.* The values of $\tilde{\beta}_k$ and $\tilde{\gamma}_k$ are listed in columns 6 to 10 of Table II. The comparison of these data with (12) shows that the difference between our case and the case of μ^- capture in mesic hydrogen is, roughly speaking, that the asymmetry coefficient α_H for mesic hydrogen with neglect of the hyperfine structure is here multiplied by some positive factor smaller than unity. For an absorption corresponding to $\xi = -0.15$, α_H is reduced to about half its value. It is easily shown that $\beta_k(E_N) \sim E_N^{1/2}$ for $E_N \to 0$.

To exhibit the sensitivity of our results to a variation in the parameters, we repeated the calculations for O^{16} with $r_0 = 1.3 \times 10^{-13}$ cm and $U_N = 52$ Mev (in agreement with reference 19, we assumed $r_0 U_N^{1/2} = \text{const}$). The proton parameters were the following: $U_P = 36.6$ Mev, binding energies $\epsilon_P^{1p} = 12.1$ Mev, and $\epsilon_P^{1s} = 24.3$ Mev. Within the limits of error of the calculation, the results agree with the data obtained with our earlier value $r_0 = 1.45 \times 10^{-13}$ Mev (for example, taking $\xi = -0.15$ we get $\tilde{\beta}_0 = 0.491$, and the ratio of the probabilities of μ^- capture in the 1s and 1p shells is equal to 1 : 31, etc.).

*As shown in a number of papers devoted to the calculation of the scattering neutrons from nuclei, the choice of an absorption coefficient corresponding to $\xi = -0.15$ is, for our neutron well parameters and for energies $E_N \sim 5\text{-}15$ Mev, quite reasonable.

4. CONCLUSION

The calculations for the absorption of μ^- mesons in oxygen and calcium lead to the following results.

1. The energy spectrum of the neutrons emitted from the nucleus immediately after the μ^- capture has a clearly expressed maximum at neutron energies of $E_N \sim 5$ Mev.

2. The overwhelming majority of the μ^- mesons is absorbed by protons in the outer shells of the nuclei.

3. The angular distribution $q(\theta)$ of the neutrons emitted during the absorption of polarized μ^- mesons by nuclei can, in first approximation, be represented in the form

$$q(\theta) = 1 + P_\mu \tilde{\beta} \alpha_H \cos\theta,$$

where α_H is the asymmetry coefficient for μ^- meson capture in mesic hydrogen (with neglect of its hyperfine structure), and the factor $\tilde{\beta}$ ($|\tilde{\beta}| \le 1$) depends on the properties of the nucleus. $\tilde{\beta} \approx +0.5$ for μ^- capture in O^{16} and Ca^{40}.

4. Since the degree of polarization of the μ^- mesons in the K orbit of the mesic atoms reaches the value $P_\mu \sim 0.15$ to 0.20 (see references 14 and 22), while α_H is enclosed within the limits[5] $-1 \le \alpha_H \le +1/3$, one may, for $\tilde{\beta} \sim 0.5$, expect an asymmetry on the order of 3 to 10% in the angular distribution of the neutrons from the direct process.

Recently there appeared a preliminary account[23] of measurements of the angular distribution of neutrons for the μ^- capture by S, Zn, and Pb. The results are apparently isotropic. It should be noted, however, that the experimental errors of these measurements are comparatively large (2% for S, 3% for Zn, and 10% for Pb). Since the expected asymmetry effect may amount to only a few percent, reliable conclusions can be drawn only from measurements with accuracy of at most a few tenths of a percent.

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APPENDIX

We give a list of the symbols used in the article:

$$f_{ab} = g_a^* g_b + g_a'^* g_b', \quad h_{ab} = g_a^* g_b' + g_a'^* g_b \quad (a, b = s, v, p, t, a); \tag{A.1}$$

$$A_k(E_N) = C \sum_{njl} [(2j+1)/(2l+1)] (E_\nu^{njl}/2Mc^2)^k A_{njl}(E_N) \rho_{njl}(E_N); \tag{A.2}$$

$$B_k(E_N = C \sum_{njl} [(2j+1)/(2l+1)] (E_\nu^{njl}/2Mc^2)^k B_{njl}(E_N) \rho_{njl}(E_N); \tag{A.3}$$

$$G_k(E_N) = C \sum_{njl} [(2j+1)/(2l+1)] (E_\nu^{njl}/2Mc^2)^k G_{njl}(E_N) \rho_{njl}(E_N); \tag{A.4}$$

$$\beta_k(E_N) = - B_k(E_N)/A_k(E_N), \quad \gamma_k(E_N) = A_k(E_N)/A_0(E_N), \quad \delta_k(E_N) = G_k(E_N)/A_0(E_N); \tag{A.5}$$

$$\widetilde{A}_k = \int A_k(E_N)\, dE_N; \quad \widetilde{B}_k = \int B_k(E_N)\, dE_N, \quad \widetilde{G}_k = \int G_k(E_N)\, dE_N; \tag{A.6}$$

$$\widetilde{\beta}_k = -\widetilde{B}_k/\widetilde{A}_k, \quad \widetilde{\gamma}_k = \widetilde{A}_k/\widetilde{A}_0, \quad \widetilde{\delta}_k = \widetilde{G}_k/\widetilde{A}_0, \quad (k = 0, 1, 2); \tag{A.7}$$

$$A_{njl}(E_N) = \sum_{L\Lambda} (2L+1)(2\Lambda+1)(C_{L0\Lambda0}^{l0})^2 | b_{L\Lambda njl}(E_N) |^2; \tag{A.8}$$

$$B_{njl}(E_N) = \mathrm{Re} \sum_{L\Lambda} [(2L+1)(2L+2)(2L+3)(2\Lambda+1)(2\Lambda+2)(2\Lambda+3)]^{1/2} [C_{L0\Lambda0}^{l0} C_{L+10\Lambda+10}^{l0} W(L+1l1\Lambda;\ \Lambda+1L)$$
$$\times b_{L\Lambda njl}(E_N) b_{L+1\Lambda+1njl}^*(E_N) + C_{L+10\Lambda0}^{l0} C_{L0\Lambda+10}^{l0} W[Ll1\Lambda;\ \Lambda+1L+1) b_{L+1\Lambda njl}(E_N) b_{L\Lambda+1njl}^*(E_N)]; \tag{A.9}$$

$$G_{njl}(E_N) = 6(2l+1) W\left(lj1\tfrac{1}{2};\ \tfrac{1}{2}l\right) \mathrm{Im} \sum_{L\Lambda} [(2L+1)(2L+2)(2L+3)(2\Lambda+1)(2\Lambda+2)(2\Lambda+3)]^{1/2}$$
$$\times [C_{L0\Lambda0}^{l0} C_{L+10\Lambda+10}^{l0} X(L\Lambda l;\ L+1\Lambda+1l;\ 111) b_{L\Lambda njl}^*(E_N) b_{L+1\Lambda+1njl}(E_N)$$
$$+ C_{L0\Lambda+10}^{l0} C_{L+10\Lambda0}^{l0} X(L\Lambda+1l;\ L+1\Lambda l;\ 111) b_{L+1\Lambda njl}^*(E_N) b_{L\Lambda+1njl}(E_N)]; \tag{A.10}$$

$$b_{L\Lambda njl}(E_N) = \int_0^\infty R_\mu(r) a_L^*(r) j_\Lambda(k_\nu r) R_{njl}(r) r^2 dr; \tag{A.11}$$

$$C = (2M)^{3/2} \hbar^{-7} c^{-3};$$
$$E_\nu^{njl} = E_{N\max}^{njl} - E_N; \quad E_{N\max}^{njl} = m_\mu c^2 - \varepsilon_\mu - \varepsilon_p^{njl};$$
$$\rho_{njl}(E_N) = E_N^{1/2}(E_{N\max}^{njl} - E_N)^2;$$

$j_\Lambda(x) = (\pi/2x)^{1/2} J_{\Lambda+1/2}(x)$ is the spherical Bessel function; $R_\mu(r)$ is the normalized radial wave function of the μ^- meson for the ground state of the mesic atom; W are the Racah coefficients;[24] the definition of the quantities X is given in reference 25; P_μ is the degree of polarization of the μ^- mesons at the moment of absorption; ϵ_μ is the binding energy of the ground state of the mesic atom; m_μ is the mass of the μ^- meson; ϵ_P^{njl} is the binding energy of the proton in the subshell (n, j, l); M is the mass of the nucleon (we neglect the proton-neutron mass difference).

Leaving out the sum over (n, j, l) in formulas (A.2), (A.3), and (A.4) and substituting these expressions in (9), we obtain a formula describing the absorption of the μ^- meson in a given filled subshell (n, j, l).

We note that $G_{n\,1/2\,0}(E_N) = 0$ for $l = 0$.

$$W_{njl}^{svta}(E_N, \theta) = (2^{5/2}/3)(-)^{j-s}(2j+1) \sum_{IL\Lambda I'L'\Lambda' gfh} i^{L+\Lambda-L'-\Lambda'} (-)^{I+f+h} (2I+1)(2I'+1)(2L+1)(2L'+1)(2\Lambda+1)(2\Lambda'+1)$$
$$\times (2g+1)(2f+1)(2h+1) C_{L0\Lambda0}^{l0} C_{L'0\Lambda'0}^{l0} C_{\Lambda0\Lambda'0}^{10} C_{L0h1}^{11} C_{L'0h1}^{11} W\left(I\tfrac{1}{2}\Lambda l;\ Lj\right) X\left(L'\Lambda' l;\ I'gj;\ \tfrac{1}{2}1\tfrac{1}{2}\right)$$
$$\times X(jI'g;\ If1;\ \Lambda 1\Lambda') X\left(I1f;\ Lh1;\ \tfrac{1}{2}L'I'\right) \mathrm{Re}\{[h_{st} + h_{sa} + h_{vt} + h_{va}]\, b_{IL\Lambda njl}^*(E_N)\, b_{I'L'\Lambda' njl}(E_N)\} \cos\theta. \tag{A.12}$$

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