SCATTERING OF NEUTRONS BY ORIENTED NONSPHERICAL NUCLEI

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We consider the scattering of neutrons by oriented nonspherical nuclei. To calculate the scattering cross section we use the black-nucleus model. It is shown that nonsphericity effects are much more pronounced on oriented nuclei than on unoriented ones. It is also shown that a considerable azimuthal asymmetry appears in the angular distribution of neutrons scattered by oriented nonspherical nuclei.

1. INTRODUCTION

THE investigation of neutron scattering by nonspherical nuclei may yield much valuable information on the structure of atomic nuclei. However, as shown in many papers,[1,2] the nonsphericity under ordinary experimental conditions does not manifest itself strongly enough to permit reliable interpretation of the experimental results. For example, at neutron energies of several tens of Mev, the total neutron cross sections are changed by nonsphericity of the nuclei by only two or three percent (at the experimentally-observed values of nonsphericity).

The effects due to nonsphericity increase if the targets employed are oriented nuclei. Actually, we consider, for example, a black nucleus in the shape of an ellipsoid of revolution with semi-axes a and b (a is the major semi-axis, directed along the axial symmetry axis of the nucleus). If the nuclei are now oriented so that the symmetry axis of the nucleus coincides with the direction of the incident beam of neutrons, we obtain for the total cross section $\sigma_t^{\parallel} = 2\pi b^2$, but if the symmetry axis is directed perpendicular to the incident beam, we get $\sigma_t^{\perp} = 2\pi ab$. Thus $\sigma_t^{\perp}/\sigma_t^{\parallel} = a/b$. We can thus determine directly whether the nucleus is prolate or oblate. In the former case $\sigma_t^{\perp}/\sigma_t^{\parallel} > 1$, and in the latter $\sigma_t^{\perp}/\sigma_t^{\parallel} < 1$. For the nonsphericities observed experimentally, a typical ratio of the semi-axis is 1.3 — 1.4 and the estimate made here shows that the nonsphericity effects may reach 30 or 40%.

Actually this estimate is somewhat too high, since it is impossible in practice to attain complete orientation of the nuclear spins along a specified direction. Furthermore, the symmetry axis of the nucleus undergoes a quantum-mechanical precession about the direction of the spin and this leads to a further decrease in the effect. It is obvious that the last circumstance will manifest itself less, the greater the nuclear spin. In very large spins, i.e., in the quasi-classical case, the precession can be neglected.

If the direction of the nuclear spin orientation does not coincide with the direction of the incident beam, an azimuthal asymmetry may occur in the angular distribution of the scattered neutrons. This phenomenon is connected with the fact that there exists a definite plane, determined by the direction of the incident beam and by the direction of the nuclear spins of the target. The appearance of azimuthal asymmetry is directly connected with the nonsphericity of the nuclei and therefore a study of this phenomenon can yield interesting information on the parameters of the nucleus.

In the present paper we examine a neutron scattering by oriented nonspherical nuclei. In the calculations we use the adiabatic approximation,[1-3] i.e., the motion of the neutron is assumed sufficiently fast to permit neglecting the rotation of the nucleus during the collision time. This is true if the neutron energy exceeds several Mev.

In the calculation of the specific examples we used the model of the black nucleus, which limits the applicability of the numerical results obtained to neutron energies of several tens of Mev. When using the corresponding expressions for the scattering amplitude, similar calculations can be performed in complete analogy for other energies, too.

2. DESCRIPTION OF THE SPIN STATE OF THE NUCLEUS

The spin state of an ensemble of nuclei with spin I is characterized by a (2I + 1)-row density matrix,[4] which can be represented in the form of an expansion in the irreducible spin tensors

T^{JM}, which transform under rotations of the quantization axis according to the irreducible representation of the rotation group D_J. Defining the mm′-th matrix element of T^{JM} in the form

$$T^{JM}_{mm'} = (-1)^{I+m'} (Im - m' \mid IIJM), \quad (1)$$

so that the normalization condition assumes the form

$$\mathrm{Sp}\, T^{JM} (T^{J'M'})^{+} = \delta_{JJ'} \delta_{MM'}, \quad (2)$$

we obtain the following expansion of the density matrix ρ in irreducible spin tensors

$$\rho = \sum_{J=0}^{2I} \sum_{M=-J}^{J} \langle (T^{+})^{JM} \rangle T^{JM}. \quad (3)$$

In cases of practical interest, when the spin orientation of the nuclei is caused by an axially-symmetrical field, the density matrix (assuming that the direction of the orienting field is taken to be the same as the quantization axis) is diagonal and can be written

$$\rho = \sum_{J=0}^{2I} \langle (T^{+})^{J0} \rangle T^{J0}. \quad (4)$$

The 2I quantities $<(T^{+})^{J_0}>$, which characterize the spin state of a system of nuclei with spin I, can be expressed in terms of the parameters f_k, which describe the degree of orientation of the nuclei, for example, for $I = 1$:

$$T^{10} = (1/\sqrt{2})\, f_1,$$
$$T^{20} = (1/\sqrt{6})\, f_2.$$

De Groot[5] and Khutsishvili[6] give the explicit form and a detailed description of the properties of the parameters f_k. We note that the system in which at least one parameter $f_{2p+1} \neq 0$ (p is an integer) is called polarized; but if all $f_{2p+1} = 0$, but at least one parameter $f_{2p+1} \neq 0$, the system is called aligned. The alignment is possible only if $I \geq 1$. For non-oriented nuclei all $f_k = 0$; the normalization is so chosen that the maximum values of f_k are $+1$.

As is known, the average value of a certain physical quantity A is defined with the aid of the density matrix in the following form:

$$\bar{A} = \mathrm{Sp}\, \rho A.$$

Taking into account the fact that the density matrix is diagonal, we can write the average value of the cross section of any scattering process in the following form:

$$\bar{\sigma} = \sum_{M=-I}^{I} \rho_{MM} \sigma_M = \sum_{M=-I}^{I} \sum_{J=0}^{2I} \langle T^{J0} \rangle T^{J0}_{MM} \sigma_M, \quad (5)$$

where σ_M is the cross section of the particular process on a nucleus with spin and spin projection along the quantization axis M. Since $\sigma_M = \sigma_{-M}$, the expression for $\bar{\sigma}$ will contain only f_k with even k, i.e., the scattering cross section will depend only on the degree of alignment, but not on the degree of polarization of the target nuclei.

The effect of the orientation of the nuclear spins on the cross section of any scattering process is best characterized by a ratio of the scattering cross section $\sigma(f_k)$ on nuclei with an orientation specified by a certain set of parameters f_k, to the section $\sigma(0)$ of the same process on unoriented nuclei.

We introduce the notation

$$\gamma^I_{MK} = (\sigma^I_{MK} - \sigma^I_{M_0K}) / \sigma^I_{M_0K}, \quad (6)$$

where M is the projection of the nuclear spin on the direction of the orienting field, M_0 vanishes for integral spin and equals ½ for half-integral spin, and K is the projection of the nuclear spin along the direction of the symmetry axis of the ellipsoid. The indices I and K we shall omit in the future wherever it causes no misunderstanding. Using (1), (5), and (6) we obtain for the quantities $\bar{\sigma}(f_k)/\bar{\sigma}(0)$, under specific cases of different spins, the expressions given in the table. The explicit form of the expression for the multiplier of f_6 is not given, since an estimate has shown that the terms containing f_6 give a negligible contribution.

3. TOTAL CROSS SECTION

Drozdov[1] and Inopin[2] have shown that in the adiabatic approximation the total cross section of all process σ_t is determined by the amplitude for scattering on a stationary nucleus. If the nucleus is characterized by quantum numbers I, M, and K, the total cross section can be expressed as follows

$$\sigma^I_{tMK} = \frac{4\pi}{k} \mathrm{Im} \int f(\Omega, \omega) \,|\Psi^I_{MK}(\omega)|^2 d\omega \,|_{\theta=0}, \quad (7)$$

where ω is a unit vector that defines the orientation of the nucleus in space, Ω a unit vector defining the direction of the wave vector $\mathbf{k}'$ of the scattered neutron, and θ is the scattering angle. The wave functions that describe the rotational state of the nucleus is of the form

$$\Psi^I_{MK}(\omega) = \sqrt{(2I+1)/8\pi^2}\, D^I_{MK}(\omega), \quad (8)$$

where $D^I_{MK}(\omega)$ is an element of the $(2I+1)$-dimensional irreducible representation of the rotation group. The expression for the scattering amplitude, assuming a black nucleus, can be obtained from Eq. (4) of reference 7, and is of the form

$$f(\Omega,\ \omega) = ikb^2 \zeta(\vartheta)\, J_1(\varkappa)/\varkappa, \quad \zeta(\vartheta) = \sqrt{1 + \varepsilon \sin^2\vartheta},$$

$$\varepsilon = \frac{a^2}{b^2} - 1, \quad \varkappa = 2kb \sin\frac{\theta}{2}\sqrt{1 + \varepsilon\cos^2\gamma}, \qquad (9)$$

where k is the wave vector of the incident neutron, and γ is the angle between the vector $\mathbf{k}' - \mathbf{k}$ and the symmetry axis of the nucleus.

Putting the scattering angle θ equal to zero and using an expansion in Legendre polynomials

$$\sqrt{1 + \varepsilon\sin^2\vartheta} = \sum_l (2l + 1) A_l P_l(\cos\vartheta), \qquad (10)$$

where the expansion coefficients are determined by the expression

$$A_l = \frac{1}{2}\int_0^\pi \sqrt{1 + \varepsilon\sin^2\vartheta}\, P_l(\cos\vartheta)\sin\vartheta\, d\vartheta, \qquad (11)$$

we get

$$\sigma^I_{tMK} = \frac{2I+1}{4\pi}\frac{R_0^2}{(1+\varepsilon)^{1/3}}$$

$$\times \sum_l \int (2l + 1) A_l P_l(\cos\vartheta)\, |D^I_{MK}(\omega)|^2\, d\omega, \qquad (12)$$

where R_0 is the radius of a spherical nucleus of equal volume. Using the properties of the functions D^I_{MK}:

$$|D^I_{MK}|^2 = \sum_{L=0}^{2I} (-1)^{M-K}(IIM - M\,|\,L0)$$

$$\times (IIK - K\,|\,L0)\, D^L_{00}, \qquad (13)$$

$$D^L_{00}(\omega) = P_L(\omega), \qquad (13a)$$

we obtain after simple transformations

$$\sigma^I_{tMK} = 2\pi(2I + 1)\frac{R_0^2}{(1+\varepsilon)^{1/3}}(-1)^{M-K}$$

$$\times \sum_{L=0}^{2I} (IIM - M\,|\,L0)(IIK - K\,|\,L0)\, P_L(\cos\psi)\, A_L, \qquad (14)$$

where ψ is the angle between the vector $\mathbf{k}$ and the direction of the orienting field $\mathbf{H}$.

Correspondingly, the quantities γ^I_{MK} can be represented as follows:

$$\gamma^I_{MK} = (-1)^{M-M_0}$$

$$\times \frac{\sum_L (IIM - M\,|\,L0)(IIK - K\,|\,L0) P_L(\cos\psi)\, A_L}{\sum_L (IIM_0 - M_0\,|\,L0)(IIK - K\,|\,L0)\, P_L(\cos\psi) A_L} - 1. \qquad (15)$$

As already mentioned in the introduction, the total neutron cross section for an unoriented nucleus depends very little on the deformation. Furthermore, it is entirely independent of the nuclear spin[8] if the nucleus is not oriented. It is therefore convenient to study the behavior of the quantity

$$\Lambda_t = \bar{\sigma}_t(f_k)/\bar{\sigma}_t(0),$$

i.e., of the ratio of the total neutron cross section on an oriented nucleus to the total neutron cross section on an unoriented one.

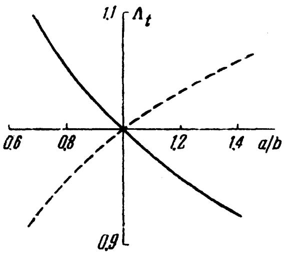

FIG. 1

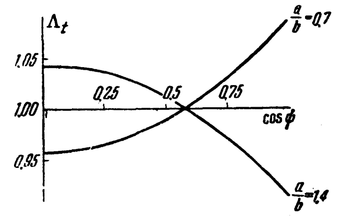

FIG. 2

After calculating the quantity γ^I_{MK} and using the expressions in the table, we obtain the numerical values of the ratio Λ_t of interest to us. The calculation of the coefficients A_l can be either by direct computation of the integrals (11) or by expanding the expression $\sqrt{1 + \epsilon \sin^2\vartheta}$ in powers of ϵ.

An estimate shows that the contribution due to the term containing f_4 is only several percent of the main contribution of the term containing f_2. The term containing f_6, as already mentioned, yields a negligible contribution. It is also clear that the effect is nonlinearly related to the value of f_2.

The dependence of Λ_t on the ratio of semi-axis a/b is illustrated in Fig. 1. The solid curve corresponds to the case $I = K = 5/2$, $\psi = 0$, and $f_2 = +1$. We see that the ratio of the cross section diminishes monotonically with increasing a/b; when $a/b < 1$, we get $\Lambda_t > 1$, and vice versa, in full agreement with what was said in the introduction. The dotted curve corresponds to the same parameters, but for $K = 1/2$ rather than $5/2$ as in the first case. Such a case, as is known, occurs when the sequence of the levels in the rotational band is reversed. The resultant curve can be obtained from the curve for $K = 5/2$ by reversing the sign of the deformation. In other words, in this case a prolate nucleus behaves like an oblate one, and vice versa. This result is connected with the fact that when $K = 1/2$ and $I > 1/2$ the spin is perpendicular to the symmetry axis of the nucleus.

The characteristic variation of Λ_t with ψ, the angle between the orienting field and the incident beam, is shown in Fig. 2 for the case $I = K = 5/2$, $f_2 = +1$ and for two values of the semi-axis ratio a/b, namely 1.4 and 0.7 (i.e., $b/a = 1.4$). In both cases the effect reaches a maximum value when the orienting field is parallel to $\mathbf{K}$ (i.e., $\psi = 0$). Obviously the effect manifests itself most fully if the measurements are made at $\psi = 0$ and $\psi = \pi/2$. In accordance with the remarks made in the introduction, the curves corresponding to the different signs of deformation differ in the sign of the effect.

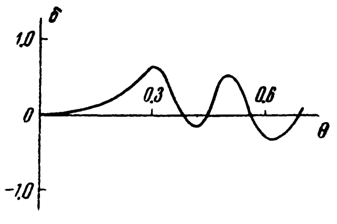

FIG. 3

We give the values of Λ_t for $a/b = 0.7$, $\psi = 0$, and $f_2 = +1$ and for various nuclear spins:

$I = 1$	$^3/_2$	2	$^5/_2$	$^7/_2$
$\Lambda_t = 1.024$	1.049	1.070	1.087	1.113

As expected, the effect increases monotonically with increasing spin.

In the foregoing examples a typical value of the effect was $\sim 10\%$. However, the attainment of 100% orientation of nuclei in the experiment is of little likelihood, i.e., we always have $f_2 < 1$, and the observed effects are correspondingly decreased.

4. ANGULAR DISTRIBUTION OF SCATTERED NEUTRONS

Let us consider the differential cross section of scattering of neutrons in a direction defined by a unit vector $\mathbf{\Omega}$, by a nucleus characterized by quantum numbers I, M, and K. This cross section is given by

$$\sigma^I_{MK}(\Omega) = \int |f(\Omega, \omega)|^2 \, |\Psi^I_{MK}(\omega)|^2 \, d\omega. \tag{16}$$

The scattering amplitude f and the wave function of the rotational state of the nucleus Ψ^I_{MK} are determined as before by expressions (9) and (8). The calculations yield the following expression for the sought cross section

$$\sigma^I_{MK}(\Omega) = k^2 R_0^4 (1+\varepsilon^2)^{1/2} \sum_{L=0}^{2I} (-1)^{M-K}$$

$$\times (IIM - M \mid IIL0)(IIK - K \mid IIL0)$$

$$\times \Big\{ \Big(1 - \frac{1}{3}\frac{\varepsilon}{1+\varepsilon}\Big) P_L(\cos\beta) B_L - \frac{2}{3}\frac{\varepsilon}{1+\varepsilon} \Big[P_L(\cos\beta) P_2(\cos\eta)$$

$$\times \sum_{l=|L-2|}^{L+2} (L200|L2l0)^2 B_l + 2 \sum_{\mu=1}^{2} (-1)^\mu$$

$$\times \sqrt{\frac{(L-\mu)!\,(2-\mu)!}{(L+\mu)!\,(2+\mu)!}} P_L^\mu(\cos\beta)$$

$$\times P_2^\mu(\cos\eta) \cos\mu$$

$$\times \sum_{l=|L-2|}^{L+2} (L200 \mid L2l0)(L2\mu - \mu \mid L2l0) B_l \Big] \Big\}. \tag{17}$$

Here

$$B_\lambda = \int Y_{\lambda 0}(\omega) \frac{J_1^2 (kb\theta \sqrt{1+\varepsilon\cos^2\gamma})}{(kb\theta\sqrt{1+\varepsilon\cos^2\gamma})^2} \, d\omega, \tag{18}$$

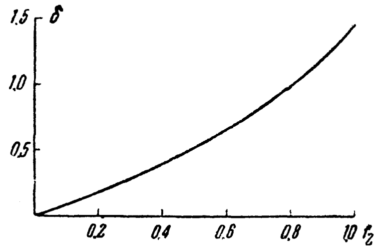

FIG. 4

β is the angle between $\mathbf{k}' - \mathbf{k}$ and $\mathbf{H}$, η is the angle between $\mathbf{k}' - \mathbf{k}$ and $\mathbf{k}$. The angle ξ is determined by the relation

$$\cos\xi = -\cot\beta \cot\eta. \tag{19}$$

The angles β and γ can be expressed in terms of the scattering angle θ, the azimuth angle φ, and the angle ψ

$$\cos\eta = -\sin(\theta/2),$$

$$\cos\beta = -\cos\psi\sin(\theta/2) + \sin\psi\cos(\theta/2)\cos\varphi. \tag{20}$$

If the orienting field is perpendicular to the direction of the incident neutron beam, i.e., if $\psi = \pi/2$, we get

$$\cos\eta = -\sin(\theta/2),$$

$$\cos\beta = \cos(\theta/2)\cos\varphi, \quad \tan\xi = \tan\varphi/\sin(\theta/2). \tag{20a}$$

The expressions (16) and (20) yield the dependence of $\sigma^I_{MK}(\Omega)$ on the azimuth angle φ. In case of unoriented nuclei this dependence vanishes in the averaging over M, but in the case of oriented nuclei an azimuthal asymmetry occurs in the angular distribution of the scattered neutrons. If we denote by $\bar\sigma(\theta, \varphi)$ the angular distribution of the neutrons scattered in a direction defined by the angles θ and φ, averaged over M, then the azimuthal asymmetry can be characterized by the following quantity

$$\delta = (\bar\sigma(\theta, \pi/2) - \bar\sigma(\theta, 0)) / \bar\sigma(\theta, 0). \tag{21}$$

After calculating the differential cross section with equation (17) (in which the coefficients B_λ are calculated numerically) and averaging it over M, for which we must use expression (15) for γ'_{MK} and the expressions for $\bar\sigma(f_k)/\bar\sigma(0)$, listed in the table, we obtain the value of the azimuthal asymmetry from (21). Figure 3 shows the dependence of the azimuthal asymmetry on the scattering angle θ for the case $I = K = 5/2$, $a/b = 1.3$, $f_2 = +0.5$, and $kR = 12$. The azimuthal asymmetry reaches a considerable value at $\theta \approx 0.3$, i.e., near the first diffraction minimum.

The dependence of the azimuthal asymmetry on the degree of alignment of the nuclei f_2 is shown in figure 4 for $\theta = 0.5$, $I = K = 5/2$, $a/b = 1.3$, and $kR = 12$. It is seen there that the azimuthal asym-

I	$\bar{\sigma}(f_k)/\bar{\sigma}(0)$
1	$1+f_2\,\frac{\gamma_1}{3+2\gamma_1}$
$^3/_2$	$1+f_2\,\frac{\gamma_{3/2}}{2+\gamma_{3/2}}$
2	$1+\frac{10}{7}f_2\,\frac{2\gamma_2-\gamma_1}{5+2\gamma_1+2\gamma_2}+\frac{1}{7}f_4\,\frac{\gamma_2-4\gamma_1}{5+2\gamma_1+2\gamma_2}$
$^5/_2$	$1+\frac{5}{14}f_2\,\frac{5\gamma_{5/2}-\gamma_{3/2}}{3+\gamma_{3/2}+\gamma_{5/2}}+\frac{3}{14}f_4\,\frac{\gamma_{5/2}-3\gamma_{3/2}}{3+\gamma_{3/2}+\gamma_{5/2}}$
$^7/_2$	$1+\frac{1}{3}f_2\,\frac{7\gamma_{7/2}+\gamma_{5/2}-3\gamma_{3/2}}{4+\gamma_{3/2}+\gamma_{5/2}+\gamma_{7/2}}+\frac{1}{11}f_4\,\frac{7\gamma_{7/2}-13\gamma_{5/2}-3\gamma_{3/2}}{4+\gamma_{3/2}+\gamma_{5/2}+\gamma_{7/2}}+\frac{1}{33}c_6f_6$

metry increases monotonically with increasing f_2, and at $f_2 \lesssim 0.4$ this dependence is close to linear.

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