RADIATION FROM A CURRENT-CARRYING RING MOVING UNIFORMLY IN A PLASMA SITUATED IN A MAGNETIC FIELD

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Energy losses due to Vavilov-Cerenkov radiation are computed for a current-carrying ring moving uniformly in a plasma perpendicular to its plane and parallel to the external magnetic field.

IT is known that the Vavilov-Cerenkov effect is possible in a plasma placed in a magnetic field. The problem of the radiation from a charge moving uniformly in a magnetized plasma has been discussed in detail by Kolomenskii.[1,2] In connection with the coherent method of acceleration of charged particles proposed by V. I. Veksler[3] it is of interest to investigate the radiation from a current filament in an active an isotropic medium. Morozov[4] has obtained the radiation from an infinite current in a plasma in a magnetic field. In this paper we investigate the radiation from a ring current.

We consider a perfectly conducting ring of radius a and carrying a current I_0 moving in a plasma placed in a magnetic field. The velocity of the motion of the ring is perpendicular to the plane of the ring and is parallel to the externally applied magnetic field (which we later choose for the direction of the z axis).

If the external magnetic field is considerably greater than the field produced by the ring itself, then the electron plasma behaves as a gyrotropic crystal whose properties are described by the dielectric constant tensor

$$\tilde{\varepsilon} = \begin{pmatrix} \varepsilon & -i\gamma & 0 \\ i\gamma & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix},$$

$$\varepsilon = (\omega^2 - \omega_0^2 - \omega_H^2)/(\omega^2 - \omega_H^2);$$

$$\varepsilon_z = 1 - (\omega_0/\omega)^2; \quad \gamma = \omega_0^2\omega_H / \omega(\omega^2 - \omega_H^2);$$

$$\omega_0^2 = 4\pi e^2 N/m, \quad \omega_H = eH/mc. \tag{1}$$

To obtain the field of the moving ring it is necessary to solve the following system of equations for the potentials

$$\Delta \mathbf{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\tilde{\varepsilon} - \mathbf{A}) - \operatorname{grad}\operatorname{div}\mathbf{A} = -\frac{4\pi}{c}\mathbf{j} + \frac{\tilde{\varepsilon}}{c}\operatorname{grad}\frac{\partial\varphi}{\partial t},$$

$$(\nabla\tilde{\varepsilon}\nabla)\varphi = -4\pi\rho$$

subject to the subsidiary condition

$$\operatorname{div}(\tilde{\varepsilon}\mathbf{A}) = 0.$$

Since the current-carrying ring is moving perpendicularly to its own plane, then $\rho = 0$ and we may set $\varphi = 0$. Consequently it is necessary to find the solution of the equation

$$\Delta \mathbf{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\tilde{\varepsilon}\mathbf{A}) - \operatorname{grad}\operatorname{div}\mathbf{A} = -\frac{4\pi}{c}\mathbf{j}.$$

The solution of this equation in general form may be written as follows:[5]

$$\mathbf{A} = -\frac{4\pi}{c}\int \Lambda^{-1}\left(\mathbf{j}_{\mathbf{k}} - \mathbf{k}\frac{(\mathbf{k}\tilde{\varepsilon}\Lambda^{-1}\mathbf{j}_{\mathbf{k}})}{(\mathbf{k}\tilde{\varepsilon}\Lambda^{-1}\mathbf{k})}\right)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{v}t)}\,d\mathbf{k}, \tag{2}$$

where

$$\mathbf{j} = \int \mathbf{j}_{\mathbf{k}} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{v}t)}\,d\mathbf{k},$$

while the matrices Λ^{-1} and $\tilde{\varepsilon}\Lambda^{-1}$ are of the form:

$$\Lambda^{-1} = \begin{pmatrix} \lambda & -i\lambda_1 & 0 \\ i\lambda_1 & \lambda & 0 \\ 0 & 0 & \lambda_2 \end{pmatrix}, \qquad \tilde{\varepsilon}\Lambda^{-1} = \begin{pmatrix} p & -ip_1 & 0 \\ ip_1 & p & 0 \\ 0 & 0 & p_2 \end{pmatrix},$$

$$\lambda = (k^2 - \varkappa^2\varepsilon)/L_1; \quad \lambda_1 = \varkappa^2\gamma/L_1;$$

$$\lambda_2 = 1/L_2, \quad p = [\varkappa^2(\gamma^2 - \varepsilon^2) + \varepsilon k^2]/L_2$$

$$p_1 = k^2\gamma/L_1, \quad p_2 = \varepsilon_z/L_2, \quad L_1 = \gamma^2\varkappa^4 - (\varkappa^2\varepsilon - k^2)^2;$$

$$L_2 = \varkappa^2\varepsilon_z - k^2; \qquad \varkappa = \mathbf{k}\cdot\mathbf{v}/c.$$

We shall solve the problem in the cylindrical system of coordinates with the z axis directed along the direction of motion of the ring. Therefore it is convenient to write $\mathbf{j}$ in the form

$$\mathbf{j} = I_0\delta(z - vt)\operatorname{curl}\mathbf{B}_z, \quad \mathbf{B}_z = \mathbf{z}\begin{cases} 1 & r < a, \\ 0 & r > a. \end{cases}$$

Then

$$\mathbf{j} = \frac{iI_0 a}{(2\pi)^2}\int \frac{[\mathbf{k}\times\mathbf{z}]}{k_r} J_1(k_r a)\, e^{i\mathbf{k}\,(\mathbf{x}-\mathbf{v}t)} d\mathbf{k}, \quad (3)$$

$$\mathbf{k} = \mathbf{k}_r + \mathbf{z}(\mathbf{k}\cdot\mathbf{v})/v.$$

Vavilov-Cerenkov waves will be radiated when the following condition is fulfilled

$$\mathbf{k}\tilde{\varepsilon}\Lambda^{-1}\mathbf{k} = 0.$$

This condition leads to two values n_1 and n_2 for the indices of refraction in the directions of maximum radiation determined by the condition for the Cerenkov radiation:

$$\cos^2\vartheta_{1,2} = 1/\beta^2 n_{1,2}^2;$$

$$n_{1,2}^2 = \frac{1}{2\varepsilon}\Big\{\varepsilon\varepsilon_z + \frac{\varepsilon - \varepsilon_z}{\beta^2} + \varepsilon^2 - \eta^2$$
$$\pm \Big[\Big(\varepsilon\varepsilon_z + \frac{\varepsilon-\varepsilon_z}{\beta^2} + \varepsilon^2 - \eta^2\Big)^2$$
$$+ 4\varepsilon\Big[\frac{\varepsilon\varepsilon_z}{\beta^2} + (\eta^2 - \varepsilon^2)\Big(\varepsilon_z + \frac{1}{\beta^2}\Big]\Big]^{1/2}\Big]\Big\}. \quad (4)$$

On substituting (3) into (2) and on integrating over φ and k_r, we can obtain expressions for the components of the vector potential $\mathbf{A}$ which we do not give here. We simply note that the optical activity of the medium leads to the appearance of the components A_r and A_z, which are absent when the current-carrying ring is moving in an inactive medium.[4]

The power losses in the form of Vavilov-Cerenkov radiation are given by

$$P = 2\pi a I_0 E_\varphi|_{r=a,\, z=vt}.$$

As a result of some calculation we obtain

$$P = \frac{4\pi^2 a^2 I_0^2}{c^2}\frac{1}{v}\,\mathrm{Re}$$
$$\times\Big\{\int\limits_{n_1^2\beta^2>1} \frac{n_1^2 - n_0^2}{n_1^2 - n_2^2} H_1^{(\varepsilon)}\Big(\frac{|\omega|a}{v}\sqrt{n_1^2\beta^2 - 1}\Big) J_1\Big(\frac{|\omega|a}{v}\sqrt{n_1^2\beta^2-1}\Big)\omega d\omega$$
$$- \int\limits_{n_2^2\beta^2>1} \frac{n_2^2 - n_0^2}{n_1^2 - n_2^2} H_1^{(\varepsilon)}\Big(\frac{|\omega|}{v} a\sqrt{n_2^2\beta^2 - 1}\Big)$$
$$\times J_1\Big(\frac{|\omega|}{v} a\sqrt{n_2^2\beta^2 - 1}\Big)\omega d\omega\Big\},$$
$$n_0^2 = \varepsilon_z + \beta^{-2} - \varepsilon_z/\varepsilon\beta^2. \quad (5)$$

The integration extends over the range of frequencies for which $n_{1,2}^2(\omega)\beta^2 \gg 1$.

From (5) we can obtain the energy losses due to radiation from a magnetic dipole with a moment directed parallel to the direction of motion. To do this we set $a \to 0$ and by considering that the magnetic moment is given by $M = \pi a^2 I_0/c$ we obtain

$$P = \frac{M^2}{v^3}\Big[\int\limits_{n_1^2\beta^2>1} \frac{n_1^2 - n_0^2}{n_1^2 - n_2^2}(n_1^2\beta^2 - 1)\,\omega^3 d\omega$$
$$- \int\limits_{n_2^2\beta^2>1} \frac{n_2^2 - n_0^2}{n_1^2 - n_2^2}(n_2^2\beta^2 - 1)\,\omega^3 d\omega\Big]. \quad (6)$$

The magnitude of the indices of refraction $n_{1,2}$ in a plasma placed in a magnetic field is a complicated function of ω, ω_0, ω_H, β. $n_{1,2}$ may be determined by substituting into (4) the specific dependence on ω of $\epsilon(\omega)$, $\epsilon_z(\omega)$, $\eta(\omega)$. It is of interest to note that when a magnetic field is applied to the plasma there exists no frequency for which all the components of the dielectric constant tensor would vanish simultaneously. Therefore in a plasma placed in a magnetic field polarization losses are not possible. Cerenkov losses occur in their place, so that the integral of the losses given by (5) determines the total losses of the ring current due to so-called distant collisions. The conditions $n_{1,2}^2(\omega)\beta^2 \gg 1$ define the range of frequencies radiated. When $\beta \to 1$ waves are radiated whose frequencies lie within the limits

$$\omega_0 < \omega < \sqrt{\omega_H^2 + \omega_0^2}. \quad (7)$$

When $\beta < 1$ waves of frequency $\omega < \omega_0$ can also be radiated.

Let us now consider the losses accompanying the relativistic motion of the ring. This case is of the greatest interest for the coherent method of accelerating the rings, since it corresponds to the initial instant of acceleration (the inverse of the Vavilov-Cerenkov effect). Taking into account the fact that as $\beta \to 1$ $n_1^2\beta^2 > 1$, $n_2^2\beta^2 < 1$ we are able to obtain in a general form the integral of the losses given by (5), viz.:

$$P = \frac{2\pi^2 a^2 I_0^2 \omega_H^2}{c^3}\Big\{\frac{\omega_0 a}{c}\Big[I_1'\Big(\frac{\omega_0 a}{c}\Big)K_1\Big(\frac{\omega_0 a}{c}\Big) + I_1\Big(\frac{\omega_0 a}{c}\Big)K_1'\Big(\frac{\omega_0 a}{c}\Big)\Big]$$
$$+ \Big(\frac{\omega_0 a}{c}\Big)^2\Big[I_1''\Big(\frac{\omega_0 a}{c}\Big)K_1\Big(\frac{\omega_0 a}{c}\Big)$$
$$+ 2I_1'\Big(\frac{\omega_0 a}{c}\Big)K_1'\Big(\frac{\omega_0 a}{c}\Big) + I_1\Big(\frac{\omega_0 a}{c}\Big)K_1''\Big(\frac{\omega_0 a}{c}\Big)\Big]\Big\}. \quad (8)$$

From (8) it is easy to obtain the losses for a rarefied plasma and for small rings by utilizing expansions of $I_1(\omega_0 a/c)$, $K_1(\omega_0 a/c)$ in the case $\omega_0 a/c \ll 1$

$$P = 2\pi^2 a^4 I_0^2 c^{-5}\omega_H^2\omega_0^2 \ln(2c/\omega_0 a). \quad (9)$$

In the case of a magnetic dipole this integral diverges logarithmically as $a \to 0$. This divergence is associated with the fact that the upper limiting frequency $\omega_{lim} = \sqrt{\omega_H^2 + \omega_0^2}$ for a point dipole corresponds to zero wave length. Therefore at the upper limit we cannot utilize the results of macroscopic electrodynamics. The integral (6) for a magnetic dipole should be cut off at a certain maximum frequency ω_m. If close collisions are taken into account the total losses will not depend on the indefinite choice of the cutoff parameter ω_m.

For a dense plasma and for rings of large radius, i.e., $\omega_0 a/c \gg 1$ the losses will be given by the following expression:

$$P = \pi^2 a^3 I_0^2 \omega_H^2 \omega_0 / c^4. \tag{10}$$

One of the characteristic features of the medium under investigation is the possibility of Cerenkov radiation for $\beta \ll 1$. It may be shown that losses due to Cerenkov radiation at nonrelativistic velocities are proportional to β^2

$$P = \frac{\pi^2 I_0^2}{c^2}\left(\frac{a\omega_0}{v}\right)^4 \beta^2 \frac{v\omega_0^2}{\omega_H^2},$$

with $\omega_0 \ll v/a$, $\omega_H > \omega_m > \omega_0$.

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