ON THE ROLE PLAYED BY EXCHANGE EFFECTS IN STRIPPING REACTIONS

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A general expression has been obtained for the cross-section for "heavy particle stripping" as one of the exchange effects in stripping theory. Results of numerical computations of exchange effects in stripping reactions are presented for some of the simplest cases.

TAKING exchange effects into account in stripping reactions[1-3] leads to the result that in addition to the "ordinary" mechanism of stripping it is also necessary to consider two other processes: (a) the ejection (we are considering the (p, d) reaction) by a proton of a deuteron from the nucleus accompanied by proton capture into a bound state ("knock-out") in a manner similar to what may occur in the (p, n) reaction; (b) "heavy particle stripping."[2-4] In reference 3 a general discussion is given of the problem of determining the cross section for the stripping reaction with an antisymmetric wave function for the system. In this paper we present and compare with experiment results of specific calculations of exchange effects in stripping reactions for the simplest cases.

We designate the amplitude for the "ordinary" stripping reaction by I_1, the amplitude for the process (a) by I_2, and for the process (b) by I_3'. Then the amplitude for the reaction is given by

$$I = I_1 + (n-1) I_2 + (n-1) I_3', \qquad (1)$$

where n is the number of nucleons outside a closed shell in the initial nucleus.

The expressions for I_1^2 and I_2^2 are given in reference 3. For the calculation of $I_3'^2$ we have used the method of calculation presented in reference 3 after replacing the interaction V_{pn} by the interaction $V_{p\xi}$, where ξ represents the nuclear "core." This enabled us to obtain a more complete expression for the amplitude of the process (b):

$$\begin{aligned} I_3'^2 = (4\pi)^2 & (T_1 M_{T_1} \tau m_\tau | T_2 M_{T_2})^2 \sum \langle j^n \alpha_1 | j^{n-2}\alpha_3;\ j^2\alpha_d\rangle \\ & \times \langle j^n\alpha_1 | j^{n-2}\alpha_3^*;\ j^2\alpha_d^*\rangle \langle j^{n-1}\alpha_2 | j^{n-2}\alpha_3\rangle \langle j^{n-1}\alpha_2 | j^{n-2}\alpha_3^*\rangle \\ & \times \langle lsj,\ lsj,\ J_d | llL_d,\ ssS_d,\ J_d\rangle \langle lsj,\ lsj,\ J_d^* | llL_d^*, ssS_d,\ J_d^*\rangle \\ & \times [J_1][J_2][j][l][L_d]^{1/2}[L_d^*]^{1/2}[J_d]^{1/2}[J_d^*]^{1/2}(-1)^R C_{000}^{ll\Lambda} C_{000}^{L_d L_d^* \Lambda} \\ & \times W(ljlj;\ s\Lambda) W(jJ_3 jJ_3^*;\ J_2\Lambda) W(J_3 J_d J_3^* J_d^*;\ J_1\Lambda) \\ & \times W(J_d L_d J_d^* L_d^*;\ S_d\Lambda) A_q(l^2,\ L_d) \\ & \times A_q^*(l^2,\ L_d^*) B_k(l) B_k^*(l) P_\Lambda(\cos\beta). \end{aligned} \qquad (2)$$

Here $j^n\alpha_1$ is the state of the initial nucleus [α_1 is the set of quantum numbers required to specify it uniquely, $\alpha_1 = (J_1, T_1, T_{1z}$, seniority etc.)]; $j^{n-1}\alpha_2$ is the state of the final nucleus, $\alpha_2 = (J_2, T_2, T_{2z}$ etc.); $j^{n-2}\alpha_3$ is the state of the nuclear "core," $\alpha_3 = (J_3, T_3, T_{3z}$ etc.); $j^2\alpha_d$ is the state of the deuteron, $\alpha_d = (L_d, S_d = 1; J_d; T_d = 0)$; s, l, j are the spin, orbital and total angular momentum of the nucleon in the nucleus; $\langle lsj,\ lsj | llL_d,\ ssS_d,\ J_d\rangle$ is the matrix element of the unitary transformation of the two-nucleon wavefunction from jj- to LS-coupling,[5] expressed by means of the 9j-symbol;[5,6] $\langle j^n\alpha_1 | j^{n-2}\alpha_3;\ j^2\alpha_d\rangle$ is the fractional parentage coefficient; $[x] \equiv 2x+1$; $R = s - J_1 + J_2 + 2J_3 + 2J_3^*$; C are the Clebsh-Gordan coefficients; W are the Racah coefficients;

$$4\pi Y_{L_d M_d}(q) A_q(l^2,\ L_d) = \langle \mathbf{k}_d(1,\ 2) | l^2 L_d M_d(1,\ 2)\rangle;$$

$$4\pi Y_{lm_l}^*(k) B_k(l) = \langle lm_l(0) | V_{0\xi} | \mathbf{k}_p(0)\rangle;$$

$$\mathbf{q} = \mathbf{k}_d + (M_d/M_f)\mathbf{k}_p; \quad \mathbf{k} = \mathbf{k}_p + (M_p/M_i)\mathbf{k}_d;$$

$\mathbf{k}_d$, $\mathbf{k}_p$ are the propagation vectors for the deuteron and for the proton in the center of mass system; M_d, M_p, M_i, M_f are the masses of the deuteron, the proton, the initial and the final nuclei (for an infinitely heavy nucleus $\mathbf{q} = \mathbf{k}_d$ and $\mathbf{k} = \mathbf{k}_p$); β is the angle between $\mathbf{k}$ and $\mathbf{q}$. The summation is carried out over α_3, α_3^*, α_d, α_d^*, Λ.

An explicit expression for $B_k(l)$ in terms of the spherical Bessel functions is obtained in a manner similar to the way this is done in the case of "ordinary" stripping. In the same way, by regarding the deuteron as a whole and by utilizing the deuteron reduced width, one may evaluate the integral $A_q(l^2, L_d)$. An expression for this integral may also be obtained by utilizing oscillator wave functions.[7] Both methods of calculation lead to closely similar results.

A numerical calculation was carried out of the differential cross-section for neutron production in the reaction (d, n). The expressions for I_1^2, I_2^2 and $I_3'^2$ obtained for the reaction (p, d) are also valid for the reaction (d, p) and, consequently, for the reaction (d, n), if the Coulomb interaction is not taken into account. For the calculation we have chosen the simplest case when $n = 2$, i.e., when in the target nucleus there is only one nucleon outside a closed shell. In this case the spin of the nuclear "core" (J_3) is equal to zero and the formulas are considerably simplified.

The following cases have been considered:

(a) $l = 1,\quad j = {}^1/_2,\quad J_1 = 1,\quad T_1 = 0$ (*jj*-coupling),

(b) $l = 1,\quad L_1 = 0,\quad S_1 = 1,\quad T_1 = 0$ (*LS*-coupling),

(c) $l = 0,\quad J_1 = 1,\quad T_1 = 0.$

Specific reaction parameters for cases a and b have been taken corresponding to the case $C^{13}(d, n)N^{14}$, in particular, $Q = 5.19$ Mev, $R = 4.5 \times 10^{-13}$ cm; for case c the reaction parameters were taken corresponding to $Si^{29}(d, n)P^{30}$ (ground state), i.e., $Q = 3.27$ Mev, $R = 5.5 \times 10^{-13}$ cm. The deuteron energy in all these cases was taken to be equal to 8 Mev. For both reactions the differential cross sections $\sigma_1(\vartheta)$ and $\sigma_2(\vartheta)$ were calculated corresponding to the amplitudes I_1 and I_2; in the case $C^{13}(d, n)N^{14}$ we have also calculated $\sigma_3(\vartheta)$ which corresponds to the amplitude I_3'.

The results of the calculations are shown in Figs. 1 and 2 from which it may be seen that the shapes of the angular distributions of σ_1 and σ_2 are similar to each other, particularly in the case of LS coupling. With reference to the angular distribution of σ_3 we see that it practically coincides with the angular distribution calculated on the assumption of LS-coupling by a simpler method which was used in the papers of Owen and Madansky.[2,4]

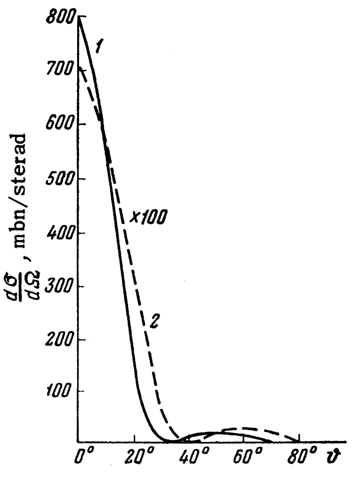

FIG. 1. Differential cross sections for the reaction Si^{29} (d, n) P^{30}: 1 – "ordinary" stripping and 2 – "knockout."

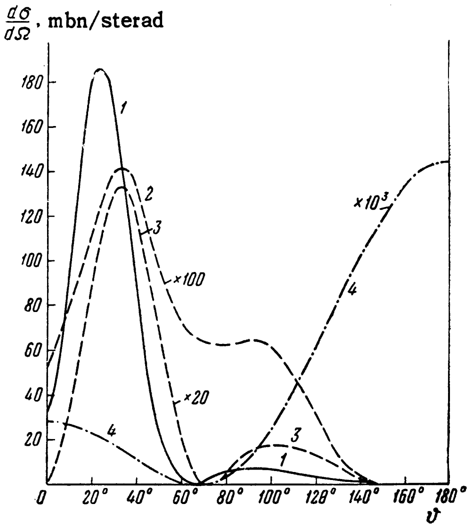

FIG. 2. Differential cross sections for the reaction C^{13} (d, n) N^{14}: 1 – "ordinary" stripping, 2 – "knockout" (jj-coupling), 3 – "knockout" (LS coupling), 4 – "heavy particle stripping."

The absolute cross section was calculated on the assumption that the reduced widths for a single nucleon are equal to the Wigner limit. The results have shown (Figs. 1 and 2) that the cross section σ_2 is lower by approximately one-two orders of magnitude compared to the cross section of "ordinary stripping. This is not accidental, since the relative smallness of σ_2 is explained by the fact that the integrals E_l, which appear in the expression for I_2^2 are practically always smaller than the integrals D_l, upon which I_1^2 depends.

Because of the difference in the way angular momenta combine the cross section σ_3 calculated according to formula (2) turns out to be smaller by a factor of two or three compared to the cross section calculated by a formula similar to (2) but derived on the assumption of LS coupling. In this last case the results are close to the results of calculation using formulas from the papers of Owen and Madansky if we assume that the reduced nucleon widths are equal to the Wigner limit. (We note that in references 2 and 4 there are certain inaccuracies in the formulas.) The reason for the smallness of the magnitude of σ_3 compared to σ_1 (the ratio of the cross sections at their maxima is approximately 10^{-3}) may be easily understood if we examine the formulas of Owen and Madansky. According to these formulas the cross section is approximately proportional to $j_{L_d}^2(qR) \times j_l^2(kR)$. Since in our case $M_d \ll M_f$, q varies slowly as the angle ϑ between $\mathbf{k}_d$ and $\mathbf{k}_p$ is varied, and is approximately equal to k_d. Therefore, for not too low energies of the deuteron the quantities qR are not small (for the reaction with C^{13} $qR \approx 3.5$) and, consequently, they correspond to comparatively small values of $j_{L_d}(qR)$.

At their maxima the cross sections σ_1 and σ_3 are comparable in magnitude if we assume that $L_d = 0$ and that the reaction parameters are such that at large angles qR is small while kR is near the value of the argument corresponding to the first maximum of the function j_l. In particular, this in many cases should occur at low deuteron energies, since as the energy is reduced $qR \rightarrow 0$, while $kR \rightarrow \sim 2$, if $Q \approx 5$ Mev and $R \approx 4 \times 10^{-13}$ cm. For some specific reactions the cross sections σ_1 and σ_3 are of the same order of magnitude at not very low energies; for example, in the case $B^{11}(d, n)C^{12}$ this occurs up to deuteron energy of 4 or 5 Mev. However, for deuteron energies exceeding 7 or 8 Mev the cross section σ_3 is considerably smaller than σ_1 for all the reactions. Consequently, at these energies the angular distribution of particles in stripping reactions should be very well described by the formulas of "ordinary" stripping.

The effect of the Coulomb interaction must be approximately the same for "ordinary" stripping and for "heavy particle stripping." Therefore, one can hope that if σ_1 and σ_3 are of the same order of magnitude, then by measuring the relative values of the maxima of the differential cross section at small and at large angles one could obtain the ratio of the deuteron and the proton (in the case of the (d, n) reaction) reduced widths. This is possible due to the fact that in formula (2) there is essentially contained the expression for the reduced deuteron widths in the shell model (jj coupling).

It must be noted that there is one circumstance which is difficult to understand on the basis of the concepts adopted by us above. In a number of cases "heavy particle stripping" practically does not show up even at low energies, for example, in the reactions (d, p) in Li^6, B^{10}, and N^{14}.[8-10] In specific reactions there may be reasons for this. For example, in those cases when the isotopic spin of the nuclear "core" is $T_3 = 0$, while the isotopic spin of the final nucleus is $T = 1$, "heavy particle stripping" is forbidden, since $T_\alpha = 0$ and, consequently, the equality $T_1 = T_3$ must hold. Example: the reaction $O^{17}(d, n)F^{18*}$, $T^* = 1$. However, for the reactions enumerated above in Li^6, B^{10}, and N^{14} it is difficult to formulate any reason for the reaction being forbidden. Perhaps in these cases the explanation lies in the smallness of the fractional parentage coefficients.

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