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### STATISTICAL ELECTRON CAPTURE MECHANISM IN BETATRONS

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RECENT papers have shown that the capture of injected electrons into orbits is due to the Coulomb interaction and that the capture of part of the electrons becomes possible as a result of the loss of the remaining ones. Kovrizhnykh and Lebedev succeeded by an ingenious formulation of the kinetic equations in obtaining some general result.<sup>1</sup> However, the mathematical difficulties of integrating the kinetic equations hide the physical picture of the process.

Physically, the considered capture mechanism, which was proposed by Matveev,<sup>2,3</sup> is of importance in the initial phases of the capture process, although it in itself cannot lead to capture. This is a direct consequence of Poincaré's theorem on conservative systems, if the following experimental data are considered.

1) The time dependence of the magnetic field does not have an important influence on the capture. A direct experimental proof is contained in our earlier work,<sup>4,5</sup> where all experiments on the capture were carried out in a dc magnetic field.

2) The capture takes place also on the flat portion of the injection pulse.<sup>6</sup> In contrast to Logunov et al., we find<sup>5</sup> that the pulse fronts do not seem to be important in the injection into a dc field.

3) The captured charge increases with increasing injection pulse length and reaches its maximum value at a pulse length corresponding to several tens of revolutions. The larger the injection current the sooner saturation sets in. However, if the injection time is of the order of one revolution the captured charge is very small even for very large injection currents.<sup>5</sup>

From this one obtains the following picture of

the process of injection into a dc magnetic field. A few revolutions after the begin of the injection there appears a stationary state in the doughnut — the number of the injected electrons almost equals the number of lost electrons. Let us take a look at the Hamiltonian for an arbitrary electron which moves in the field of all the other electrons. As long as we do not take into account the microstructure of the charge distribution, the Hamiltonian does not depend on the time. If the electron does not strike the walls it will hit the injector after a few revolutions.

The system is not conservative if one takes into account the statistical fluctuations of the charge density. As a result of the collective interaction of the electrons, their density distribution approaches statistical equilibrium.

The author<sup>7</sup> has investigated the equilibrium state of a toroidal electron beam employing several simplifying assumptions. The equilibrium state is determined by two parameters:

$$a = 2kT/E_0, \quad b = P/P_0, \quad (1)$$

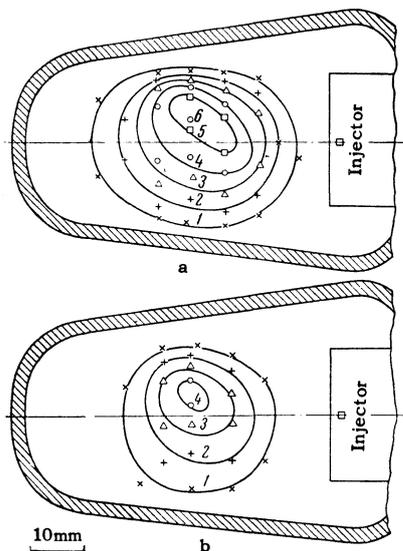
where  $k$  is Boltzmann's constant,  $kT$  is the mean kinetic energy of the transversal electron motion,  $E_0 = qV$  ( $q$  = electron charge,  $V$  = injection voltage) is the energy of the azimuthal motion, and  $P = I/V^{3/2}$ , where  $I$  = beam current;  $P_0 = 3.33 \times 10^{-5}$  amp/(volt)<sup>3/2</sup>.

In the case  $a \gg b$  the effective radius of the beam cross section is (in terms of the radius of the equilibrium orbit,  $r_0$ )  $\rho_0 = \sqrt{a}$  and the distribution of the charge density is given by

$$\sigma/\sigma_{\max} = (b/\rho_0^2) \exp\{-\rho/\rho_0\}, \quad (2)$$

where  $\rho$  is the relative distance from the beam center and  $\sigma_{\max} = 2\epsilon_0 V/r_0^2$  is the maximum possible charge density (in MKS units). Experiments show that the distribution of the charge density agrees well with Eq. (2) and becomes established a short time after the injection. The figure shows the experimentally-observed equal-density curves in percent of  $\sigma_{\max}$ ; the values shown in  $a$  and  $b$  were measured after 20 and 100 revolutions after the termination of the injection, respectively.

The statistical electron capture mechanism can be understood if one considers the cooling process of the beam. The loss of electrons to the walls and to the injector does not only decrease the number of the electrons in the beam but also decreases the mean transversal energy of the remaining electrons. As a result of this cooling the cross section of the beam decreases. The change of the number of electrons,  $\Delta N$ , and the temperature parameter,  $\Delta a$ , during the time



Curves of equal charge density. a—measured after 20 revolutions; b—measured after 100 revolutions after termination of injection. The numbers at the curves denote the charge density in percent of the maximum possible charge density.

interval  $\Delta t$  is approximately given by the expressions

$$\frac{\Delta N}{N} = -e^{-u} \frac{\Delta t}{\tau_p}, \quad \frac{\Delta a}{a} = -\frac{ue^{-u}}{1-e^{-u}} \frac{\Delta t}{\tau_p}, \quad (3)$$

where  $u = (\rho_m/\rho_0)^2$ ,  $\rho_m$  is the smallest relative distance of the doughnut walls from the beam center, and  $\tau_p$  is the relaxation time. For the conditions of our experiment  $\tau_p \sim 1 \mu\text{sec}$ .

The following capture picture emerges from the foregoing. The injected electrons, which start out in a nonequilibrium distribution, try to approach the equilibrium distribution. As a result of the described cooling mechanism, the beam cross section decreases. Since the relaxation time is much larger than the time of one revolution, the number of the captured electrons depends on the mean life of the injected electrons. Matveev's mechanism increases this lifetime. For small values of the injected current this time increases, owing to a suitable ratio of the frequency of the betatron oscillations<sup>5</sup> and the orbital frequency.

<sup>5</sup> M. Seidl, Inst. for Vacuum Electronics, Prague, Report 27.32.58.

<sup>6</sup> Logunov, Ovchinnikov, and Rusanov, J. Tech. Phys. (U.S.S.R.) **27**, 1135 (1957), Soviet Phys. JTP **2**, 1023 (1957).

<sup>7</sup> M. Seidl, Czechosl. J. Phys. (in press).

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### ANISOTROPY OF THE ABSORPTION OF ULTRASOUND IN METALS IN A MAGNETIC FIELD

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THE absorption of ultrasound in metals at low temperatures has been the subject of relatively few investigations, and principal attention has been paid to an investigation of the difference in absorption coefficients in the superconducting and normal states.<sup>1,2</sup> It has been noted that for longitudinal sound, the coefficient of absorption in tin depends on the magnetic field, but if the field is perpendicular to the sound wave vector  $\mathbf{k}$ , a weakly pronounced maximum of absorption is observed in certain fields. A similar phenomenon was observed in polycrystalline copper<sup>3</sup> and in indium<sup>4</sup> and was not observed in zinc.

We report in this communication experiments set up to study the influence of the magnetic field on the absorption of ultrasound in single-crystal specimens of very pure metals. We prepared tin specimens with a residual resistivity less than  $1.6 \times 10^{-5}$  and of zinc with  $R_{4.2}/R_{300} = 2 \times 10^{-4}$ . The specimens were 12 mm in diameter and 12 and 15 mm long, respectively. The absorption coefficients were measured by known<sup>5</sup> pulse techniques at 17.3, 23.3, 51, and 70 Mcs.

Figure 1 shows the measured values of the coefficient of absorption of longitudinal sound in tin and zinc as a function of the magnetic field intensity at 4.2° K. The magnetic field is perpendicular to the wave vector. The ordinates represent the difference in the absorption coefficients with

<sup>1</sup> L. M. Kovrkzhnykh and A. N. Lebedev, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 984 (1958), Soviet Phys. JETP **7**, 679 (1958).

<sup>2</sup> A. N. Matveev, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1331 (1958), Soviet Phys. JETP **7**, 918 (1958).

<sup>3</sup> A. N. Matveev, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 372 (1958), Soviet Phys. JETP **8**, 259 (1959).

<sup>4</sup> M. Seidl, Czechosl. J. Phys. (in press).