THE EXCITATION OF COLLECTIVE STATES IN NUCLEI BY SCATTERING OF CHARGED PARTICLES

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Elastic and inelastic scattering of fast charged particles on black nonspherical nuclei is investigated in the diffraction approximation. The radius of the nucleus and its nonsphericity parameter can be determined by comparing the calculations with experimental data.

1. INTRODUCTION

EXCITATION of collective states in the nucleus takes place during the scattering of nucleons on nonspherical nuclei by the process of direct interaction of the incident particle with the nuclear surface. In this connection we will investigate the scattering of charged particles (protons, alpha particles) with an energy E which considerably exceeds the coulomb barrier $ZZ'e^2/R$ when condition (1) is fulfilled

$$kR \gg \eta, \tag{1}$$

where $\eta = ZZ'e^2/\hbar v$ is the Coulomb parameter, k is the wave number of the incident particle, and R is the radius of the nucleus. Assuming that the nucleus is black, we find the differential cross section for elastic and inelastic scattering with excitation of the first rotational level of an even-odd nucleus. If, in addition to (1), condition (2) is fulfilled

$$kR\Delta E/E \ll 1, \tag{2}$$

where ΔE is the energy of the excited level, then in the process of scattering the nucleus can be considered immovable and the change in the energy of the particles in inelastic scattering can be ignored. In this case the adiabatic approximation is applicable, according to which the solution of the scattering problem reduces to finding the scattering amplitudes $f(\Omega, \omega)$ of particles from a fixed nucleus, where the angles $\Omega = (\theta, \phi)$ determine the direction of scattering and the angles $\omega = (\vartheta, \varphi)$ show the orientation of the axis of symmetry of the nucleus. Formula (3) gives the differential scattering cross section with excitation of the rotational level of the even-odd nucleus with momentum λ.

$$\sigma_\lambda(\theta) = \sum_\mu |\langle Y^*_{\lambda\mu}(\omega) f(\Omega, \omega) Y_{00}\rangle|^2. \tag{3}$$

If condition (1) is fulfilled, it is possible to calculate the scattering amplitude $f(\Omega,\omega)$ from the fixed black nucleus using the diffraction theory method[1-4] in which the energy of interaction is considered as excitation.

2. SCATTERING AMPLITUDE

If the energy of the particles considerably exceeds the coulomb barrier (1) then the wave function describing the scattering of charged particles from a black nucleus, in cylindrical coordinates with the polar axis z along the wave vector k of the incident particles, has the form[1]

$$\Psi_k(\rho, z) = \Omega(\rho) \exp i \left[kz - \frac{1}{\hbar v} \int_{-z_0}^{z} U(\rho, z)\, dz \right]. \tag{4}$$

Here, $\Omega(\rho)$ is the function which accounts for the characteristics of a black nucleus: to the right of the nucleus $(z > 0)$, $\Omega(\rho) = 1$ on all planes $z = \text{const}$, aside from the shadow of the nucleus within whose limits $\Omega(\rho) = 0$. The function $U(\rho_2 z)$ is the energy of the electric interaction of the particle with the nucleus. If the equation of the nuclear surface, in the coordinate system connected with the axis of symmetry of the nucleus, can be written in the form $r(\mu) = R(1 + \alpha_\lambda P_\lambda(\mu))$, $\lambda = 2$, then for small nonsphericity parameters α_λ we have

$$U(\rho, z) = ZZ'e^2\left(\frac{1}{r} + \frac{3\alpha_\lambda}{2\lambda+1} r^{-\lambda-1} P_\lambda(\mu)\right). \tag{5}$$

in formula (4) z_0 signifies a rather large scattering to the left of the nucleus for which the wave function satisfies the condition

$$\Psi_k(\rho, -z_0) = e^{-ikz_0}. \tag{6}$$

According to Akhiezer and Sitenko[2] the scattering amplitude for particles from a fixed nucleus can be represented in the following form:

$$f(\Omega,\omega) = -\frac{ik}{2\pi}\int d\rho e^{-i\mathbf{k}'\rho}[\Psi_{\mathbf{k}}(\rho, z_0)e^{-ikz_0} - 1], \tag{7}$$

where $\mathbf{k}'$ is the wave vector of the scattered particle, and the integration is done over the entire plane. The square brackets contain the expression which represents the scattered wave.

For scattering through the angles $\theta \neq 0$, the last term in square brackets can be omitted. In addition, using (4), (5) with $z_0 \to \infty$ we have

$$e^{-ikz_0}\Psi_{\mathbf{k}}(\rho, z_0) = \Omega(\rho)\exp\left\{-i\eta\left[2\ln 2kz_0 - 2\ln k\rho + \sum_{\mu}\frac{12\pi\alpha_\lambda R^\lambda}{(2\lambda+1)^2}Y^*_{\lambda\mu}(\omega)\int_{-\infty}^{\infty} r^{-\lambda-1}Y_{\lambda\mu}\left(\frac{\mathbf{r}}{r}\right)dz\right]\right\}. \tag{8}$$

In discarding the inessential phase factor $\exp(-i2\eta\ln 2kz_0)$ and changing the variable of integration, we obtain with the help of (7) and (8) the following expression for the scattering amplitude:

$$f(\Omega,\omega) = -\frac{ik}{2\pi}\int_0^{2\pi} d\varphi' \int_{\rho(\varphi')}^{\infty} d\rho\, \rho\, (k\rho)^{2i\eta}$$
$$\times \exp\left\{-i\left[k\rho\theta\cos(\varphi' - \phi) + \eta\sum_{\mu}\frac{12\pi\alpha_\lambda R^\lambda\rho^{-\lambda}}{(2\lambda+1)^2}Y^*_{\lambda\mu}(\omega) \times \int_{-1}^{1}(1-\mu'^2)^{\lambda/2-1}Y_{\lambda\mu}(\mu',\varphi')d\mu'\right]\right\}. \tag{9}$$

In this formula the integration is done over the entire plane except for the nuclear shadow. It is not difficult to show that in the linear approximation for small non-sphericity parameters α_λ the nuclear shadow on the plane perpendicular to the vector $\mathbf{k}$ has the following form

$$\rho(\varphi') = R + R\alpha_\lambda\sum_{\mu}\frac{4\pi}{2\lambda+1}Y^*_{\lambda\mu}(\omega)Y_{\lambda\mu}\left(\frac{\pi}{2},\varphi'\right). \tag{10}$$

To find the cross section for elastic or inelastic scattering (3), we expand amplitude (9) into a series for small non-sphericity parameters α_λ, which are limited by the linear approximation. According to reference 5 this can be done if

$$\alpha_\lambda kR\theta \ll 1. \tag{11}$$

As a result of this expansion the integrals appear in the form

$$\frac{1}{2\pi}\int_0^{2\pi} d\varphi \exp i(\mu\varphi - z\cos\varphi) = i^{-\mu}J_\mu(z),$$

which reduce to Bessel functions. Thus we obtain the following expression for the scattering amplitude of charged particles from a fixed nucleus ($\lambda = 2$):

$$f(\Omega,\omega) = \frac{i}{k}(kR)^{2(1+i\eta)}\left\{-(kR\theta)^{-2(1+i\eta)}\int_{kR\theta}^{\infty} x^{1+2i\eta}J_0(x)\,dx\right\}$$
$$+\frac{i}{k}(kR)^{2(1+i\eta)}\sum_{\mu}\frac{4\pi\alpha_\lambda}{2\lambda+1}Y^*_{\lambda\mu}(\omega)e^{i\mu\phi}i^{-\mu}$$
$$\times\left\{Y_{\lambda\mu}\left(\frac{\pi}{2},0\right)J_\mu(kR\theta) + i\eta\frac{3}{2\lambda+1} \times \int_{-1}^{1}Y_{\lambda\mu}(\mu',0)\,d\mu'\,(kR\theta)^{-2i\eta}\int_{kR\theta}^{\infty}x^{-1+2i\eta}J_\mu(x)\,dx\right\}. \tag{12}$$

Using the formula in reference 6

$$\int_0^\infty x^{-2\nu+\mu-1}J_\mu(x)\,dx = 2^{-2\nu+\mu-1}\Gamma(\mu-\nu)/\Gamma(1+\nu), \tag{13}$$

we obtain from (12) the following expression for the elastic scattering amplitude

$$\langle Y_{00}f(\Omega,\omega)Y_{00}\rangle = f_{E0}(\theta) + \frac{i}{k}(kR)^{2(1+i\eta)}(kR\theta)^{-2(1+i\eta)}\int_0^{kR\theta}x^{1+2i\eta}J_0(x)\,dx, \tag{14}$$

where $f_{E0}(\theta)$ is the scattering amplitude in the field $ZZ'e^2/r$:

$$f_{E0}(\theta) = -\frac{2\eta}{k\theta^2}\exp\left(-2i\eta\ln\frac{\theta}{2}\right)\frac{\Gamma(1+i\eta)}{\Gamma(1-i\eta)}. \tag{15}$$

Thus the amplitude of elastic scattering consists of the amplitude of elastic scattering in the electric field of the nucleus and the nuclear part of the elastic scattering amplitude.

In the linear approximation for a small non-sphericity parameter α_λ the angular distributions for elastic scattering on spherical[4] and non-spherical nuclei are the same.

The amplitude of inelastic scattering, as a result of which the nucleus transfers from the ground state to the rotational state $Y_{\lambda\mu}(\omega)$ has the following matrix element:

$$\langle Y^*_{\lambda\mu}(\omega)f(\Omega,\omega)Y_{00}\rangle$$
$$= e^{-i\mu\phi}i^{\mu+1}\frac{(kR)^{2(1+i\eta)}}{k}\frac{(4\pi)^{1/2}\alpha_\lambda}{2\lambda+1}\left[Y_{\lambda\mu}\left(\frac{\pi}{2},0\right)J_\mu(kR\theta) + i\eta\frac{3}{2\lambda+1}\int_{-1}^{1}Y_{\lambda\mu}(\mu',0)\,d\mu'\,(kR\theta)^{-2i\eta}\int_{kR\theta}^{\infty}x^{-1+2i\eta}J_\mu(x)\,dx\right]. \tag{16}$$

The integral in this formula can be put in the form $\int_0^\infty f(x)\,dx - \int_0^\infty f(x)dx$ after which the first integral is done with the help of formula (13). We obtain

$$\langle Y^*_{\lambda\mu}f(\Omega,\omega)Y_{00}\rangle = \langle Y^*_{\lambda\mu}f_E(\Omega,\omega)Y_{00}\rangle + \langle Y^*_{\lambda\mu}f_n(\Omega,\omega)Y_{00}\rangle. \tag{17}$$

The first term in the right hand part of this formula as shown in the Appendix is the amplitude of electric quadrupole excitation. According to formulas (13) and (16) it looks like

$$\langle Y^*_{\lambda\mu}(\omega) f_E(\Omega, \omega) Y_{00}\rangle = \delta_{\mu\pm 2} e^{-i\mu\Phi}\left(\frac{3}{2}\right)^{1/2} \frac{\alpha_\lambda k R^2}{(2\lambda+1)^{3/2}} \frac{\eta(1+i\eta)}{1+\eta^2} \times \exp\left(-2i\eta \ln\frac{\theta}{2}\right)\frac{\Gamma(1+i\eta)}{\Gamma(1-i\eta)}. \quad (18)$$

The second term of formula (17), which can be called the nuclear part of the inelastic scattering amplitude, can be described by the expression

$$\langle Y^*_{\lambda\mu}(\omega) f_n(\Omega, \omega) Y_{00}\rangle = e^{-i\mu\Phi} i^{\mu+1}\frac{(kR)^{2(1+i\eta)}}{k}\frac{\alpha_\lambda}{(2\lambda+1)^{1/2}} \times \Big[\left(\frac{4\pi}{2\lambda+1}\right)^{1/2} Y_{\lambda\mu}\left(\frac{\pi}{2}, 0\right) J_\mu(kR\theta) - \delta_{\mu\pm 2}\frac{6^{1/2}}{2\lambda+1} i\eta (kR\theta)^{-2i\eta}\int_0^{kR\theta} x^{-1+2i\eta} J_2(x)\,dx\Big]. \quad (19)$$

With $\eta = 0$ formula (19) describes the inelastic scattering of neutrons.[7]

3. SCATTERING CROSS SECTION

The differential cross section for inelastic scattering of charged particles with excitation of the first rotational level of the even-odd nucleus is determined by the square of the absolute value of the amplitude (17):

$$\sigma_\lambda(\theta) = \sigma_{E\lambda}(\theta) + \sigma_{n\lambda}(\theta) + \sigma_{\mathrm{int}\,\lambda}(\theta); \qquad \lambda = 2;$$

$$\sigma_{E\lambda}(\theta) = \alpha_\lambda^2 \frac{(kR)^4}{k^2(2\lambda+1)^3}\frac{3\eta^2}{1+\eta^2};$$

$$\sigma_{n\lambda}(\theta) = \alpha_\lambda^2\frac{(kR)^4}{k^2(2\lambda+1)}\Big\{\frac{1}{4}[J_0^2(kR\theta) + 3J_2^2(kR\theta)] + \frac{6\eta}{2\lambda+1}\Big[\frac{2\eta}{2\lambda+1}|F(kR\theta)|^2 + J_2(kR\theta)\,\mathrm{Im}\,F(kR\theta)\Big]\Big\};$$

$$\sigma_{\mathrm{int}\,\lambda}(\theta) = -\alpha_\lambda^2\frac{(kR)^4}{k^2(2\lambda+1)^2}\frac{6\eta}{1+\eta^2}\Big\{(\eta\sin\chi + \cos\chi)\frac{2\eta}{2\lambda+1}\mathrm{Re}\,F(kR\theta) + (\eta\cos\chi - \sin\chi)\Big[\frac{2\eta}{2\lambda+1}\mathrm{Im}\,F(kR\theta) + \frac{1}{2}J_2(kR\theta)\Big]\Big\}. \quad (20)$$

Here

$$F(a) = a^{-2i\eta}\int_0^a x^{-1+2i\eta} J_2(x)\,dx;$$

$$\chi = 2\eta\ln\frac{kR\theta}{2} - 2\delta, \qquad \delta = \arg\Gamma(1+i\eta).$$

Thus, the cross section for inelastic scattering $\sigma_\lambda(\theta)$ is the sum of the cross section for coulomb excitation $\sigma_{E\lambda}(\theta)$, the nuclear part of the inelastic scattering cross section $\sigma_{n\lambda}(\theta)$ and the interference term $\sigma_{\mathrm{int}\lambda}(\theta)$. At sufficiently great magnitudes of ZZ' all these functions are of the first order of magnitude with the exception of the interference term which, as can be seen from formula (20), can be disregarded in the region of very small scattering angles $kR\theta \ll 1$. We should also mention that the form of the angular distribution of inelastically scattered particles (20), is not dependent on the nonsphericity parameter α_λ of the nucleus and is determined by the parameters kR and η. With $\eta = 0$ formula (2) describes the angular distribution of neutrons scattering inelastically with excitation of the first collective level of the even-odd nucleus:[7]

$$\sigma_{n\lambda}(\theta) = \alpha_\lambda^2\frac{(kR)^4}{4k^2(2\lambda+1)}[J_0^2(kR\theta) + 3J_2^2(kR\theta)], \quad (21)$$

assuming that $kR \gg 1$ and $\alpha_\lambda kR\theta \ll 1$.

The angular distribution of particles scattering inelastically with excitation of the first rotational level of the even-odd nucleus as shown in reference 5, under corresponding conditions, coincides with the angular distribution of particles scattering with excitation of the first vibrational level.

4. RESULTS OF CALCULATION AND COMPARISON WITH EXPERIMENTAL DATA

The complex function F(a) in formula (20) for angular distributions, on the basis of the known relationship

$$J_2(x) = \left(\frac{x}{a}\right)^2\sum_{m=0}^{\infty}\frac{a^m J_{2+m}(a)}{m!\,2^m}\left(1-\frac{x^2}{a^2}\right)^m$$

and the characteristics of Γ functions, can be represented in the form

$$F(a) = \sum_{m=0}^{\infty}\frac{a^m J_{2+m}(a)}{2^{m+1}}\prod_{p=0}^{m}\frac{p+1-i\eta}{(p+1)^2+\eta^2}. \quad (22)$$

For instance, with $a \leq 8$, for calculating the real and imaginary part of the function F(a) with good accuracy it is sufficient to use the first eight terms of sum (22).

The angular distributions of charged particles from inelastic scattering on non-spherical nuclei calculated according to formulas (20) and (22) are shown in Figs. 1—3. A comparison of theoretical and experimental[8,9] data for angular distributions of protons and alpha-particles inelastically scattered by the nucleus Mg^{24}_{12} (Figs. 1 and 2) makes it possible to evaluate the non-sphericity parameter of this nucleus: α_2 = 0.17 to 0.20.

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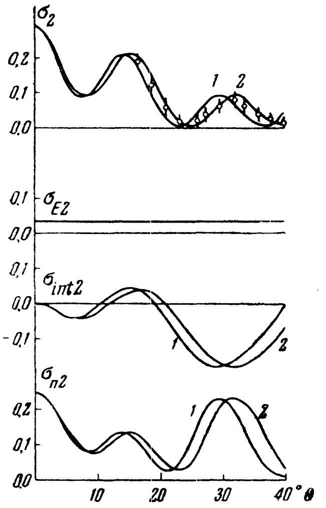

FIG. 1. Functions $\sigma_2(\theta)$, $\sigma_{E2}(\theta)$, $\sigma_{\text{int }2}(\theta)$, $\sigma_{n2}(\theta)$ in units $\alpha_2^2(kR)^4/5k^2$, describing the angular distribution of 31.5-Mev alpha particles inelastically scattered from M_{12}^{24} with excitation of its first collective level. The calculation was done with $R = 4.9 \times 10^{-13}$ cm (curve 2) and $R = 5.3 \times 10^{-13}$ cm (curve 1). Small circles indicate experimental data.[8] Comparison of theoretical and experimental data gives the value of the parameter of nonsphericity of the nucleus Mg_{12}^{24}: $\alpha_2 = 0.17$ to 0.20

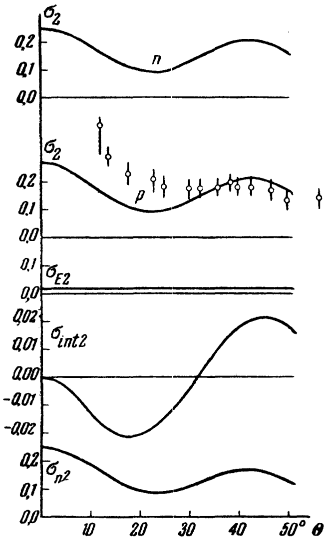

FIG. 2. Functions $\sigma_2(\theta)$, $\sigma_{E2}(\theta)$, $\sigma_{\text{int }2}(\theta)$, $\sigma_{n2}(\theta)$ in units $\alpha_2^2(kR)^4/5k^2$, describing the angular distribution of 18-Mev protons (p) inelastically scattered from Mg_{12}^{24} with excitation of its first collective level. For comparison the angular distribution of inelastically scattering neutrons $\sigma_2(\theta)$ are presented (n) in the same units. The calculation was done with $R = 4.7 \times 10^{-13}$ cm. The small circles show experimental data.[9] The discrepancy between theoretical and experimental data at $\theta < 20°$ can possibly be explained by the fact that we were unable to separate experimentally the group of elastically scattered protons from the inelastically scattered group. Therefore the comparison of experimental and theoretical data with $\theta = 30$ to $50°$ gives a value of the parameter of non-sphericity of the nucleus Mg_{12}^{24}, $\alpha_2 = 0.31$, which exceeds α_2 obtained from an analysis of experiments on the scattering of alpha particles (see Fig. 1).

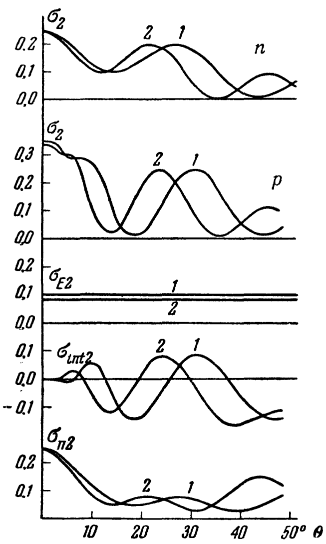

FIG. 3. Graphs of the same quantities as in Fig. 2, for the case of inelastic scattering of protons and neutrons, with energies of 20 Mev (1) and 30 Mev (2), from the nucleus Gd_{64}^{160}: $R = 1.3 \times 10^{-13} A^{1/3}$ cm.

APPENDIX

We shall show that formula (18) determines the amplitude of electric quadrupole excitation for small scattering angles $\theta \ll 1$ in the adiabatic approximation when the corresponding parameter[10] is $\xi = \eta\Delta E/2E \ll 1$. Actually, if the condition $\alpha_2\eta \ll 1$ is fulfilled[5] then the amplitude of scattering in the electrical field of the nucleus (5) in the first approximation of the theory of excitation can be presented in the form ($\lambda = 2$):

$$f_E(\Omega, \omega) = f_{E0}(\theta) - \eta k \sum_{\mu} \frac{6\alpha_\lambda R^\lambda}{(2\lambda+1)^2} Y_{\lambda\mu}^*(\omega) \left\langle \mathbf{k}_f \left| r^{-\lambda-1} Y_{\lambda\mu}\left(\frac{\mathbf{r}}{r}\right) \right| \mathbf{k}_i \right\rangle, \quad (1.1)$$

where k_i, k_f are the wave vectors of the incident and scattered particle; $|k\rangle$ is the Coulomb wave function. From (1.1) we obtain the following expression for the amplitude of inelastic scattering in the adiabatic approximation:

$$\langle Y_{\lambda\mu}^*(\omega) f_E(\Omega, \omega) Y_{00}\rangle = -e^{-2i\mu\phi} \frac{6\alpha_\lambda \eta k R^\lambda}{(2\lambda+1)^2 (4\pi)^{1/2}} \left\langle \mathbf{k}_f \left| r^{-\lambda-1} Y_{\lambda\mu}\left(\frac{\mathbf{r}}{r}\right) \right| \mathbf{k}_i \right\rangle. \quad (1.2)$$

Selecting k_i as a polar axis and using expansions of Coulomb functions $|k\rangle$ in spherical functions we have (see reference 10, page 449):

$$\left\langle \mathbf{k}_f \left| r^{-\lambda-1} Y_{\lambda\mu}\left(\frac{\mathbf{r}}{r}\right) \right| \mathbf{k}_i \right\rangle = \left(\frac{4\pi}{2\lambda+1}\right)^{1/2} \sum_{l_i l_f} i^{l_i - l_f} e^{i(\delta_i + \delta_f)} (2l_i+1)(2l_f+1) \times C_{l_i 0 l_f 0}^{\lambda 0} C_{l_i 0 l_f \mu}^{\lambda\mu} \left[\frac{(l_f-\mu)!}{(l_f+\mu)!}\right]^{1/2} P_{l_f}^{\mu}(\cos\theta) M_{l_i l_f}^{-\lambda-1}, \quad (1.3)$$

where $M_{l_i l_f}^{-\lambda-1}$ is the radial matrix element $C_{l_i m_i l_f m_f}^{\lambda\mu}$ are the Clebsh-Gordan coefficients,

$\delta_i = \arg\Gamma(1 + l_i + i\eta)$.

The large magnitudes of l_i, l_f play a basic role in sum (1.3) when we observe scattering through small angles. Therefore, independently of the magnitude of η (according to reference 10, page 456) the radial matrix elements are

$$M_{l_i l_f}^{-\lambda-1} = \frac{k^{\lambda-2}}{4\eta^\lambda} I_{\lambda m}(\vartheta, 0), \qquad \tan\frac{\vartheta}{2} = \frac{\eta}{l},$$

$$l = \frac{l_f + l_i}{2}, \ m = l_f - l_i, \qquad (1.4)$$

Where $I_{\lambda m}(\vartheta, \xi)$ are the orbital integrals of the theory of coulomb excitation (reference 10, page 482) where in this case the parameter of adiabaticity is $\xi = 0$. In addition we can use the following asymptotic characteristics of the Γ function, spherical functions and Clebsh-Gordan coefficients[10,11]:

$$e^{i(\delta_i+\delta_f)} = l^{2i\eta}; \qquad \left[\frac{(l_f - \mu)!}{(l_f + \mu)!}\right]^{1/2} P_{l_f}^\mu(\cos\theta) = J_\mu(l\theta),$$

$$C_{l_i 0 l_f 0}^{\lambda 0} C_{l_i 0 l_f \mu}^{\lambda\mu} = \frac{2\lambda+1}{2l+1} D_{0m}^\lambda\left(0, \frac{\pi}{2}, 0\right) D_{\mu m}^\lambda\left(0, \frac{\pi}{2}, 0\right), \qquad (1.5)$$

Where $D_{\mu m}^\lambda(\varphi_1, \vartheta_1, \varphi_2)$ are generalized spherical functions.[11] Substituting (1.4) and (1.5) into (1.3) and changing the double sum over l_i, l_f to the integral over $l = (l_f + l_i)/2$ and changing the sum over $m = l_f - l_i$ we obtain

$$\left\langle \mathbf{k}_f \left| r^{-\lambda-1} Y_{\lambda\mu}\left(\frac{\mathbf{r}}{r}\right) \right| \mathbf{k}_i \right\rangle = (-1)^\mu e^{i\mu\Phi} \frac{k^{\lambda-2}}{\eta^\lambda} [\pi(2\lambda+1)]^{1/2} \cdot$$

$$\times \sum_m i^{-m} D_{0m}^\lambda\left(0, \frac{\pi}{2}, 0\right) D_{\mu m}^\lambda\left(0, \frac{\pi}{2}, 0\right)$$

$$\times \int_0^\infty x^{1+2i\eta} J_\mu(x\theta) I_{\lambda m}(\vartheta, 0)\, dx, \qquad (1.6)$$

where $\tan(\vartheta/2) = \eta/x$. Since in the integral over x, $x \sim 2\eta/\theta$ with $\theta \ll 1$ plays an important role, the function $x^\lambda I_{\lambda m}(\vartheta, 0)$ can be expanded in a series for small η/x. According to reference 10 (page 482) with $\lambda = 2$ we have

$$x^2 I_{2\pm2}(\vartheta, 0) = \tfrac{2}{3}\eta^2, \qquad x^2 I_{20}(\vartheta, 0) = 2\eta^2.$$

By applying the well-known values of the functions[11] $D_{\mu m}^\lambda(0, \pi/2, 0)$ we obtain from (1.6) and (13) formula (18) for the amplitude of electric quadrupole excitation with scattering through small angles in adiabatic approximation.

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