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ON THE POSSIBILITY OF DETERMINING THE AMPLITUDE FOR CHARGE EXCHANGE PION-PION SCATTERING FROM AN ANALYSIS OF THE $\pi^- + p = N + \pi^+ + \pi^-$ *REACTIONS NEAR THRESHOLD*

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It is shown that an analysis of experimental data on the energy distribution and angular correlations in the $\pi^- + p \to n + \pi^+ + \pi^-$, $n + \pi^0 + \pi^0$, and $p + \pi^- + \pi^0$ reactions makes it possible to determine the amplitude for charge-exchange scattering of charged mesons into neutral ones: $\pi^+ + \pi^- \to 2\pi^0$.

IN a previous paper by the authors[1] it was shown that the experimental study of the photoproduction of two π mesons near threshold may give information on the charge exchange amplitude $(\pi^+, \pi^-) \to 2\pi^0$ at zero energy. In this note analogous results are presented for the case of production of a π-meson pair in a pion-proton collision. In this case knowledge of pion-nucleon scattering phase shifts (δ_{31} and δ_{11}) makes it possible to indicate a somewhat different method for analyzing the experimental data.

Three reactions accompanied by production of two π mesons are possible in the collision of a π^- meson with a proton:

$$\pi^- + p \to n + \pi^+ + \pi^-, \quad \text{(1a)}$$
$$\pi^- + p \to n + \pi^0 + \pi^0, \quad \text{(1b)}$$
$$\pi^- + p \to p + \pi^- + \pi^0. \quad \text{(1c)}$$

The squares of the matrix elements for reactions (1), with final-state interactions taken into account, can be written, analogously to the case of photoproduction of two π mesons, as follows (accurate up to terms linear in kr_0):[1]

$$|\langle\pi^+\pi^-n\,|\,S\,|\,\pi^-p\rangle|^2 = \rho_1^2 [1 + \rho_{12}\sin\varphi_{12}\cdot \tfrac{2}{3}(a_2 - a_0)\,k_{12} + \rho_{13}\sin\varphi_{13}\cdot \tfrac{2}{3}\sqrt{2}\,(b_{3/2} - b_{1/2})\,k_{13}],$$
$$|\langle\pi^0\pi^0n\,|\,S\,|\,\pi^-p\rangle|^2 = \rho_2^2 [1 + \rho_{21}\sin\varphi_{21}\cdot \tfrac{4}{3}(a_2 - a_0)\,k_{12} + \rho_{23}\sin\varphi_{23}\cdot \tfrac{2}{3}\sqrt{2}\,(b_{3/2} - b_{1/2})\,(k_{13} + k_{23})],$$
$$|\langle\pi^-\pi^0p\,|\,S\,|\,\pi^-p\rangle|^2 = \rho_3^2 [1 + \rho_{31}\sin\varphi_{31}\cdot \tfrac{2}{3}\sqrt{2}\,(b_{3/2} - b_{1/2})\,k_{13} + \rho_{32}\sin\varphi_{32}\cdot \tfrac{2}{3}\sqrt{2}\,(b_{3/2} - b_{1/2})\,k_{23}],$$
$$\rho_{ik} = \rho_k/\rho_i, \qquad \varphi_{ik} = \varphi_i - \varphi_k. \quad (2)$$

Here ρ_i and φ_i are determined by the relations $\lambda_i = \rho_i \exp(i\varphi_i)$ where λ_i are the matrix elements of reactions (1a) – (1c) at threshold; $(a_2 - a_0)/3$ and $\sqrt{2}\,(b_{3/2} - b_{1/2})/3$ are, as before, charge exchange amplitudes for π-π and π-N scattering at zero energy; k_{lm} is the absolute value of the relative momentum of the l-th and m-th particles, which are numbered in the order in which they are written out in the left hand sides of Eq. (2).

To determine $a_2 - a_0$ it is sufficient, as in reference 1, to study the angular or energy distribution of the reaction 1a because the coefficients $\rho_{12}\sin\varphi_{12}$ and $\rho_{13}\sin\varphi_{13}$ are related due to charge independence by

$$\rho_{12}\sin\varphi_{12} = -\sqrt{2/3}\,\rho_{13}\sin\varphi_{13}. \quad (3)$$

Below we discuss in some detail the quantity $\rho_{12}\sin\varphi_{12}$ which enables us to give a rough estimate of the magnitude of the effect and indicate a method for a determination of $(a_2 - a_0)$ without a measurement of the ratio of the coefficients of k_{12} and k_{13}.

Near threshold the contribution to the matrix elements of reactions (1a) – (1c) comes from the $P_{1/2}$ state of the (π^-, p) system. The (π^-, p) system is a superposition of isospin $T = 1/2$ and $3/2$ states. Consequently, if we characterize the system (N, π, π) by its total isospin T and the isospin of the two mesons T_{12}, then in the final state there are only the following possibilities: $T = 1/2$, $T_{12} = 0$ or $T = 3/2$, $T_{12} = 2$ ($T_{12} = 1$ is forbidden for zero-energy π mesons). Therefore the three amplitudes λ_i may be expressed in terms of two isospin invariant matrix elements $\langle 1/2\; 0\,|\,S\,|\,1/2\rangle$ and $\langle 3/2\; 2\,|\,S\,|\,3/2\rangle$.

It is easy to show (see reference 2) that the phases of these matrix elements arise from initial-state interactions and coincide with the scattering phase shifts of π mesons on nucleons, δ_{11} and δ_{31}, in the $P_{1/2}$ state with isospin $T = 1/2$ and $3/2$ at the energy corresponding to the threshold of the reactions under study. We can write

$$\langle \tfrac{1}{2}0|S|\tfrac{1}{2}\rangle = F_{11}e^{i\delta_{11}}, \qquad \langle \tfrac{3}{2}0|S|\tfrac{3}{2}\rangle = F_{31}e^{i\delta_{31}}, \tag{4}$$

where F_{11}, F_{31} are real (but may be positive as well as negative).

It is then easy to find for λ_1, λ_2, and λ_3:

$$\begin{aligned}\lambda_1 &= \rho_1 e^{i\varphi_1} = -(\sqrt{2}/3)\,F_{11}e^{i\delta_{11}} + (1/3\sqrt{5})\,F_{31}e^{i\delta_{31}},\\ \lambda_2 &= \rho_2 e^{i\varphi_2} = (\sqrt{2}/3)\,F_{11}e^{i\delta_{11}} + (2/3\sqrt{5})\,F_{31}e^{i\delta_{31}},\\ \lambda_3 &= \rho_3 e^{i\varphi_3} = -\sqrt{3/10}F_{31}e^{i\delta_{31}}.\end{aligned} \tag{5}$$

Equation (5) leads in particular, to Eq. (3).

Let us express $\rho_{12}\sin\varphi_{12}$ in terms of F and δ:

$$\rho_{12}\sin\varphi_{12} = \frac{3\sin(\delta_{31}-\delta_{11})}{x\sqrt{10}+1/x\sqrt{10}-2\cos(\delta_{31}-\delta_{11})}, \quad x = \frac{F_{11}}{F_{31}}. \tag{6}$$

Thus the quantity $\rho_{12}\sin\varphi_{12}$, which determines the order of magnitude of the effect, depends on $\delta_{31}-\delta_{11}$ and x. The scattering phase shifts δ_{31} and δ_{11}, at energies 140-220 Mev in the center of mass system, are but poorly known.[3] It is expected, however, that $|\delta_{11}-\delta_{31}| \lesssim 10$ deg (B. M. Pontecorvo, private communication).

The quantity $\rho_{12}\sin\varphi_{12}$ behaves as a function of x (with $\delta_{31}-\delta_{11} = 10$ deg) as shown in the figure. For $x < 0$ the effect is small. In that case it would be more favorable to study the reaction (1b).

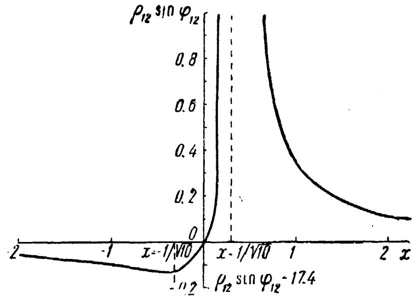
In order to determine $\rho_{12}\sin\varphi_{12}$, assuming the phase shifts δ_{11} and δ_{31} to be known, it is necessary to know x. A measurement of the ratio (ρ_k^2/ρ_i^2) of the rates of any two of the reactions (1a — 1c) at threshold would determine x, using Eq. (5). However, the x so determined will be two-valued. One may also determine x by measuring the ratio of the rate of reaction (1a) to the rate of either of the reactions $\pi^+ + p \to \pi^+ + \pi^0 + p$, $\pi^+ + \pi^+ + n$ (the cross sections for these reactions are $\sim F_{31}^2$). Here again x will be two-valued. To obtain a unique value for x it must be measured by any two of the indicated methods. Otherwise two alternatives for the sign of $a_2 - a_0$ will be obtained.

Once the quantity $\rho_{12}\sin\varphi_{12}$ has been determined, it is sufficient to study, for example, the dependence of the total cross section for reaction (1a) on the energy of the incident π^- meson. This cross section is given by (see reference 3):

$$\sigma = \rho_1^2T^2\left\{1 + \frac{64}{45\pi}\rho_{12}\sin\varphi_{12}\left[(a_2-a_0)\sqrt{2\mu_{12}T} - (b_{3/2}-b_{1/2})\sqrt{6\mu_{13}T}\right]\right\},$$

$$\mu_{12} = \mu/2, \quad \mu_{13} = m\mu/(m+\mu), \tag{7}$$

where m and μ are the nucleon and meson masses, and T is the kinetic energy of the three particles in the center of mass system.



The method described for determining $a_2 - a_0$ can also be used in principle in the photoproduction of two π mesons on a proton.[1] Instead of Eq. (6) we have in that case

$$\rho_{12}\sin\varphi_{12} = \frac{3\sin(\alpha_{31}-\alpha_{11})}{y\sqrt{5}+1/y\sqrt{5}-2\cos(\alpha_{31}-\alpha_{11})}, \quad y = \frac{G_{11}}{G_{31}}, \tag{8}$$

where $G_{31}e^{i\alpha_{31}}$ and $G_{11}e^{i\alpha_{11}}$ are the matrix elements for photoproduction in the isospin states $3/2$ and $1/2$ (with a total angular momentum $1/2$). The phases α_{11} and α_{31} differ from zero because of the existence of a real intermediate state $\gamma + N \to N + \pi \to N + \pi + \pi$, and can be expressed, with the help of the unitarity condition,[2] in terms of photoproduction amplitudes and the matrix elements F_{11} and F_{31} (see reference 4) for the π into 2π transition:

$$\begin{aligned}G_{11}\sin\alpha_{11} &= \pm|M_{11}||F_{11}|\Gamma^{1/2},\\ G_{31}\sin\alpha_{31} &= \pm|M_{31}||F_{31}|\Gamma^{1/2}.\end{aligned} \tag{9}$$

Here M_{11} and M_{31} are the amplitudes for the photoproduction of a single π meson by an M1 photon at a photon energy $E = m + 2\mu$ in a state of total angular momentum $1/2$ and isospin $1/2$ and $3/2$ respectively.[4] Γ is the phase space volume $(k^2\,dk/dE)$ of the nucleon + pion system at energy $E = \sqrt{k^2+m^2} + \sqrt{k^2+\mu^2} = m + 2\mu$. If the photoproduction amplitudes M are normalized so that the cross section is given by $\sigma = \int|M|^2 d\Omega$, then $\sqrt{\Gamma} = 1.5\mu$.

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