ROTATION OF THE PLANE OF POLARIZATION OF ELASTIC WAVES IN MAGNETICALLY POLARIZED MAGNETOELASTIC MEDIA

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Some peculiarities of the propagation of magnetoelastic waves in magnetically polarized media are studied; the medium considered is one with uniaxial symmetry. It is shown that a magnetoelastic wave propagated along the axis of symmetry consists of three waves: a longitudinal wave and two circularly polarized waves, whose speeds of propagation are different and are determined by the magnetic state of the medium — the magnitude of the magnetization or of the polarization field. The last circumstance should lead to rotation of the plane of polarization of linearly polarized elastic waves. The treatment is based on phenomenological "equations of state" that describe the dynamic properties of magnetoelastic media. Some remarks are made regarding the physical meaning of the constants that determine the rotation of the plane of polarization in certain specific types of magnetoelastic media; and an estimate is made of the frequencies at which an appreciable effect is to be expected.

1. The elastic, magnetic, and magnetoelastic properties of a magnetoelastic medium in the dynamic range can be described by equations of the form[1]

$$\sigma^*_{ij} - \left(d'_{ijkl} \frac{\partial u_k}{\partial x_l} + h'_{ij,n} J_n \right) = d''_{ijkl} \frac{\partial \dot{u}_k}{\partial x_l} + h''_{ij,n} \dot{J}_n ,$$

$$H_m + a_{mnpq} \frac{\partial^2 J_p}{\partial x_n \partial x_q} - \left(h'_{m,kl} \frac{\partial u_k}{\partial x_l} + \gamma'_{mn} J_n \right)$$

$$= h''_{m,kl} \frac{\partial \dot{u}_k}{\partial x_l} + \gamma''_{mn} \dot{J}_n, \qquad (1)$$

where H_m, J_n, and u_k are the components of the vector magnetic field intensity, the vector magnetization, and the vector displacement at points of the elastic medium, and where the σ^*_{ij}'s are the components of a certain tensor related to the elastic stresses; d'_{ijkl}, $h'_{ij,n} = h'_{n,ij}$, γ'_{mn}, and a_{mnpq} are components of the tensors formed, respectively, by the elastic moduli, the magnetostrictive constants, the inverse magnetic susceptibilities, and the exchange-interaction constants.

By way of illustration we consider a homogeneous magnetoelastic medium with uniaxial symmetry. Let the medium furthermore be magnetically polarized: that is, let it be subject to a constant, homogeneous polarizing magnetic field H_0, or let it possess a constant, homogeneous polarization magnetization J_0. We shall also suppose that the polarization magnetization or the polarizing field is directed along the axis of symmetry, the x_3 axis. We assume in addition that $J \ll J_0$ and $H \ll H_0$.

In this case it can be shown[1] that the tensor constants that appear in (1) have the form

$$\begin{pmatrix} \gamma'_{11} & \gamma''_{12} & 0 \\ -\gamma''_{12} & \gamma'_{11} & 0 \\ 0 & 0 & \gamma'_{33} \end{pmatrix}, \qquad (2)$$

$$\begin{pmatrix}
d'_{1111} & d'_{1122} & d'_{1133} & 0 & 0 & 0 & 0 & d''_{1112} & d''_{1121} \\
d'_{1122} & d'_{1111} & d'_{1133} & 0 & 0 & 0 & 0 & -d''_{1121} & -d''_{1112} \\
d'_{1133} & d'_{1133} & d'_{3333} & 0 & 0 & 0 & 0 & d''_{3312} & -d''_{3312} \\
0 & 0 & 0 & d'_{2323} & d'_{2332} & d''_{2331} & d''_{2313} & 0 & 0 \\
0 & 0 & 0 & d'_{2332} & d'_{3232} & d''_{3231} & d''_{2331} & 0 & 0 \\
0 & 0 & 0 & -d''_{2331} & -d''_{3231} & d'_{3232} & d'_{2332} & 0 & 0 \\
0 & 0 & 0 & -d''_{2313} & -d''_{2331} & d'_{2332} & d'_{2323} & 0 & 0 \\
-d''_{1112} & d''_{1121} & -d''_{3312} & 0 & 0 & 0 & 0 & d'_{1212} & d'_{1221} \\
-d''_{1121} & d''_{1112} & d''_{3312} & 0 & 0 & 0 & 0 & d'_{1221} & d'_{1212}
\end{pmatrix}. \qquad (3)$$

$$d'_{1221} = d'_{1111} - d'_{1122} - d'_{1212};$$

$$\begin{pmatrix} 0 & 0 & 0 & h''_{1,23} & h''_{1,32} & h'_{1,31} & h'_{1,13} & 0 & 0 \\ 0 & 0 & 0 & h'_{1,13} & h'_{1,31} - h''_{1,32} & -h''_{1,23} & & 0 & 0 \\ h'_{3,11} & h'_{3,11} & h'_{3,33} & 0 & 0 & 0 & 0 & h''_{3,12} & -h''_{3,12} \end{pmatrix},$$

$$h'_{11,3} = h'_{22,3} = h'_{3,11}, \quad h'_{33,3} = h'_{3,33}, \quad h'_{31,1} = h'_{32,2} = h'_{1,31},$$
$$h'_{13,1} = h'_{23,2} = h'_{1,13}, \quad h''_{23,1} = -h''_{13,2} = -h''_{1,23},$$
$$h''_{32,1} = -h''_{31,2} = -h''_{1,32}, \quad h''_{12,3} = -h''_{21,3} = -h''_{3,12}. \quad (4)$$

We remark that such a simultaneous entry, in a single table, of the single-primed and double-primed components of the tensor constants became possible by virtue of the fact that in this work we did not take account of those tensor components (double-primed) that describe energy absorption. The nature of the dependence of the components of the tensor constants on the polarizing field or magnetization was discussed earlier.[1]

As is easily seen, the "stress" tensor σ^*_{ij} is unsymmetrical and differs from the stress tensor that appears in the equation of motion of elasticity theory,

$$\rho \ddot{u}_i = \partial \sigma_{ij} / \partial x_j, \quad (5)$$

the latter, by definition (as the momentum flow tensor, taken with reversed sign), must be symmetric. An expression for the tensor σ_{ij} can be obtained by symmetrizing the tensor σ^*_{ij}, i.e., by separating it into symmetric and antisymmetric terms

$$\sigma_{ij} = {}^1/_2 (\sigma^*_{ij} + \sigma^*_{ji}), \quad T_{ij} = {}^1/_2 (\sigma^*_{ij} - \sigma^*_{ji}). \quad (6)$$

The second term determines the force couple that acts on unit volume. This couple is always equilibrated by some other couple that acts on unit volume; its nature can vary. In anisotropic media it can be, for example, a couple produced by the noncoincidence of the orientations of the vector magnetic field intensity and of the magnetization (for a more detailed discussion of this question, see references 2 and 3).

It appears that the following additional conditions[4] must still be imposed on the constants d'_{ijkl}:

$$d'_{ijkl} + d'_{jilk} = d'_{ijlk} + d'_{jikl}. \quad (7)$$

2. We now consider the specific problem of the propagation of plane magnetoelastic waves along the direction of the symmetry axis. Hereafter we shall neglect displacement currents and conduction currents. Therefore in the joint solution of the equations of elasticity (5) and the equations of electrodynamics, we shall need only the following two of Maxwell's equations:

$$\operatorname{curl} \mathbf{H} = 0, \quad \operatorname{div} \mathbf{B} = 0.$$

Furthermore we shall suppose that all variable quantities vary as $\exp[i(\omega t - kx_3)]$; and we shall also assume that the exchange interaction, for a medium with uniaxial symmetry, is described by just two independent tensor components a_{mnpq} (a_1 and a_2). As a result we get the following two independent systems of homogeneous equations:

$$\omega^2 \rho u_3 = \rho (c_l^0)^2 k^2 u_3 + i h_{33} k J_3,$$
$$-4\pi J_3 = -i h_{33} k u_3 + (\gamma'_{33} + a_2 k^2) J_3; \quad (8)$$

$$\omega^2 \rho u_1 = \rho (c_t^0)^2 k^2 u_1 - i\omega B k^2 u_2 + i h_{15} k J_1 - \omega h''_{14} k J_2,$$
$$\omega^2 \rho u_2 = i\omega B k^2 u_1 + \rho (c_t^0)^2 k^2 u_2 + \omega h''_{14} k J_1 + i h_{15} k J_2,$$
$$0 = -i (h_{15} - h^*_{15}) k u_1 + \omega (h''_{14} + h^{*''}_{14}) k u_2 + (\gamma'_{11} + a_1 k^2) J_1 + i\omega \gamma''_{12} J_2,$$
$$0 = -\omega (h''_{14} + h^{*''}_{14}) k u_1 - i (h_{15} - h^*_{15}) k u_2 - i\omega \gamma''_{12} J_1 + (\gamma'_{11} + a_1 k^2) J_2, \quad (9)$$

Here the following symbols have been introduced:

$$c_t^0 = [(d'_{2323} + d'_{2332})/2\rho]^{1/2}, \quad c_l^0 = [d'_{3333}/\rho]^{1/2},$$
$$B = c''_{45} - c^{*''}_{45} = {}^1/_2 (d''_{2313} + d''_{2331}),$$
$$h_{33} = h'_{3,33}, \quad h_{15} = {}^1/_2 (h'_{1,13} + h'_{1,31}), \quad h^*_{15} = h_{15} - h'_{1,13},$$
$$h''_{14} = \tfrac{1}{2} (h''_{1,23} + h''_{1,32}), \quad h^{*''}_{14} = h''_{1,23} - h''_{14}. \quad (10)$$

The system of homogeneous equations (8) determines the propagation of a longitudinal magnetoelastic wave. For the speed of propagation of this wave, $c_l = \omega/k$, we get the expression

$$c_l = c_l^0 [1 - h_{33}^2 / \rho (c_l^0)^2 (\gamma'_{33} + a_2 k^2 + 4\pi)]^{1/2}. \quad (11)$$

On setting the determinant of the system of homogeneous equations (9) equal to zero, we get a characteristic equation of the sixth degree in ω; it separates into the two equations

$$\rho\omega^2 - \rho (c_t^0)^2 k^2 \mp \omega B k^2 + k^2 (\gamma'_{11} + a_1 k^2 \mp \omega \gamma''_{12})^{-1} (h_{15} \mp \omega h''_{14}) \times [(h_{15} - h^*_{15}) \mp \omega (h''_{14} + h^{*''}_{14})] = 0, \quad (12)$$

We shall henceforth neglect the term $a_1 k^2$; this is permissible when $a_1 k^2 \ll |\gamma'_{11} \mp \omega \gamma''_{12}|$. Then it is easy to find two positive roots k_+ and k_- of Eq. (12). To these there will correspond two transverse magnetoelastic waves that are circularly polarized: that is, $u_1/u_2 = \mp i$ and $J_1/J_2 = \mp i$. Since the speeds of propagation of these waves are different, this should lead to rotation of the plane of polarization of a linearly polarized elastic wave.

The angle through which the plane of polarization rotates upon propagation of a linearly polarized elastic wave through unit distance is $\varphi = (k_+ - k_-)/2$, or

$$\varphi = \frac{\omega^2}{2\rho (c_t^0)^3} \left\{ B + \varkappa'_{11} \frac{[h_{15}(h''_{14} + h^{*''}_{14}) + h''_{14}(h_{15} - h^*_{15})]}{1 - (\omega/\omega_0)^2} - \varkappa'_{11} \frac{\omega_0^{-1} [h_{15}(h_{15} - h^*_{15}) + \omega^2 h''_{14}(h''_{14} + h^{*''}_{14})]}{1 - (\omega/\omega_0)^2} \right\}, \quad (13)$$

where $\kappa'_{11} = 1/\gamma'_{11}$ is the susceptibility measured in a direction perpendicular to the symmetry axis, and $\omega_0 = \gamma'_{11}/\gamma''_{12}$ is the magnetic resonance frequency. Here it has been assumed that the expression in curly brackets, multiplied by $\omega/\rho(c_t^0)^2$, is small in comparison with unity.

3. We shall now discuss the results obtained. As is evident from relation (11), the speed of propagation of a longitudinal magnetoelastic wave along the symmetry axis in a magnetically polarized medium is dependent not only on the elastic constants but also on the magnetic state of the medium. In connection with this we recall that in ferromagnetics both h_{33} and γ'_{33} depend appreciably on the magnetization. Formally this phenomenon can be described by introducing an elastic-modulus tensor d^H_{ijkl} measured at constant field. For quasistationary processes, a definite relation[5] holds between the tensor component $d^J_{ijkl} = d'_{ijkl}$ measured at constant magnetization and the tensor component d^H_{ijkl} measured at constant field; in our case this takes the form

$$d^H_{3333}/d^J_{3333} = 1 - h^2_{33}/d^J_{3333}\gamma'_{33}.$$

In the dynamic range, as is evident from (11), two additional terms a_2k^2 and 4π must be added to γ'_{33}. This may be one of the reasons for the differences between the values of elastic moduli obtained experimentally in measurements made in the quasistationary and in the dynamic ranges. We recall that in the case of ferromagnets, the dependence of the elastic moduli on magnetic state bears the name "ΔE-effect."

As is evident from (13), the rotation of the plane of polarization is determined by several constants. Their physical meaning for specific types of medium can be established on the basis of a microscopic theory of the kinetic processes that occur in a specific magnetoelastic medium. We consider several types of medium.

The role of the constant B was considered earlier,[6] and it has been shown[7] that in metals the constant c''_{45} that appears in B may be dependent on the effect of the polarizing magnetic field on the electron distribution function, as perturbed by the elastic wave. A specific calculation[7] of the constant c''_{45}, carried out within the framework of the free-electron model, gives, in particular, for low frequencies ($kl \ll 1$) or for strong polarizing fields ($\omega_c\tau \gg kl$), the relation

$$c''_{45} = -c''_{54} = -c''_{44}\omega_c\tau, \quad (14)$$

where $\omega_c = eH_0/cm$; e, m, τ, and l are, respectively, the charge, mass, collision time, and free path length of an electron; $c''_{44} = \frac{1}{5} nml v_0 \times (1 + \omega_c^2\tau^2)^{-1}$ is the imaginary part of the complex elastic-modulus tensor and determines the losses caused by transfer of the energy of the elastic wave to the conduction electrons;[8] v_0 is the speed of electrons at the Fermi surface. Calculation shows that appreciable rotation of the plane of polarization (through angles φ of the order of a few degrees or more in a field $H_0 \sim 10^3$ oe) may be expected in pure metals at low temperatures at frequencies of the order of tens of megacycles/sec.[7]

Concerning the constants that appear in the subsequent terms in the curly brackets in (13), it is possible to draw some conclusions from relations (1) and (6). The constant h^*_{15} determines the torque that acts on an anisotropic body because of noncoincidence of the directions of the vector magnetization and of the vector magnetic field intensity, or the field that must be applied in order to keep the magnetization unchanged upon rotation of the volume element. The value of this constant is determined by the magnetocrystalline anisotropy constant. In magnetically uniaxial ferromagnetics, the constant h^*_{15} and the magnetostrictive constant h_{15} (which depends on the magnetostriction constants) are in order of magnitude approximately the same and equal to 10^4 oe. The constant $h^{*''}_{14}$ describes gyromagnetic effects, and the constant h''_{14} is apparently related to possible anisotropy of the gyromagnetic effects. The value of the constant $h^{*''}_{14}$ is determined by the gyromagnetic ratio and is of order of magnitude 10^{-7} oe sec. In uniaxial ferromagnetics, the ferromagnetic resonance frequency is $\omega_0 \sim 10^9$ sec^{-1}.

Thus in ferromagnetics, the second and third terms in curly brackets in (13) can provide an appreciable amount of rotation of the plane of polarization at elastic-wave frequencies of order 10^9 sec^{-1}.

We emphasize that the different constants appearing in (13) have different types of temperature dependence.

We remark also that in the works of Akhiezer et al.[9] and of Kittel,[10] only one possible cause of rotation of the plane of polarization was taken into account — namely, the magnetostrictive constant h_{15}. The expression derived by Kittel for the rotation of the plane of polarization (which is correct for frequencies $\omega > \omega_0$) can be obtained from formula (13) by setting B, $h_{14}^{*\prime\prime}$, $h_{14}^{\prime\prime}$, and h_{15}^{*} equal to zero.

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