# ON THE INFLUENCE OF COHERENT MAGNETIC DIPOLE RADIATION ON MAGNETIC RESONANCE

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Corrections to the relaxation time, necessitated by the effect of the radiation field, are computed. The effect of the resonator on the nature of the observed phenomenon is taken into account.

**1.** When a spin system radiates electromagnetic quanta of wavelength exceeding the dimensions of the system coherent phenomena occur which lead to a considerable increase in the radiation line width.[1,2] The role of this phenomenon first noted by Mandel'shtam[3] and investigated by Dicke[1] was discussed in detail in connection with magnetic resonance in the paper by Bloembergen and Pound[4] where an estimate was made of the effect of coherent phenomena on the damping of radiation in the sample placed in the receiver coil of the circuit or in a resonator. Faĭn[5] noted that if the interaction of spins through their common radiation field is taken into account this leads in the radio frequency region to a shift of the resonance frequency.

In this paper we give a consistent calculation of corrections to the relaxation times and to the additional shift of the resonance frequency due to the influence of the coherent radiation field. In the course of this calculation it turns out, as expected, that within the limits of the approximation considered, quantum theory leads to the same results as classical theory. In conclusion the question is discussed of the changes in the characteristics of the signal due to the finite figure of merit of the resonator. The conditions are established under which radiation corrections will be significant.

**2.** Ginzburg[6] has shown that the classical equation of motion for the magnetic moment $\boldsymbol{\mu}$ of a homogeneously magnetized sample of dimensions small compared to the wavelength of the radiation has the form

$$\dot{\boldsymbol{\mu}} = \gamma\,[\boldsymbol{\mu}\times\mathbf{H}] - \frac{4\gamma\omega_m}{3\pi v^3}\,[\boldsymbol{\mu}\times\ddot{\boldsymbol{\mu}}] + \frac{2\gamma}{3v^3}\,[\boldsymbol{\mu}\times\dddot{\boldsymbol{\mu}}], \qquad (1)$$

where $v = c/\sqrt{\epsilon\mu}$ is the phase velocity of light in the material of the sample, while $\omega_m \approx cV^{-1/3}$.

On introducing the magnetization $M = \mu/V$ and including terms due to other relaxation mechanisms, in terms of the circularly polarized variables $M_\alpha$ ($\alpha = 0, \pm 1$):

$$M_{\pm 1} = \mp (M_x \pm iM_y)/\sqrt{2}, \qquad M_0 = M_z,$$

we obtain

$$\dot{M}_\alpha + \frac{M_\alpha}{T_\alpha} = \gamma\,[\mathbf{M}\times\mathbf{H}]_\alpha + \frac{\chi_0 h_\alpha(t)}{T_\alpha} - \frac{4\gamma\omega_m V}{3\pi v^3}\,[\mathbf{M}\times\ddot{\mathbf{M}}]_\alpha + \frac{2\gamma V}{3v^3}\,[\mathbf{M}\times\dddot{\mathbf{M}}]_\alpha. \qquad (2)$$

We shall assume that the sample in addition to being situated in a constant field $H_0 = H_z$, is also placed into a weak variable magnetic field $h(t) \ll H_0$ ($\mathbf{h} \perp \mathbf{H}_0$). By linearizing (2) near the equilibrium position for the case $1/T_\alpha \ll \omega_0 = \gamma H_0$, we obtain

$$\dot{M}_\alpha + (1/T_\alpha^* + i\alpha\omega_0^*)\,M_\alpha = i\alpha\omega_0\chi_0 h_\alpha(t), \qquad (3)$$

where

$$1/T_\alpha^* = 1/T_\alpha + 1/T_\alpha^r, \qquad 1/T_\alpha^r = {}^2/_3\,(2-\alpha^2)\,\chi_0\omega_0^4 v^{-3} V, \qquad (4)$$

$$\omega_0^* - \omega_0 = {}^4/_3\,\chi_0\,(\omega_0^3\omega_m/\pi v^3)\,V. \qquad (5)$$

As can be seen from (4), the longitudinal relaxation time $T_{\parallel}^r = T_0^r$ is by a factor of two smaller than the transverse relaxation time $T_{\perp}^r = T_{\pm 1}^r$. The expression for the shift of the resonance frequency $\omega_0^* - \omega_0$ in the form (5) was obtained by Faĭn.[5] Corrections due to the interaction of the particles through their common radiation field increase rapidly with the field $H_0$ and are proportional to the volume of the sample. Under favorable conditions in the range of strong magnetic fields $T_\alpha^r$ may become comparable to $T_\alpha$.

**3.** The quantum theory of the phenomenon under consideration for the case of weak radio frequency fields may be developed on the basis of the method of Kubo and Tomita.[7]

If we restrict ourselves to taking into account only the interactions of the magnetic moments through the radiation field then the part of the Hamiltonian independent of the time may be written in the form

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_1 + \hat{\mathcal{H}}_2 + \hat{\mathcal{H}}' = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}',$$

$$\hat{\mathcal{H}}_1 = -\hbar\omega_0 \sum_j^N \hat{I}_{j0}, \quad \hat{\mathcal{H}}_2 = \sum_{k\lambda} (\hat{a}_{k\lambda}^+ \hat{a}_{k\lambda} + 1/2)\hbar v k, \tag{6}$$

where $\hat{\mathcal{H}}_1$ describes the interaction of the magnetic moments with an external constant magnetic field, $\hat{\mathcal{H}}_2$ is the Hamiltonian of the radiation field, $\lambda = \pm 1$ corresponds to the two possible values of the polarization, while $v$ is the phase velocity of light in the medium into which the magnetic moments are immersed. We shall treat the Hamiltonian of the interaction of the magnetic moments with the radiation field as a perturbation. In the case when the dimensions of the system are considerably smaller than a wavelength it has the form

$$\hat{\mathcal{H}}' = -i\gamma\hbar \sum_{k,\lambda} \sqrt{2\pi\hbar v k/V} \sum_{j\alpha}^N (-1)^\alpha \hat{I}_{j\alpha}\varepsilon_{k-\lambda-\alpha} (\hat{a}_{k\lambda} - \hat{a}_{k\lambda}^+). \tag{7}$$

The relaxation times and the shift of the resonance frequency may be determined in a manner similar to the way this is done in our earlier paper.[8] By utilizing equations (40), (46), and (47) of that article for the case $\hbar\omega_0 \gg kT$ we shall obtain expressions for $T_\alpha^*$ and $\omega_0^*$ which coincide with those obtained above by a classical method.

**4.** The foregoing calculations have been carried out for the case when the sample is situated in free space. In making observations of magnetic resonance the sample is placed either into a coil or into a resonator of finite figure of merit. The resonator may affect in an essential manner the nature of the phenomenon being observed.

The figure of merit of an ideal resonator of volume $V_p$, in which the losses are due only to the presence of a magnetic sample of volume $V$ situated in a constant field $H_0$ and which absorbs energy from the radio-frequency field $h(t)$, is given by*

$$Q'(\omega) = V_p/4\pi\chi''(\omega)V. \tag{8}$$

If a small sample is placed within such a region inside the resonator where the variable magnetic field of resonance frequency $\omega_0$ may be regarded as homogeneous and linearly polarized, then $2\chi''(\omega_0) = \chi_0\omega_0 T_\perp^*$ and

$$Q'(\omega_0) = (V_p/V)/2\pi\chi_0\omega_0 T_\perp^*. \tag{9}$$

Further, we denote by $Q_0(\omega_0)$ the figure of merit of a real resonator with the sample in the absence of the constant magnetic field $H_0$.

Then $Q(\omega_0)$ of a real resonator with the sample situated in a magnetic field $H_0$ in conditions of resonance is related to $Q'(\omega_0)$ and $Q_0(\omega_0)$ by the relation

$$Q'(\omega_0) = \frac{Q_0(\omega_0)}{Q_0(\omega_0)/Q(\omega_0) - 1}. \tag{10}$$

The ratio $Q_0/Q$ and $Q_0$ can be measured directly in an experiment and, thus, the dependence of $1/T_\perp^*$ on the frequency $\omega_0$ may be found.

*The figure of merit of a system is defined as $Q(\omega) = \omega \times$(total energy of the system/energy absorbed per second).


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