ON THE DETERMINATION OF THE POTENTIAL IN QUANTUM FIELD THEORY

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The problem of the determination of the potential in quantum field theory is studied in relation to the limitations that are imposed on the transition amplitude by the conditions of orthonormality and completeness for the systems of states of the noninteracting and interacting particles. A nonlinear integral equation for the transition amplitude is used for the construction of the potential. It is shown that the potential so constructed correctly describes the scattering of particles in the range of energies in which production of new particles does not occur, and also the bound states. Problems associated with the nonuniqueness of the potential are discussed.

In the theoretical literature of recent years much space has been given to papers devoted to the study of the interaction of particles in quantum field theory by means of some sort of effective potential (cf., e.g., the survey article by Klein,[1] which gives an extensive bibliography, and also references 2—6). Nevertheless, the problem of the potential in quantum field theory cannot be regarded as solved. The methods proposed for its construction are not satisfactory. Some of them are in essence approximate (for example, the particles are assumed to be bound, and as the potential one takes their energy when they are a given distance apart). In other papers the authors get a potential that depends on the energy, so that they do not arrive at the Schrödinger equation for the eigenvalues, but at an equation of the form $[h_0 + V(E)]\psi_E = E\psi_E$. The methods for eliminating the dependence of the potential on the energy that have been proposed in a number of papers (cf. reference 1) are extremely artificial and complicated, and are not very effective, in the opinion of the authors themselves.

We shall be guided by an idea due to Novozhilov, that a potential independent of the energy can be determined by means of a nonlinear equation for the transition amplitude.[3] The present paper develops and refines this idea.

For our purposes the basic quantities are $\langle\varphi_\alpha|\psi_\beta^{(+)}\rangle$ in the quantum mechanics of two particles and $\langle\Phi_\alpha|\Psi_\beta^{(+)}\rangle$ in quantum field theory, where φ_α and Φ_α are states of the two particles without interaction, and $\psi_\alpha^{(+)}$ and $\Psi_\alpha^{(+)}$ are states of the interacting particles with the outgoing-wave condition. The nonlinear equations for the transition amplitude relate $\langle\varphi_\alpha|\psi_\beta^{(+)}\rangle$ to the potential in a simple way. Thus the problem is to relate $\langle\varphi_\alpha|\psi_\beta^{(+)}\rangle$ to $\langle\Phi_\alpha|\Psi_\beta^{(+)}\rangle$ (in the construction of the potential the latter quantities are regarded as known). As is shown in Sec. 1 of the present paper, direct identification of $\langle\varphi_\alpha|\psi_\beta^{(+)}\rangle$ with $\langle\Phi_\alpha|\Psi_\beta^{(+)}\rangle$ is impossible. In virtue of the orthonormality and completeness of the systems φ_α and $\psi_\alpha^{(+)}$, the $\langle\varphi_\alpha|\psi_\beta^{(+)}\rangle$ satisfy definite relations. On the other hand, the $\langle\Phi_\alpha|\Psi_\beta^{(+)}\rangle$ do not satisfy these relations, because the system of states $\Psi_\beta^{(+)}$ (α is a two-particle state) is not complete, i.e., because of the possibility of the production of new particles.

In Sec. 2 the $\langle\Phi_\alpha|\Psi_\beta^{(+)}\rangle$ are replaced by other quantities $\langle\tilde{\Phi}_\alpha|\Psi_\beta^{(+)}\rangle$, where the $\tilde{\Phi}_\alpha$ are chosen in such a way that, firstly, they agree asymptotically with the Φ_α, and secondly, the relations mentioned above are satisfied. On the basis of the $\langle\tilde{\Phi}_\alpha|\Psi_\beta^{(+)}\rangle$ it is possible to construct a potential which, as will be rigorously proved, correctly describes the scattering of the particles at energies below the threshold for the production of new particles, and makes it possible to find the levels of the bound states.

At the end of Sec. 2 we give expressions for the potential that are suitable for concrete calculations. They are an improvement on the corresponding formulas given by Novozhilov,[3] who did not examine the question of the possibility of identifying the $\langle\varphi_\alpha|\psi_\beta^{(+)}\rangle$ and the $\langle\Phi_\alpha|\Psi_\beta^{(+)}\rangle$. It must be emphasized, however, that this improvement does not extend to the results of the calculation of the nucleon-nucleon potential made by Novozhilov and Terent'ev,[4] since it is outside the scope of the approximation used in that paper (the correction terms fall off as $e^{-3\mu r}$ and faster with increase of the distance r between the nucleons; μ is the mass of the π meson).

Section 3 contains a brief discussion of some peculiarities of the potential constructed here. It turns out that the very statement of the problem of describing a quantum-field system by potentials does not permit an unambiguous definition of the potential. All the possible potentials are connected by transformations of a special type, and differ only at small distances between the particles. Considerations of simplicity and convenience enable us to single out uniquely the potential that is most suitable for applications. Here also it is shown that, generally speaking, the potential so obtained has a nonlocal character.

In our work we use some formulas of the general theory of scattering, which can be found, for example, in a paper by Ekstein.[7]

1. STATEMENT OF THE PROBLEM

Let us consider the Schrödinger equation for a system of two particles that interact with the potential V:

$$i\partial\psi/\partial t = (h_0 + V)\psi = h\psi. \tag{1}$$

Let us introduce the eigenvectors φ_α of the unperturbed Hamiltonian, where α denotes the spins and momenta of the two particles. Let $h_0\varphi_\alpha = E_\alpha\varphi_\alpha$ and $\langle\varphi_\alpha|\varphi_\beta\rangle = \delta_{\alpha\beta}$. We denote the corresponding orthonormal eigenvectors of the total Hamiltonian, with outgoing waves, by $\psi_\alpha^{(+)}$. We define $t_{\alpha\beta}$ by the equation

$$\langle\varphi_\alpha|\psi_\beta^{(+)}\rangle = \delta_{\alpha\beta} - t_{\alpha\beta}/(E_\alpha - E_\beta - i0). \tag{2}$$

Let there exist also bound states ψ_m, so that $h\psi_m = E_m\psi_m$, where $E_m < E_\alpha$ when the total momenta of these states are the same. We set

$$\langle\varphi_\alpha|\psi_m\rangle = t_{\alpha m}/(E_m - E_\alpha). \tag{3}$$

A matrix element of the S matrix is defined by the formula

$$S_{\alpha\beta} = \delta_{\alpha\beta} - 2\pi i\delta(E_\alpha - E_\beta)\,t_{\alpha\beta}. \tag{4}$$

For $t_{\alpha\beta}$ we can write the linear equation

$$t_{\alpha\beta} = V_{\alpha\beta} - \sum_\gamma \frac{V_{\alpha\gamma}t_{\gamma\beta}}{E_\gamma - E_\beta - i0} \tag{5}$$

and the nonlinear equation

$$t_{\alpha\beta} = V_{\alpha\beta} - \sum_\gamma \frac{t_{\alpha\gamma}t^+_{\gamma\beta}}{E_\gamma - E_\beta - i0} - \sum_m \frac{t_{\alpha m}t^+_{m\beta}}{E_m - E_\beta}. \tag{6}$$

A peculiarity of the nonlinear equation (6) is its simple dependence on the potential. In reference 3 it is therefore suggested that one insert in (6) instead of $t_{\alpha\beta}$ the corresponding expressions from quantum field theory, whereupon the potential in momentum space is determined from (6). Of course, to determine the potential in terms of $t_{\alpha\beta}$ one could start from (5), as Klein did.[1] This, however, would have given nothing new, since (6) in fact is the solution of (5) that gives the potential explicitly. We thus get

$$V_{\alpha\beta} = t_{\alpha\beta} + \sum_\gamma \frac{t_{\alpha\gamma}t^+_{\gamma\beta}}{E_\gamma - E_\beta - i0} + \sum_m \frac{t_{\alpha m}t^+_{m\beta}}{E_m - E_\beta}. \tag{7}$$

The question arises: can one insert arbitrary quantities for the $t_{\alpha\beta}$ in (7)? It turns out that they cannot be arbitrary. The point is that the $t_{\alpha\beta}$ must satisfy certain integral relations that arise from the orthonormality and completeness of the systems $\psi_\alpha^{(+)}$, ψ_m and φ_α. These relations were first obtained by Wigner and Eisenbud.[8] As an example we give one of these relations here:

$$\frac{t^+_{\alpha\beta} - t_{\alpha\beta}}{E_\alpha - E_\beta - i0} + \sum_\gamma \frac{t_{\alpha\gamma}t^+_{\gamma\beta}}{(E_\alpha - E_\gamma - i0)(E_\beta - E_\gamma + i0)}$$
$$+ \sum_m \frac{t_{\alpha m}t^+_{m\beta}}{(E_m - E_\alpha)(E_m - E_\beta)} = 0 \tag{8}$$

[hereafter when we refer to (8), we shall take it to mean all the relations of this type].

It can be shown by direct verification that the relations (8) for the $t_{\alpha\beta}$ are not only necessary, but also sufficient conditions for the quantity V determined from (7) to be capable of being interpreted as a potential. One of these relations guarantees the Hermitian character of V, and the others assure that (5) with the potential V will have as its solution the same $t_{\alpha\beta}$ as appear in (7).

Thus we conclude that the quantum-field quantities inserted in (7) instead of the $t_{\alpha\beta}$ must satisfy the conditions (8).

Let us now turn to the quantum-field system. The usual definition of the transition amplitude in field theory is*

$$T_{\Gamma\Delta} = \langle\Phi_\Gamma|H - E_\Gamma|\Psi_\Delta^{(+)}\rangle. \tag{9}$$

Here Φ are the so-called asymptotically stationary states.[9] The quantities that correspond to them in ordinary quantum mechanics are the states of noninteracting particles. H is the field-theory Hamiltonian, and $\Psi_\Delta^{(+)}$ are its eigenstates with outgoing waves, corresponding to the asymptotically stationary states Φ_Δ and energies E_Δ. The S matrix is expressed in terms of the $T_{\Gamma\Delta}$ by a

*We denote quantities relating to the quantum field theory by capital letters.

formula analogous to (4). We further define the quantities $T_{\Gamma M} = \langle \Phi_\Gamma | H - E_\Gamma | \Psi_M \rangle$, where the Ψ_M are bound states in the field theory. The indices Γ and M characterize all the states of the quantum-field system, including both the states corresponding to the two particles with which we are concerned ($\Gamma = \gamma$ and $M = m$) and also states corresponding to other sets of particles ($\Gamma \neq \gamma$ and $M \neq m$).

For the quantities $T_{\alpha\beta}$ and $T_{\alpha m}$ we can write relations of the type of (8), but with one extremely important difference. In (8) the summations are over the spins and momenta of the two particles and their bound states. In the relations for the $T_{\alpha\beta}$, on the other hand, the summations must be taken over all the states in the field theory, i.e., as compared with (8) we have additional terms corresponding to the summations over all $\Gamma \neq \gamma$ and $M \neq m$.

From this there follows our most important conclusion: $T_{\alpha\beta}$ and $T_{\alpha m}$ do not satisfy the required conditions (8) and cannot be inserted instead of $t_{\alpha\beta}$ in (5) – (7).

We shall show, however, that there exist quantities $\widetilde{T}_{\alpha\beta}$ and $\widetilde{T}_{\alpha m}$ that on one hand satisfy (8) and on the other hand are equal to $T_{\alpha\beta}$ for $E_\alpha = E_\beta$, if processes of creation and annihilation of particles do not occur. Substitution of $\widetilde{T}_{\alpha\beta}$ and $\widetilde{T}_{\alpha m}$ in (7) gives the desired potential. The construction of the $\widetilde{T}_{\alpha\beta}$ and $\widetilde{T}_{\alpha m}$ will be carried out in the next section.

2. CONSTRUCTION OF THE POTENTIAL

Let us separate out from all the states $\Psi_\Gamma^{(+)}$ the states with $\Gamma = \gamma$. Let P be the operator for projection onto these states, so that

$$\langle \Psi' | P\Psi'' \rangle = \sum_\gamma \langle \Psi' | \Psi_\gamma^{(+)} \rangle \langle \Psi_\gamma^{(+)} | \Psi'' \rangle + \sum_m \langle \Psi' | \Psi_m \rangle \langle \Psi_m | \Psi'' \rangle. \quad (10)$$

If the two particles under consideration form certain bound states, then in the sum over m we may include not all of these states m but an arbitrary selection from them.

Let us orthogonalize and normalize the Φ_α with respect to the metric (10). If

$$\langle \Phi_\alpha | P\Phi_\beta \rangle = \delta_{\alpha\beta} + N_{\alpha\beta}, \quad (11)$$

then the state vectors

$$\widetilde{\Phi}_\alpha = \sum_\gamma [1 + N]_{\gamma\alpha}^{-1/2} \Phi_\gamma \quad (12)$$

will obviously have the required properties:

$$\langle \widetilde{\Phi}_\alpha | P\widetilde{\Phi}_\beta \rangle = \delta_{\alpha\beta}. \quad (13)$$

By N in (12) we mean the Hermitian integral operator whose kernel is defined by (11). We shall assume that $[1+N]\Phi_\alpha \neq 0$.

We assume that the orthonormal system $\widetilde{P}\widetilde{\Phi}_\alpha$ so obtained is complete in space determined by the projection operator P. In fact, if this were not so there would exist a linear combination of the $\Psi_\gamma^{(+)}$ and Ψ_m that would be orthogonal to all the $P\widetilde{\Phi}_\alpha$, and consequently to all the Φ_α (in view of the fact that $1+N$ cannot vanish). But the expression $\langle \Psi_\gamma^{(+)} | \Phi_\alpha \rangle$ is analogous in meaning to the phenomenological function of the state γ in the momentum representation, and for such quantities it is natural to assume linear independence.

Let us set

$$\langle \widetilde{\Phi}_\alpha | \Psi_\beta^{(+)} \rangle = \delta_{\alpha\beta} - \frac{\widetilde{T}_{\alpha\beta}}{E_\alpha - E_\beta - i0},$$

$$\langle \widetilde{\Phi}_\alpha | \Psi_m \rangle = \frac{\widetilde{T}_{\alpha m}}{E_m - E_\alpha}. \quad (14)$$

In virtue of the orthonormality and completeness of the systems of states $\Psi_\gamma^{(+)}$, Ψ_m, and $P\widetilde{\Phi}_\gamma$, the quantities $\widetilde{T}_{\alpha\beta}$ and $\widetilde{T}_{\alpha m}$ defined by (14) satisfy the conditions (8). Substitution of these quantities in (7) gives us a Hermitian potential V for which solution of the Schrödinger equation (1) at once gives the quantities $\widetilde{T}_{\alpha\beta}$ and $\widetilde{T}_{\alpha m}$.

Let us now verify that the $\widetilde{T}_{\alpha\beta}$ correctly describe the scattering, i.e., that on the energy surface $E_\alpha = E_\beta$ they are identical with $T_{\alpha\beta}$. According to (9) and (14), it is sufficient for this to show that

$$\lim_{E_\alpha \to E_\gamma} (E_\alpha - E_\gamma) \langle \widetilde{\Phi}_\alpha | \Psi_\gamma^{(+)} \rangle = \lim_{E_\alpha \to E_\gamma} (E_\alpha - E_\gamma) \langle \Phi_\alpha | \Psi_\gamma^{(+)} \rangle. \quad (15)$$

It follows from (11) that

$$N_{\alpha\beta} = \langle \Phi_\alpha | \Phi_\beta \rangle - \delta_{\alpha\beta} - \sum_{\Gamma \neq \gamma} \langle \Phi_\alpha | \Psi_\Gamma^{(+)} \rangle \langle \Psi_\Gamma^{(+)} | \Phi_\beta \rangle. \quad (16)$$

If processes of creation and annihilation are impossible, then in the sum over $\Gamma \neq \gamma$ none of the terms has a denominator that gives a singularity, and $N_{\alpha\beta}$ is not singular for $E_\alpha = E_\beta$. Under these conditions (15) indeed holds, since the singular terms of $\langle \widetilde{\Phi}_\alpha | \Psi_\beta^{(+)} \rangle$ and $\langle \Phi_\alpha | \Psi_\beta^{(+)} \rangle$ are the same.

The potential determined in terms of $\widetilde{T}_{\alpha\beta}$ and $\widetilde{T}_{\alpha m}$ by (7) is

$$V_{\alpha\beta} = \widetilde{T}_{\alpha\beta} + \sum_\gamma \frac{\widetilde{T}_{\alpha\gamma} \widetilde{T}_{\gamma\beta}^{+}}{E_\gamma - E_\beta - i0} + \sum_m \frac{\widetilde{T}_{\alpha m} \widetilde{T}_{m\beta}^{+}}{E_m - E_\beta}. \quad (17)$$

We transform (17) to an expression more convenient for study:

$$\widetilde{T}_{\alpha\beta} = \langle\widetilde{\Phi}_\alpha | H - E_\alpha | \Psi_\beta^{(+)}\rangle = \langle\widetilde{\Phi}_\alpha | H - E_\alpha | P\widetilde{\Phi}_\beta\rangle$$
$$+ \sum_\gamma \langle\widetilde{\Phi}_\alpha | H - E_\alpha | \Psi_\gamma^{(+)}\rangle \langle\Psi_\gamma^{(+)} | \Psi_\beta^{(+)} - P\widetilde{\Phi}_\beta\rangle$$
$$+ \sum_m \langle\widetilde{\Phi}_\alpha | H - E_\alpha | \Psi_m\rangle \langle\Psi_m | \Psi_\beta^{(+)} - P\widetilde{\Phi}_\beta\rangle$$
$$= \langle\widetilde{\Phi}_\alpha | H - E_\alpha | P\widetilde{\Phi}_\beta\rangle - \sum_\gamma \frac{\widetilde{T}_{\alpha\gamma}\widetilde{T}^+_{\gamma\beta}}{E_\gamma - E_\beta - i0} - \sum_m \frac{\widetilde{T}_{\alpha m}\widetilde{T}^+_{m\beta}}{E_m - E_\beta}.$$

Comparing this with (17), we find

$$V_{\alpha\beta} = \langle\widetilde{\Phi}_\alpha | H - E_\alpha | P\widetilde{\Phi}_\beta\rangle. \qquad (18)$$

The Hermitian character of the potential that has been constructed can be seen directly from (18). By using (18) one can check by an elementary calculation that the Schrödinger equation with the potential V leads to the scattering amplitudes $\widetilde{T}_{\alpha\beta}$. In fact,

$$\sum_\gamma V_{\alpha\gamma} \langle\widetilde{\Phi}_\gamma | \Psi_\beta^{(+)}\rangle = \sum_\gamma \langle\widetilde{\Phi}_\alpha | H - E_\alpha | P\widetilde{\Phi}_\gamma\rangle \langle\widetilde{\Phi}_\gamma | \Psi_\beta^{(+)}\rangle$$
$$= \langle\widetilde{\Phi}_\alpha | H - E_\alpha | \Psi_\beta^{(+)}\rangle = (E_\beta - E_\alpha) \langle\widetilde{\Phi}_\alpha | \Psi_\beta^{(+)}\rangle.$$

Consequently, $\langle\widetilde{\Phi}_\alpha | \Psi_\beta^{(+)}\rangle$ is an eigenfunction of the Schrödinger equation in question, and to it there indeed corresponds the scattering amplitude $\widetilde{T}_{\alpha\beta}$.

In just as simple a way one also proves the important fact that the Schrödinger equation with the potential V will not contain any extra features: to each of its eigenfunctions there will correspond an eigenstate in the field theory for the same energy. Suppose, for example, $f_\beta(\alpha)$ is an eigenfunction of the Schrödinger equation for the energy E_β. Then

$$(H - E_\beta) \sum_\gamma f_\beta(\gamma) P\widetilde{\Phi}_\gamma = \sum_{\gamma\alpha} f_\beta(\gamma) P\widetilde{\Phi}_\alpha \langle\widetilde{\Phi}_\alpha | H - E_\beta | P\widetilde{\Phi}_\gamma\rangle$$
$$= \sum_\alpha P\widetilde{\Phi}_\alpha \sum_\gamma [(E_\alpha - E_\beta)\delta_{\alpha\gamma} + V_{\alpha\gamma}] f_\beta(\gamma) = 0.$$

Thus to $f_\beta(\gamma)$ there corresponds the state $\sum_\gamma f_\beta(\gamma) P\widetilde{\Phi}_\gamma$ in the field theory.

We can summarize the properties of the potential (18) in the form of the following theorem.

The Schrödinger equation

$$E_\alpha f_\gamma(\alpha) + \sum_\beta V_{\alpha\beta} f_\gamma(\beta) = E_\gamma f_\gamma(\alpha) \qquad (19)$$

with the potential $V_{\alpha\beta}$ defined by (18) has the following properties:

1) To each eigenstate $\Psi^{(+)}_{\Gamma=\gamma}$ or $\Psi_{M=m}$ of the Hamiltonian H of the quantum field theory there corresponds an eigenfunction of (19) for the same eigenvalue $E_{\gamma,m}$, determined by the rules:

$$f_\gamma^{(+)}(\alpha) = \delta_{\gamma\alpha} - \frac{1}{E_\alpha - E_\gamma - i0} \langle\widetilde{\Phi}_\alpha | H - E_\alpha | \Psi_\gamma^{(+)}\rangle,$$
$$f_m(\alpha) = \frac{1}{E_m - E_\alpha} \langle\widetilde{\Phi}_\alpha | H - E_\alpha | \Psi_m\rangle \qquad (20)$$

[the bound states m here are only those that were used in the scalar product (10)].

2) Conversely, to every eigenfunction $f_\gamma^{(+)}(\alpha)$ (with outgoing wave) or $f_m(\alpha)$ (for a bound state) there corresponds in the quantum field theory an eigenstate $\Psi_\gamma^{(+)}$ or Ψ_m, respectively, for the same eigenvalue, which is constructed according to the rules:

$$\Psi_\gamma^{(+)} = \sum_\alpha f_\gamma^{(+)}(\alpha) P\widetilde{\Phi}_\alpha, \qquad \Psi_m = \sum_\alpha f_m(\alpha) P\widetilde{\Phi}_\alpha. \quad (21)$$

3) In the range of energies of the colliding particles in which creation and annihilation processes are impossible, the scattering cross sections obtained from (19) and from quantum field theory are the same.

In conclusion we present an expression for the potential that is convenient for concrete calculations, and that follows directly from (18):

$$V_{\alpha\beta} = \{[1 + N]^{-1/2} \widetilde{V}(E_\alpha, E_\beta) [1 + N]^{-1/2}\}_{\alpha\beta},$$
$$[\widetilde{V}(E_\alpha, E_\beta)]_{\gamma_1\gamma_2} = \left\langle\Phi_{\gamma_1}\left| H - \frac{E_\alpha + E_\beta}{2} \right|\Phi_{\gamma_2}\right\rangle$$
$$+ \sum_{\Gamma\neq\gamma} \frac{T_{\gamma_1\Gamma} T^+_{\Gamma\gamma_2} (E_\Gamma - {}^1/_2 (E_\alpha + E_\beta))}{(E_{\gamma_1} - E_\Gamma - i0)(E_\Gamma - E_{\gamma_2} - i0)}. \qquad (22)$$

The formula (22) explicitly shows the Hermitian character of the potential, its lack of singularities, and also the fact that V actually does not depend on the states $\Psi_\gamma^{(+)}$ and Ψ_m.

3. DISCUSSION

The potential V that we have constructed is of course not the only potential that correctly describes the scattering of the particles and their bound states [in the sense of Eq. (21)]. In the derivation of V we have encountered a certain arbitrariness. In making the system P orthonormal, we could have used instead of (12) the vectors

$$\widetilde{\Phi}_\alpha = \sum_\beta U_{\beta\alpha} \widetilde{\Phi}_\beta, \qquad (23)$$

where U is an arbitrary unitary Lorentz-invariant operator, which can be written in the form $U = 1 + U_1$, and $U_{1\alpha\beta}$ is a nondiagonal matrix that goes to zero if we set $N = 0$ and has no singularity at $E_\alpha = E_\beta$ when $N_{\alpha\beta}$ does not have such a singularity. All the further arguments would have remained valid. Therefore one could also use as the potential the expression

$$V'_{\alpha\beta} = \sum_{\gamma_1 \gamma_2} U^+_{\alpha\gamma_1} \langle \widetilde{\Phi}_{\gamma_1} | H - E_\alpha | P \widetilde{\Phi}_{\gamma_2} \rangle U_{\gamma_2 \beta} \qquad (24)$$

(we note that this is not a unitary transformation of V).

The cause of this sort of ambiguity is concealed in the very statement of the problem. We have been looking for a potential that gives the same scattering and the same bound-state levels as in the field theory. In ordinary quantum mechanics, however, one can have Schrödinger equations with different potentials corresponding to the same scattering and bound-state spectrum. The wave functions for these potentials will differ at small distances between the particles, and therefore the mean values of physical quantities in a bound state will be different for different potentials, and in general will not agree with those found in quantum field theory. We indeed do not demand such agreement, however, since such a requirement is scarcely realizable in practice and seems extremely artificial from the theoretical point of view. Thus we get a whole set of potentials (24) that give equally good descriptions of the behavior of the particles within the framework of the problem as stated.

The potential (18) is distinguished from the others by its simplicity and convenience for calculations and by the absence of any arbitrary quantities not determined by the theory itself. For this reason, in our opinion, it is reasonable to adopt this particular equation (18) as the definition of the potential in quantum field theory.

It must be remarked that the arbitrariness discussed here bears no relation to the well known ambiguity in the expression for the fourth-order nucleon-nucleon potential. The reasons the latter ambiguity arises have been analyzed, for example, by Fukuda, Sawada, and Taketani.[5]

A few words about the spatial properties of the potential. Equation (18) gives the potential in the momentum representation. A Fourier transformation leads to the potential in coordinate space, which is an integral operator with the kernel $V(\mathbf{r}_1 - \mathbf{r}_2, \mathbf{r}'_1 - \mathbf{r}'_2)$, where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the radii vectors of the two particles; that is, we arrive at a nonlocal interaction in the configuration space. As is well known, one cannot get a strictly local potential, if one requires that it describe the scattering correctly. In fact, the transition amplitude obtained from a local potential is analytic in the upper half of the energy plane and on the real axis; the same is true of the amplitude in quantum field theory. If we require that these amplitudes be exactly equal on some segment of the real axis of the energy plane (for which there is no production of new particles), then because they are analytic they will agree also on the entire real axis. This cannot be true, however, since at high energies new particles can be produced. Consequently, the idea of a strictly local potential must be rejected. We shall not concern ourselves here with the question of the approximately local character of the potential in individual cases.

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