Variant of the theory	Number of prongs		
	2	4	6
Without the π-π interaction	0.98±0.12	0.99±0.16	2±1.14
With the π-π interaction in Dyson's variant	1.21±0.15	0.83±0.13	0.49±0.28
With the π-π interaction in Takeda's variant	1.38±0.17	0.71±0.11	0.64±0.36

for two, four, and six-prong stars, the latter calculated for three variants of the theory. (The indicated statistical experimental error is $\Delta_n = \pm\sqrt{N_n}$ where N_n is the number of n-pronged stars.)

It can be seen that the results of calculations without account of the π-π interaction agree well with experiment. Inclusion of this resonance π-π interaction, especially with Takeda's variant, worsens this agreement. The disagreement between the theoretical and experimental values for stars with a small number of prongs is a characteristic feature of the calculations which take account of the resonance π-π interaction, not only at E = 5 Bev, but also, at other energies.

The proportion of charged strange particles produced in inelastic π^--p collisions constitutes 8.6% for the theory which neglects the π-π interaction (5.5% from K^+ and 0.3% from K^- mesons) and 6.4% and 5.7% for the variants of Dyson and Takeda. Of the 110 inelastic stars in the experiment, in only four cases (i.e., in 3.5% of all cases) were strange particles produced. However, it is not possible to differentiate between the three theoretical variants on this basis, as was proposed in reference 10, because stars in which strange particles are produced, but do not decay in the chamber, may be included in the remaining 106 stars. Considering the lack of statistics of stars with strange particles, one would expect such cases to be very probable.

Thus, available experimental data can, within the limits of experimental error, be explained without employing the hypothesis of resonance π-π interaction. Further assumptions would be necessary to bring the statistical theory, with this interaction, into agreement with experiment.

*If one is interested only in the production of ordinary particles, then all reactions with strange particles can simply be discarded (i.e., set $r_K = 0$). Such a simplification has little effect on the results obtained for pions and nucleons since the proportion of strange particles produced is small.

†Inelastic π^--p scattering at 5 Bev with account of the resonance π-π interaction was considered by Rus'kin.[10] However, only part of the possible inelastic reaction channels were included here. Thus, for reactions with strange particles, the neglected channels have approximately the same statistical weight as the reactions taken into account by Rus'kin. If these channels are included, then the ratio of the cross section for the production of strange particles to the cross section for production of the observed π mesons exceeds that indicated by Rus'kin by a factor of more than two and is several times larger than the experimental value. The agreement with experiment for the distribution of stars with number of prongs is correspondingly worsened. This well-known result (see reference 9) indicates that K mesons should be taken into account differently than π mesons in the statistical theory.

We are grateful to V. I. Rus'kin for discussion of the comparison of our numerical results with his calculations.

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ON THE DETERMINATION OF NUCLEAR DEFORMATION FROM THE ALPHA-DECAY FINE STRUCTURE

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WE have shown earlier that by studying the α-decay fine structure it is possible to determine the form of the surface of the daughter nucleus.[1] A theory was developed for the decay of even-even nuclei, by which unique analytic expressions, valid in all space, can be obtained for the wave functions of the α particle. The resultant formula for the relative probability of excitation of the rotational level of a daughter nucleus with momentum J is found to be

$$w_J = (2J+1)\, e^{-\gamma J(J+1)} \frac{\left| \int_{-1}^{1} \chi(\mu)\, e^{\beta P_2(\mu)} P_J(\mu)\, d\mu \right|^2}{\left| \int_{-1}^{1} \chi(\mu)\, e^{\beta P_2(\mu)}\, d\mu \right|^2}, \quad (1)$$

where $\chi(\mu)$ is the wave function of the alpha particle on the surface of the nucleus, described by the equation $R(\mu) = R_0\{1 + \alpha_2 P_2(\mu)\}$; $\mu = \cos\vartheta$; the parameters γ and β/α_2 depend in a definite manner on the nuclear radius R_0 (we assume in the computations $R_0 = 1.4 A^{1/3} \times 10^{-13}$ cm), the atomic number Z, the decay energy, and moment of inertia I of the daughter nucleus (it is also possible to take the right half of (1) to mean integration over some other weakly-nonspherical surface, but χ, γ, and β must then be taken to mean the corresponding quantities that pertain to that surface). Using Eq. (1) and assuming χ = const, we determined the quadrupole deformations α_2 for 22 even-even daughter nuclei from the experimentally-observed probability w_2 of the excitation of the 2^+ level.

This problem was considered anew by Strutinskiĭ[2] under the same physical assumptions, but by a different method, which led to poorly-converging series. The wave function was therefore determined by him only inside a spherical surface S′ of radius $R' = 2Ze^2/E - R_0$. He then used the formula

$$w_J = (2J+1) \left| \int_{-1}^{1} \chi'(\mu)\, P_J(\mu)\, d\mu \right|^2 \Big/ \left| \int_{-1}^{1} \chi'(\mu)\, d\mu \right|^2, \quad (2)$$

where $\chi'(\mu)$ is the wave function on the surface S′. The results of references 1 (α_2^N) and 2 (α_2^S) are compared in the table. The values α_2^S deviate systematically from α_2^N, and the deviation increases with increasing $\Delta E = 3\hbar^2/I$.

Formula (2) is equivalent to stating that the penetration of the part of the barrier located outside the surface S′ is independent of the momentum of the alpha particle. To verify (2), we apply (1) to the surface S′ ($\beta = 0$). Identifying the corresponding values of w_J and γ by primes, and comparing with (2), we obtain

$$w'_J = e^{-\gamma' J(J+1)}\, w_J. \quad (3)$$

Thus in reference 2 the deformation agrees actually not with the experimentally-observed w_J, but with some quantity w'_J which has no physical meaning. Formula (3) can also be obtained by ordinary unidimensional quasi-classical methods, since the field is practically centrally-symmetrical in the region $r > R'$.

To estimate the role of the factor $\exp\{-\gamma' J \times (J+1)\}$, which describes the dependence of the penetration on the momentum of the alpha particle, we put J = 2 in (3); the values $e^{-6\gamma'}$ listed in the table deviate substantially from unity, indicating that the deformations obtained by Strutinskiĭ are in error.

To verify the foregoing considerations, let us insert w'_2 in the left half of (1) and calculate the corresponding deformation α'_2. Inasmuch as w'_2 is that value of the probability (not equal to the experimental one) with which the deformation in reference 2 actually agrees, α'_2 should be equal to α_2^S. As seen from the table, this is indeed so (certain discrepancies apparently do not exceed the accuracy limits of Strutinskiĭ's calculations).

The present calculation thus confirms the correctness of the deformations computed in reference 1. We note that in calculating α_2^N in accordance with (1) the error is very small. Account of all the first-order correction in the expansion in powers of the parameters $1/\kappa R_0$, α_2, and $J(J+1)/\kappa^2 R_0^2$ (κ is the wave number of the alpha particle on the surface of the nucleus) corrects the deformation by not more than 3%. Therefore

Daughter nucleus	ΔE kev	α_2^N	α_2^S	$e^{-6\gamma'}$	α'_2	Daughter nucleus	ΔE kev	α_2^N	α_2^S	$e^{-6\gamma'}$	α'_2
Em^{220}	240	0.21	0.07	0.17	0.08	U^{230}	50	0.11	0.08	0.64	0.08
Em^{222}	186	0.23	0.08	0.20	0.09	U^{232}	45	0.13	0.10	0.65	0.10
Ra^{222}	109	0.20	0.13	0.44	0.12	U^{234}	46.8	0.16	0.14	0.63	0.13
Ra^{224}	84	0.24	0.17	0.48	0.15	U^{236}	45	0.15	0.13	0.63	0.12
Ra^{226}	67.2	0.21	0.15	0.51	0.13	U^{238}	43.8	0.14	0.12	0.63	0.11
Ra^{228}	60	0.24	0.17	0.51	0.15	Pu^{238}	43.8	0.15	0.12	0.66	0.12
Th^{226}	69.6	0.22	0.17	0.55	0.15	Pu^{240}	43.2	0.15	0.12	0.65	0.12
Th^{228}	60	0.21	0.18	0.57	0.16	Cm^{242}	42	0.13	0.10	0.68	0.10
Th^{230}	52.2	0.17	0.14	0.58	0.12	Cm^{246}	37.2	0.11	0.07	0.69	0.07
Th^{232}	50	0.19	0.16	0.58	0.14	Cm^{248}	37.2	0.10	0.07	0.69	0.07
Th^{234}	50	0.17	0.14	0.57	0.13	Cf^{250}	42	0.10	0.08	0.68	0.08

the only sufficient reason for changing α_2^N would be an improvement in the experimental data, to which, in particular, the review of Perlman and Rasmussen is devoted.[3]

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THE NON-ADDITIVITY OF LONDON-VAN DER WAALS FORCES

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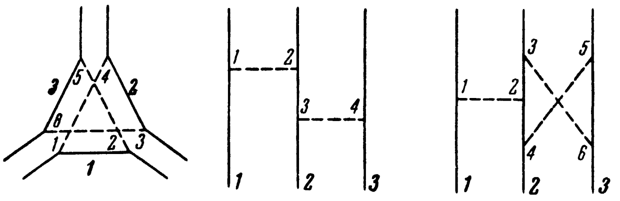
THE so-called dispersive interaction forces between neutral atoms are not additive (notwithstanding statements which are sometimes made); the additivity occurs only in the first non-vanishing perturbation theory approximation. We consider here the next terms (of third order in the coupling constant) of the perturbation series and we shall obtain an expression for the energy of the dispersive interaction of three hydrogen atoms, in its dependence on the interatomic distances R_1, R_2, R_3 as parameters and including the retardation effect.

For this purpose it is advantageous to use the Feynman-Dyson technique, as done by Dzyaloshinskiĭ[1] for the interaction of two atoms. We are interested in processes of the kind of Fig. 1a leading to a contribution



$$S^{(6)} = -\frac{4}{3}\pi^3 \int dx_1 \ldots dx_6 \, D^F(1,4)\, D^F(3,6)\, D^F(2,5) \times P[j_\delta^{(3)}(5)\, j_\nu^{(3)}(6)]\, P[j_\nu^{(2)}(3)\, j_\mu^{(2)}(4)]\, P[j_\mu^{(1)}(1)\, j_\delta^{(1)}(2)]. \quad (1)$$

To derive this expression we eliminated the variables of the intermediate electromagnetic field, used the usual, non-Heaviside system of units [$\hbar = c = 1$ up to Eq. (5)], and took into account that, for instance, for an N-particle interaction there are in all $2^{N-1}(N-1)!$ different processes of order e^{2N} corresponding to connected diagrams of the kind shown in Fig. 1a. Generally speaking we should have started our consideration with processes of the type of Fig. 1b, leading to a non-additive correction of the second order (in the coupling constant), but in the approximation chosen by us (we are only interested in the dipole interaction) all matrix elements of $S^{(4)}$ containing "propagator functions" of three interacting atoms give zero for the ground state, since two out of the three "propagator lines" are involved in only one vertex. One must drop processes depicted in Fig. 1c for similar considerations.

In the first non-vanishing order, in which the interaction energy depends simultaneously on the coordinates of all three atoms, we therefore get precisely Eq. (1). Performing in this equation the integration over all time coordinates we find:

$$U = \frac{-i}{24\pi^7} \int d^3r_1 \ldots d^3r_6 \int d^3p_1 d^3p_2 d^3p_3 \exp\{i\mathbf{p}_1(\mathbf{r}_1 - \mathbf{r}_4) + i\mathbf{p}_2(\mathbf{r}_3 - \mathbf{r}_6) + i\mathbf{p}_3(\mathbf{r}_5 - \mathbf{r}_2)\} \int_{-\infty}^{\infty} \frac{d\omega}{(p_1^2 - \omega^2)(p_2^2 - \omega^2)(p_3^2 - \omega^2)}$$
$$\times \sum_{k,m,n} \left[\frac{\langle 0 | j_\delta^{(3)}(r_5) | k\rangle \langle k | j_\nu^{(3)}(r_6) | 0\rangle}{\omega_{k0} - \omega} + \frac{\langle 0 | j_\nu^{(3)}(r_6) | k\rangle \langle k | j_\delta^{(3)}(r_5) | 0\rangle}{\omega_{k0} + \omega} \right]$$
$$\times \left[\frac{\langle 0 | j_\nu^{(2)}(r_3) | m\rangle \langle m | j_\mu^{(2)}(r_4) | 0\rangle}{\omega_{m0} - \omega} + \frac{\langle 0 | j_\mu^{(2)}(r_4) | m\rangle \langle m | j_\nu^{(2)}(r_3) | 0\rangle}{\omega_{m0} + \omega} \right]$$
$$\times \left[\frac{\langle 0 | j_\mu^{(1)}(r_1)\, n\rangle \langle n | j_\delta^{(1)}(r_2) | 0\rangle}{\omega_{n0} - \omega} + \frac{\langle 0 | j_\delta^{(1)}(r_2) | n\rangle \langle n | j_\mu^{(1)}(r_1) | 0\rangle}{\omega_{n0} + \omega} \right]. \quad (2)$$

In the non-relativistic approximation we can put

$$\langle l | j_\mu(r_1) | m\rangle \langle m | j_\mu(r_2) | l\rangle = -e^2 [\psi_l^*(r_1)\, \psi_m(r_1) - \delta(r_1)\, \delta_{lm}]\, [\psi_m^*(r_2)\, \psi_l(r_2) - \delta(r_2)\, \delta_{ml}], \quad (3)$$

and so on. If we now introduce the polarizability

$$\alpha(\omega) = \sum_n 2\omega_{n0} |d_{0n}|^2 / (\omega_{n0}^2 - \omega^2), \quad (4)$$