where d_{0n} is the dipole moment matrix element, and perform the necessary integration in (2), we get finally after elementary, but cumbersome computations

$$U(R_1, R_2, R_3) = -132\,\hbar c \alpha_1(0) \times \alpha_2(0)\,\alpha_3(0) / \pi R_1 R_2 R_3 (R_1 + R_2 + R_3)^7, \qquad (5)$$

and this formula is valid under the assumption that the distances between the atoms are much larger than the characteristic wave length λ_0 in the spectrum of the atom (R_1, R_2, $R_3 \gg \lambda_0$ so that exchange forces play no role whatever; as was already stated, we neglect the effects of higher multipoles).

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NUMERICAL SOLUTION OF STATIC DISPERSION RELATIONS OF THE PHOTOPRODUCTION P-WAVE

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WE found an exact numerical solution of the static dispersion relations obtained in references 1 and 2 for the photoproduction P-amplitudes. We used the method, proposed by Omnes[3] and based on the work of Muskhelishvili,[4] of reducing the linear singular integral equations to regular Fredholm equations. The procedure (which is not unique in the case of scattering), for the transition from the singular to the regular equations is found to be unique in this case under the following conditions: 1) the scattering phase shifts vanish at the threshold and at infinity, 2) the solution of the regular equation is bounded and has the same value at infinity as the solution of the singular equation.

The values used for the phase shifts were obtained from the Chew-Low static equations (at $f^2 = 0.08$ and a cutoff parameter $P = 7$), the solutions of which were obtained by the Salzmans[5] and repeated by Tentyukova on the "Strela" computer. The regular photoproduction equations were solved by successive approximation on the "Ural" computer of the Joint Institute for Nuclear Research.

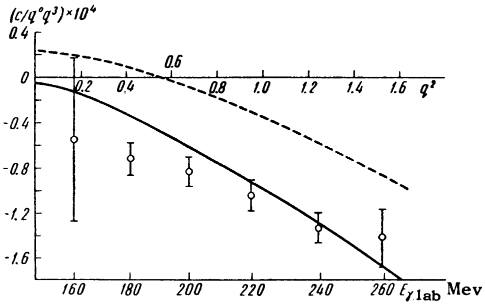
The exact solutions for the quadrupole amplitudes and the e-parts of the magnetic dipole amplitudes behave qualitatively like the corresponding Bohr terms multiplied by $\cos\delta$.[1] The μ-parts of the magnetic dipole amplitudes (including the isotope-scalar amplitudes) behave like $\sim q^{-3}\sin\delta$. This means that the meson created upon interaction of a photon with the static magnetic moment of the nucleon always experiences secondary scattering.

It is shown further that the electric dipole amplitude is independent in the static approximation of the magnetic moments. This follows from the supplementary condition and from the dispersion relations for the longitudinal amplitudes, obtained in reference 6.

As a first attempt at comparing the complete expression for the photoproduction amplitude with experiment, with allowance for the obtained corrections, we calculated the coefficient C in the photoproduction cross section of π^0 mesons at threshold:

$$d\sigma(\gamma p \to \pi^0 p)/d\Omega = A + B\cos\theta + C\cos^2\theta.$$

In the figure, C is given in $(\hbar/\mu_0 c)^2$ units, q is the meson momentum in the c.m.s. and in units of $\mu_0 c^2$, and $q^0 = \sqrt{1-q^2}$. The solid curve corresponds to the exact solution, while the dotted one corresponds to the approximate solution obtained in reference 1; the experimental points for 160 — 240 Mev are taken from reference 7, while those for 260 Mev are from reference 8. It is seen that



the exact solution leads to fair agreement with experiment near threshold. This agreement becomes somewhat worse for large energy, which can be attributed to relativistic effects. The approximate solution of reference 1 is in poor agreement with experiment, as noted by Baldin and Govorkov (private communication).

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ON THE QUESTION OF CRITICAL VELOCITIES FOR FLOW OF He II *IN CAPILLARIES*

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As is known, Onsager's[1] and Feynman's[2] ideas about vortex lines yield the right order of magnitude of critical velocities for superfluid helium rotating in a cylinder and for flow from a narrow capillary into a large beaker. In the former case the vortex lines are straight and parallel to the cylinder axis, while in the latter they are rings formed in the beaker near to the junction with the capillary.[2]

It will be shown below that similar values of the critical velocities can be calculated for the flow of helium through a long capillary. It is natural to suppose that the vortex lines will, in this case, be closed curves lying in planes perpendicular to the capillary axis. The shape of the lines will be determined by the capillary cross section, i.e., for a circular cross section the lines will be circular and for a rectangular section the lines will form closed curves nearly rectangular in shape. The angular momentum associated with such lines is evidently zero, while the linear momentum is non-zero and is directed parallel to the capillary axis, i.e., parallel to the flow velocity v. According to Landau[3] the change in energy, ΔE, of flowing helium (in a coordinate system fixed with respect to the capillary walls), associated with the formation of a vortex line, is $\Delta E = E_V - p_V v$ (E_V and p_V are the energy and momentum of a vortex line). A vortex line can be formed if $\Delta E < 0$. As superfluidity disappears when a vortex line appears, the critical velocity v_k is determined by the condition $\Delta E = 0$, i.e., $v_k = E_V/p_V$.

The momentum p_V of a narrow vortex line is given by[4] $p_V = \kappa\rho \int dF_n$, where κ is the circulation of velocity along a contour enclosing the line and ρ is the density and the integration is over a surface bounded by the vortex contour l. In calculating the line energy we shall assume that the vortex line is sufficiently far from the walls for surface effects to be neglected. Then[4]

$$E_v = \frac{\rho}{8\pi}\int \frac{(\text{curl } \mathbf{v}(\mathbf{r}), \text{curl } \mathbf{v}(\mathbf{r}'))}{|\mathbf{r}-\mathbf{r}'|}\, d\mathbf{r}\, d\mathbf{r}'.$$

Since p_V is proportional to the square of the linear dimensions of the line and E_V is directly proportional to it, the minimum ΔE corresponds to the maximum line length, coinciding with the transverse dimensions (for a rectangular cross section it is therefore not energetically profitable for circular vortices to be formed instead of rectangular vortices). If, in fact, the line is near the walls, Eq. (1) for E_V is inexact, but it is sufficient for calculating v_k.

According to Feynman,[2] the circulation κ is quantized: $\kappa = 2\pi n_s \hbar/m$, where $n_s = 1, 2, \ldots\ldots$ The smallest values of energy, E_V and of $|\Delta E|$ correspond to $n_s = 1$. By calculating the line energy and momentum we obtain for a circular cross section of radius r

$$v_k = (\hbar/mr)(\ln(r/d) + \ln 16 - 7/4)$$

(d is the diameter of the line cross section, $d \ll r$).

For a rectangular cross section

$$v_k = \frac{\hbar}{m}\left\{\frac{1}{b}\left[\ln\frac{4a(\sqrt{a^2+b^2}-a)}{bd} - \frac{7}{4}\right] + \frac{1}{a}\left[\ln\frac{4b(\sqrt{a^2+b^2}-b)}{ad} - \frac{7}{4}\right] + 2\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}\right\};$$

and for $b \ll a$

$$v_k = (\hbar/mb)\left[\ln(2b/d) + \frac{1}{4}\right].$$