ELECTRON RELAXATION TIME IN A HIGH FREQUENCY ELECTROMAGNETIC FIELD AND THE SURFACE IMPEDANCE OF A METAL

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Submitted to JETP editor August 12, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 524-528 (February, 1960)

The effect of quantization of the electromagnetic field and electron orbits in a constant magnetic field on the relaxation time of electrons in a metal due to electron-phonon interactions is studied; the effect on the surface impedance of the metal is also considered. The complete frequency region (normal skin effect, anomalous skin effect, infrared region) up to the internal photoeffect limit has been investigated. It is shown that the quantization of the orbits is significant only in the anomalous skin effect region for $\omega \sim \Omega \gtrsim kT/\hbar$ (Ω is the cyclotron frequency, T = temperature). Quantization of the electromagnetic field is always important in the infrared region and for the anomalous skin effect in a constant magnetic field (throughout the whole magnetic field region for $\omega \sim \Omega$ and only for cyclotron resonance when $\omega \gg \Omega$).

THE surface impedance of a metal in a variable electromagnetic field either in the absence or in the presence of a constant magnetic field has been investigated in a number of researches (see, for example, references 1 — 3). In almost all researches devoted to this problem, the classical collision integral for electrons and phonons has been employed.

At the same time, quantum effects can have a strong effect on the relaxation time of the electron gas in a metal, thus changing the dependence of the surface impedance of the metal on frequency, temperature and magnetic field (see, for example, the researches of one of the authors[4]). These effects become important when the characteristic energy entering into the problem is of the order of or greater than kT (k is Boltzmann's constant, and T the temperature). Such an energy can arise for several reasons.

In the first place, as a consequence of the quantization of the levels of the electron in a constant magnetic field **H**, the distance between the levels $\Delta\epsilon = \hbar\Omega$ (Ω is the frequency of revolution of the electron, see reference 5) becomes of the order of kT at helium temperatures in a field $H \sim 10^4$ oe.

In the second place, owing to the quantization of the energy of the electromagnetic field, the value of the quantum $\hbar\omega$ is already of the order of kT (ω is the frequency) at helium temperatures for centimeter waves.

In the third place, when the energy is associated with the spatial inhomogeneity of the distribution of electrons in the metal (spatial dispersion) $v_z\Delta p \sim v_z\hbar/\Delta$ (the z axis is directed along the normal to the surface of the metal, v is the velocity of the electron, **p** its momentum, and Δ a depth, characterizing the spatial inhomogeneity). In different cases, Δ has different forms; however, if we take $\Delta \sim 10^{-5}$ cm and $v_z \sim v$, then $\hbar v_z/\Delta \sim kT$ at $T = 100°$.

We shall now consider under what conditions each of the characteristic energies appears.

1. EFFECT OF QUANTUM PHENOMENA ON THE FREQUENCY OF ELECTRON-PHONON COLLISIONS

The results given below were obtained by means of a rigorous analysis of the corresponding equations, which we shall not write down because of their comparatively complicated form; we shall instead limit ourselves to qualitative considerations.

As is well known, in the consideration of the skin effect in metals, the following three important regions are usually distinguished: the region of the normal skin effect ($l \ll \delta$, where l is the mean free path, and δ the skin depth), the region of the anomalous skin effect ($l \gg \delta$), and the infrared region ($v/\omega \ll \delta$).

We first investigate the role of quantum phenomena in the region of the normal skin effect and in the infrared region since the analysis is relatively simple in these cases.

a) It is easy to prove that quantum phenomena are not important in the case of the normal skin effect. This is connected with the fact that long before any of the characteristic energies exceeds kT, one has reached the region of the anomalous skin effect.*

b) In the infrared region of the spectrum, as had been noted previously, the inequality $v/\omega \ll \delta$ is satisfied (in the case of a quadratic law of dispersion of the electrons, $\delta^{-2} = 4\pi ne^2/mc^2$, where n n is the density of the electrons and m is the electronic mass). Since in the given case $\Delta \approx \delta$, it follows from this inequality that $v\hbar/\Delta \ll \hbar\omega$. It can be shown that the effect is nonetheless important, since the energy connected with the spatial inhomogeneity frequently exceeds the energy of the phonon which can be absorbed or emitted by the electron; actually, the usual condition is $v\hbar/\delta \gtrsim k\Theta$ (Θ is the Debye temperature of the material, which characterizes the upper boundary of the phonon spectrum). However, as a quantitative analysis shows, the only determining quantity in the given case is the ratio $v/\omega\delta$; therefore, account of spatial dispersion has no effect on the time of free flight.

It is easy to establish the fact that quantization of the electron levels in a magnetic field in the infrared region is not important. Actually, at practically obtainable fields, the inequalities $\hbar\Omega \ll \hbar\omega$ and $\hbar\Omega \ll k\Theta$ are satisfied; therefore, the energy of the phonons with which the electrons interact is large in comparison with $\hbar\Omega$ (independent of the relations between the quantities kT, $k\Theta$, and $\hbar\omega$).

Account of the quantization of the electromagnetic field in the infrared region was taken earlier (see reference 4).

c) In the case of the anomalous skin effect, the fundamental contribution to the surface impedance in the classical consideration is made, as is well known, by the small fraction of the electrons which move almost parallel to the surface of the metal (see, for example, reference 6), for which $v_z/v \lesssim (\delta/l)^{1/3}$. For these electrons, the quantity $v_z\hbar/\Delta$, even if it exceeds kT, is still small, which leads to a mean free path which is large in comparison with the length l, determined, say, by the collisions with atomic impurities. For the remaining electrons, spatial dispersion leads to a significant decrease in the mean free path (it is shown that $l \sim v_z^{-3}$) and, consequently, their contribution to the impedance increases. However, as calculations show, the "slipping" electrons still play the principal role. The considerations given here evidently remain valid even in the presence of a magnetic field.*

The situation is different in the consideration of quantum effects brought about by the quantization of the electromagnetic field and electronic orbits. In the anomalous skin effect, in the absence of a constant magnetic field, the time of free flight, as is well known, generally drops out of the expression for the surface impedance (see reference 7).† However, in the presence of a constant magnetic field parallel to the boundary of the metal, the surface impedance depends essentially on τ, and the quantum effect connected with $\hbar\omega$ and $\hbar\Omega$ must generally be taken into account.

In the present work, we limit ourselves to the case $\Omega \ll \omega$; consideration of the region $\Omega \gtrsim \omega$, where it is necessary to take into account both types of quantization, will be the subject of another article.

2. RELAXATION TIME FOR CYCLOTRON RESONANCE IN METALS

The anomalous skin effect in the presence of a constant magnetic field parallel to the boundary of the metal was considered by one of the authors and Kaner.[6,8] In these researches, an expression was obtained for the surface impedance in an arbitrary magnetic field. However, if one is not speaking of the rather uninteresting case of square law dispersion of electrons, then the simple formulas for frequency and temperature dependence hold in three cases: weak magnetic fields (when the radius of revolution of the electron in the magnetic field $r \gg l^2/\delta$), strong fields ($r \ll l$) and, finally, resonance frequency ($\omega \approx \Omega n$; n = 1, 2, ...).

In the case of weak magnetic fields, the surface impedance depends upon the relaxation time τ only in the combination $l^* = l/(1+i\omega\tau)$ (see ref-

*In the region of the normal skin effect, $\hbar\Omega \sim kT$ at practically unattainable fields, $H \sim 10^6$ oe. Here we are dealing with the fundamental electron bands; the case of anomalously small bands requires special consideration (which can lead to the appearance of quantum oscillations of the impedance with the magnetic field).

*The effect of spatial inhomogeneity on the frequency of electron-phonon collisions is possibly important in the consideration of the fluctuations of the electron density in a metal. Actually, in this case, the spatial inhomogeneity of the distribution of electrons is important, and at the same time electrons with all velocity directions play a role.

†We note that in the quantum consideration ($\hbar\omega \gg kT$) this result is valid not with accuracy up to terms of higher order in δ/l (as was the case in the classical approximation), but with accuracy up to terms of higher order in $1/\omega\tau(\omega) \sim \omega^2$ [see Eq. (3) of the present article]. However, these terms become important in a frequency region of little interest — for $\lambda = 2\pi c/\omega \lesssim 10^{-2}$ cm.

erence 8). On the other hand, for $\hbar\omega \gg kT$, $\omega\tau(\omega) \gg 1$ always [this follows from Eq. (3) of the present article] and consequently τ drops out in the first approximation.

In the case of strong fields, i.e., for $r \ll l$ or, what is the same thing, for $\Omega\tau \gg 1$ (and simultaneously, $\omega\tau \gg 1$), as is easily seen, τ also does not enter into the final formula for the surface impedance.

Finally, we come to the consideration of the resonance region. Here the surface impedance depends essentially on the relaxation time. Quantization in the magnetic field has not been considered in the present research, the results obtained being applicable only to harmonics of the cyclotron resonance ($\omega \approx 2\Omega, 3\Omega, \ldots$), where $\Omega < \omega$.

In the region of frequencies of the electromagnetic field that is of most interest, $\hbar\omega \ll k\Theta$ (this inequality is equivalent to $\lambda \gg 1/\Theta$, where λ is in centimeters, and Θ is in degrees), so that in what follows we shall assume

$$k\Theta \gg \hbar\omega \gg kT. \tag{1}$$

An expression was obtained in reference 9 for the quantum collision integral of electrons with phonons. If we write the collision integral in the form

$$\left(\frac{\partial}{\partial t} f_1(\mathbf{p})\right)_{\text{col}} = f_1(\mathbf{p}) \int d\mathbf{p}' K(\mathbf{p}, \mathbf{p}') + \int d\mathbf{p}' f_1(\mathbf{p}') Q(\mathbf{p}, \mathbf{p}')$$

(here $f_1(\mathbf{p})$ is a nonequilibrium addition to the electron distribution function, $\mathbf{p}$ is the momentum of the electron) then under the conditions of the anomalous skin effect the second term on the right hand side is small; consequently the time of free flight of the electron $\tau(\mathbf{p})$ can be introduced (see reference 6);

$$\left(\frac{\partial}{\partial t} f_1(\mathbf{p})\right)_{\text{col}} \approx \frac{1}{\tau(\mathbf{p})} f_1(\mathbf{p}), \qquad \frac{1}{\tau(\mathbf{p})} = \int d\mathbf{p}' K(\mathbf{p}, \mathbf{p}').$$

In finding the surface impedance, one must calculate integrals of the form

$$\int_0^\infty d\varepsilon \frac{\partial f_0}{\partial \varepsilon} \psi\left(\frac{1}{\tau(\mathbf{p})}\right),$$

where $f_0(\epsilon)$ is the Fermi electron distribution function, $\epsilon(\mathbf{p})$ is the energy of the electron, ψ is some function of $1/\tau$ (see reference 6).

Since the function $\psi(1/\tau)$ generally has a complicated form, the calculation of similar integrals in the general case meets with difficulties (in particular, such is the case in the classical consideration). However, if the inequalities (1) are satisfied, then, as is not difficult to show, $1/\tau(\mathbf{p})$ is a relatively slowly changing function which varies appreciably when the energy changes by an amount of the order of $\hbar\omega$. It is evident that the integral under consideration in this case is simply equal to $-\psi[1/\tau(\mathbf{p}_0)]$, where $\mathbf{p}_0$ is the momentum of the electron on the Fermi surface $\epsilon(\mathbf{p}_0) = \epsilon_0$ (ϵ_0 is the limiting energy).

Starting out from the results of reference 9, it is not difficult to prove that

$$\frac{1}{\tau(\mathbf{p}_0)} = C \int d\mathbf{q} \frac{q^2}{h\nu(\mathbf{q})} \delta(\varepsilon(\mathbf{p}+\mathbf{q}) - \varepsilon_0) \left\{\coth\frac{h\nu(\mathbf{q})}{2kT} - \frac{1}{2}\left[\tanh\frac{h\nu(\mathbf{q}) + \hbar\omega}{2kT} + \tanh\frac{h\nu(\mathbf{q}) - \hbar\omega}{2kT}\right]\right\}, \tag{2}$$

where the integration is carried out over all possible momenta of the phonons $\mathbf{q}$; $h\nu(\mathbf{q})$ is the energy of the phonon, C is some constant.

As a consequence of (1), the principal contribution to the integral (2) comes from the region of low phonon energy; $h\nu(\mathbf{q}) \lesssim \hbar\omega \ll k\Theta$. In this region, the dispersion law of the phonons can be assumed to be linear (but, in general, anisotropic); $h\nu(\mathbf{q}) = qu(\mathbf{n})$, where $\mathbf{n}$ is the unit vector in the direction of $\mathbf{q}$.

Simple calculations lead to the following results:

$$1/\tau(\mathbf{p}_0) = \tfrac{1}{3}(\hbar\omega/k\Theta)^3/\tau_0(\mathbf{p}_0),$$
$$\frac{1}{\tau_0(\mathbf{p}_0)} = C(k\Theta)^3 \int d\Omega_{\mathbf{n}} \delta[(\mathbf{v}(\mathbf{p}_0), \mathbf{n})]/[u(\mathbf{n})]^4, \tag{3}$$

where $d\Omega_{\mathbf{n}}$ is the element of solid angle in $\mathbf{q}$-space.

Choosing a spherical system of coordinates with the z axis along the vector $\mathbf{v}(\mathbf{p}_0)$, we find

$$\frac{1}{\tau(\mathbf{p}_0)} = C\frac{(k\Theta)^3}{|\mathbf{v}(\mathbf{p}_0)|} \int_0^{2\pi} \frac{d\varphi}{[u(\varphi)]^4},$$

where $u(\varphi) \equiv u(\theta = \pi/2, \varphi)$.

We note that τ_0 coincides in order of magnitude with the classical high temperature time of free flight, taken at the Debye temperature τ_0^{cl} ($\tau^{\text{cl}}(T) = (\Theta/T)\tau_0^{\text{cl}}$ for $T \gg \Theta$), in particular for a square law dispersion, $\tau_0 = 3\tau_0^{\text{cl}}$.

Now the expression (3) that has been obtained should be substituted in the corresponding equation of reference 6 for resonant values of the real R and imaginary X parts of the surface impedance.*

The frequency and temperature dependence of the quantities R_{res}, X_{res}, $(X/R)_{\text{res}}$ have been given, as well as the corresponding frequencies of revolution Ω_{res} in the quantum and classical cases. These quantities also depend on the number of harmonics n; however this dependence is not connected with the form of τ, and therefore remains as before.

*In these equations, the quantity $1/\tau(\mathbf{p}_0)$ appears, averaged over the trajectory of the electron in the magnetic field.

1) Square-law dispersion:

$R_{\text{res}}^{\text{qu}} \sim \omega^2, \quad \Delta^{\text{qu}} \sim \omega^2; \quad R_{\text{res}}^{\text{cl}} \sim T^2, \quad \Delta^{\text{cl}} \sim \omega^{1/2} T^{3/2};$
$X_{\text{res}}^{\text{qu}} \sim \omega^{4/3}, \quad \Delta^{\text{qu}} \sim \omega^3; \quad X_{\text{res}}^{\text{cl}} \sim \omega^{1/3} T, \quad \Delta^{\text{cl}} \sim T^3;$
$(X/R)_{\text{res}}^{\text{qu}} \sim \omega^{-1}, \quad \Delta^{\text{qu}} \sim \omega^2; \quad (X/R)_{\text{res}}^{\text{cl}} \sim \omega^{1/2} T^{-3/2},$
$\Delta^{\text{cl}} \sim \omega^{1/2} T^{3/2}; \quad \Delta = |\omega - n\Omega_{\text{res}}|.$

2) Ω has a maximum:

$R_{\text{res}}^{\text{qu}} = X_{\text{res}}^{\text{qu}} \sim \omega, \quad \Delta^{\text{qu}} \sim \omega^3; \quad R_{\text{res}}^{\text{cl}} = X_{\text{res}}^{\text{cl}} \sim \omega^{1/2} T^{1/2},$
$\Delta^{\text{cl}} \sim T^3.$

3) Ω has a minimum:

$R_{\text{res}}^{\text{qu}} \sim \omega^{14/9}, \quad \Delta^{\text{qu}} \sim \omega^{7/3}; \quad R_{\text{res}}^{\text{cl}} \sim \omega^{2/9} T^{4/3}, \quad \Delta^{\text{cl}} \sim \omega^{1/3} T^2;$
$X_{\text{res}}^{\text{qu}} \sim \omega, \quad \Delta^{\text{qu}} \sim \omega^3; \quad X_{\text{res}}^{\text{cl}} \sim \omega^{1/2} T^{1/2}, \quad \Delta^{\text{cl}} \sim T^3;$
$(X/R)_{\text{res}}^{\text{qu}} \sim \omega^{-2/3}, \quad \Delta^{\text{qu}} \sim \omega^{7/3}; \quad (X/R)_{\text{res}}^{\text{cl}} \sim \omega^{1/3} T^{-1},$
$\Delta^{\text{cl}} \sim \omega^{1/3} T^2.$

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Translated by R. T. Beyer
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