

no charged particles in the second), which differs from the electrodynamic current  $\langle e | \gamma_\mu | e \rangle$  by the fact that the mass of the particle changes in the transition, and also by the presence of the factor  $\gamma_5$  (the current axial vector is  $\gamma_5 \gamma_\alpha$ ).

Since, as is well known, the divergent integrals in electrodynamics do not depend on the mass of the particle, the fact that it changes cannot invalidate the conclusion from Ward's theorem<sup>5</sup> that the vertex-part and self-mass divergences cancel. The factor  $\gamma_5$  can also change nothing in this connection, since the replacement of the wave function  $\psi$  by  $\gamma_5 \psi$  leads only to a change of the mass.

It follows that a finite result will be obtained when one calculates the radiative corrections to  $\mu$ -meson decay (and to any other process of interaction of  $\mu$  mesons with electrons:  $\mu \rightarrow e + \nu + \bar{\nu} + \gamma$ ,  $e + \nu \rightarrow e + \nu$ ,  $\mu + \nu \rightarrow \mu + \nu$ , and so on) in any order (in  $e^2$ ) of perturbation theory.

In the case of the  $\beta$  decay of the neutron or the capture of a  $\mu$  meson by a proton the Hamiltonian does not reduce to the electrodynamic form. In fact,

$$H = \frac{G}{\sqrt{2}} \langle p | \gamma_\alpha (1 + \gamma_5) | n \rangle \langle e | \gamma_\alpha (1 + \gamma_5) | \nu \rangle \quad (3)$$

and it is not possible by interchanging particles of the same helicity to group the charged particles in one factor — to do so one must interchange  $n$  and  $e$ . This latter interchange does not leave the Hamiltonian in the same form, but changes it to<sup>6</sup>

$$H = \sqrt{2} G \langle e | (1 - \gamma_5) | \bar{p} \rangle \langle \bar{n} | (1 + \gamma_5) | \nu \rangle, \quad (4)$$

which, as is well known, is not renormalizable (even if one does not take into account the magnetic moment of the neutron). It can be seen from this that only for processes in which no particles appear except electrons,  $\mu$  mesons, neutrinos, and photons is it possible to calculate the radiative corrections.

In this connection one cannot at the present time predict theoretically the relative size of the constants calculated, on one hand, from the lifetime of the neutron, and on the other hand from  $\beta$  transitions between nuclei of spin zero ( $0^+ \rightarrow 0^+$  transitions); the experimental determination of this ratio is an important problem.

<sup>1</sup> Behrends, Finkelstein, and Sirlin, Phys. Rev. **101**, 866 (1956).

<sup>2</sup> S. M. Berman, Phys. Rev. **112**, 267 (1958).

<sup>3</sup> T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).

<sup>4</sup> V. P. Kuznetsov, JETP **37**, 1102 (1959), Soviet Phys. JETP **10**, 784 (1960).

<sup>5</sup> J. Ward, Phys. Rev. **78**, 182 (1950).

<sup>6</sup> Ho Tso-Hsiu and Chu Hung-Yüan (preprint).

Translated by W. H. Furry  
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## ON AN ESTIMATE OF THE MINIMUM RADIUS OF TWO-PARTICLE INTERACTIONS AT HIGH ENERGIES

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IT is impossible in practice to carry out a phase-shift analysis at high energies, owing to the large number of partial waves participating in the interaction. It is therefore important to establish what information may be extracted from the experimental data.

In this note we show how to determine the minimum number of partial waves  $L_{\min}$  necessary to describe the experimentally known total elastic scattering cross section  $\sigma_{el}$  and the differential cross section at a given angle  $\sigma(\vartheta_1)$ . The following inequalities may be proved:

a) Spinless particles:

$$\Sigma_0 \geq 4\pi\sigma(\vartheta_1) / \sigma_{el}. \quad (1)$$

b) Interaction between particles of spin 0 and  $\frac{1}{2}$ :

$$\max \{ \Sigma_0, \Sigma_1 \} \geq 4\pi\sigma(\vartheta_1) / \sigma_{el}. \quad (2)$$

c) Not identical Dirac particles:

$$\max \{ \Sigma_0, \Sigma_1, \Sigma_2 \} \geq 4\pi\sigma(\vartheta_1) / \sigma_{el}. \quad (3)$$

d) Identical Dirac particles:

$$\max \{ \Sigma'_0, \Sigma''_0, \Sigma'_1, \Sigma''_1, \Sigma'_2, \Sigma''_2 \} \geq 4\pi\sigma(\vartheta_1) / \sigma_{el}. \quad (4)$$

In these inequalities

$$\Sigma_m = \sum_{l=m}^L (2l+1) \frac{(l-m)!}{(l+m)!} [P_l^{(m)}(\cos \vartheta_1)]^2,$$

$$\Sigma'_m = \sum_{l=m}^L [1 - (-1)^l] (2l+1) \frac{(l-m)!}{(l+m)!} [P_l^{(m)}(\cos \vartheta_1)]^2,$$

$$\Sigma''_m = \sum_{l=m}^L [1 + (-1)^l] (2l+1) \frac{(l-m)!}{(l+m)!} [P_l^{(m)}(\cos \vartheta_1)]^2. \quad (5)$$

The largest of the entries in the curly brackets is to be used on the left hand sides of Eqs. (2) — (4).

At large energies, when  $L$  is sufficiently large,  $\Sigma'_m \approx \Sigma''_m \approx \Sigma_m$ . If the angle  $\vartheta_1$  is small, then  $\Sigma_0$  will be larger than  $\Sigma_1$  and  $\Sigma_2$  since the latter contain the associated Legendre polynomials. Therefore in practice one can always use the inequality (1). Let us write it out in more detail:

$$\sum_0^L (2l+1) [P_l(\cos \vartheta_1)]^2 \geq 4\pi\sigma(\vartheta_1)/\sigma_{el}. \quad (1')$$

It is obvious that (1') will begin to be valid only for  $L \geq L_{\min}$ . In a quasiclassical approach one may associate with  $L_{\min}$  a minimum interaction radius  $R_{\min} \approx L_{\min}\lambda$ .

As an example we discuss  $pp$  scattering at 8.5 Bev. According to Tsyganov et al.<sup>1</sup> we have in this case

$$\begin{aligned} \sigma_{el} &= (8.6 \pm 0.8) \text{ mb}, \\ \sigma(2.5^\circ - 5.5^\circ) &= 123 \pm 18 \text{ mb/sr}. \end{aligned}$$

From the inequality (1') we find  $L_{\min} = 16 \pm 3$ . The optical model, when used to describe the same data, gives an effective  $L$  equal to 16. The corresponding interaction radius is  $R \approx 1.6 \times 10^{-13}$  cm. It follows from our results that any other model will lead to the same or larger interaction radius.

The inequality (1') may be viewed as a stronger version of the Rarita-Schwed<sup>2</sup> inequality:

$$(L+1)^2 \geq k^2 \sigma_t^2 / 4\pi\sigma_{el}, \quad (6)$$

which, as is easy to see, follows from (1') for  $\vartheta = 0$  in the case of a vanishing real part of the scattering amplitude. Thus, in the example considered above, the inequality (6) yields the weaker estimate  $L_{\min} = 8 \pm 1$  if  $\sigma_t = (30 \pm 3) \text{ mb}$ .

In conclusion we note that all our results hold as well for inelastic two-particle reactions of the type  $\pi^- + p \rightarrow \Sigma^- + K^+$ . In this case one should replace  $\sigma_{el}$  by the total cross section for the reaction under study.

The authors are indebted to L. G. Zastavenko for discussions.

<sup>1</sup>V. I. Veksler, Report at the International Conference on High Energy Physics, Kiev, 1959.

<sup>2</sup>W. Rarita and P. Schwed, Phys. Rev. **112**, 271 (1958).

## CREATION OF ANTIPROTONS IN INTERACTION OF NEGATIVE PIONS WITH NUCLEONS

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UP to now there apparently has been no observed case of direct production of antiprotons in  $\pi N$  interactions. We have found several cases of production of antiprotons by negative pions on nucleons, two of which are reported in this letter.

The work was carried out on the proton synchrotron of the Joint Institute for Nuclear Research with a propane bubble chamber<sup>1</sup> in a permanent magnetic field of 13,700 gauss.

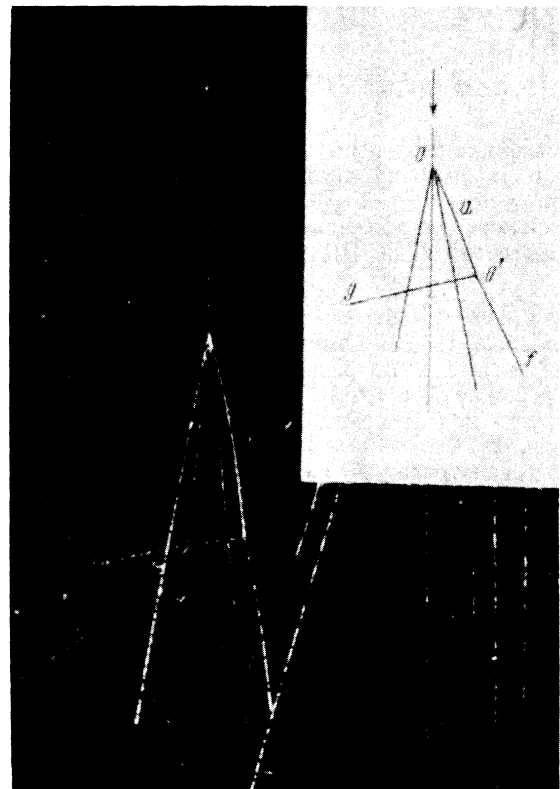


FIG. 1

Figure 1 shows a case where a primary negative pion with approximate energy 7 Bev crosses at the point O a star with four prongs. Prong a is determined unambiguously as an antiproton. The