THE MOTION OF A PISTON IN A CONDUCTING MEDIUM

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We consider magnetohydrodynamic waves which arise when a piston moves in a perfectly conducting medium in the presence of a magnetic field. If the transverse velocity component of the piston exceeds the velocity of sound in the undisturbed medium, then a magnetic field is generated; in this case, the magnetic pressure becomes comparable with the hydrostatic pressure. At supersonic velocities, a vacuum is formed between the piston and the medium (cavitation). Compared with ordinary hydrodynamics, additional cases of cavitation appear when the piston moves with supersonic velocity in the direction perpendicular to the normal, and also when the piston moves in, if the angle between its velocity vector and the normal to its surface exceeds 70° (for an ideal gas with $\gamma = 5/3$). Increase of the piston velocity component perpendicular to the normal decreases the drag. When cavitation occurs, the drag is four times less than in the case of motion of the piston in the direction normal to its surface.

I: INTRODUCTION

THE motion of a magnetohydrodynamic medium under the action of an ideally conducting piston moving in it with constant velocity was considered by Bazer[1] (for a piston moving perpendicularly to the normal, and in a longitudinal magnetic field), by Lyubarskiĭ and Polovin[2] (for motion of the piston along the normal), and by I. Akhiezer and Polovin[3] (for subsonic velocities of the piston). The special case of the problem of a piston in the absence of a longitudinal magnetic field was considered by Golitsyn.[4]

In the present paper we consider the problem of a piston in the case of an arbitrary direction of the magnetic field and for an arbitrary (constant) piston velocity. It is also assumed that the unperturbed magnetic field is small, so that the Alfvén velocity is

$$U = H/\sqrt{4\pi\rho} \ll c, \tag{1.1}$$

where $c = \sqrt{\gamma p/\rho}$ is the velocity of sound (the medium satisfies the equation of state of an ideal gas with an adiabatic exponent γ). The left side of the medium is bounded by an ideally conducting piston located in the plane $x = 0$.

At the moment $t = 0$, the piston begins to move with constant velocity $\mathbf{u}$. Since the problem involves no parameter of the dimensionality of length, the motion of the medium will be self-similar. This means that all quantities depend only on the ratio x/t. Here the motion of the medium is characterized by a succession of shock and self-similar waves traveling one after the other. It is important to note that the problem of finding these waves has a unique solution only if the existence of evolutionary shock waves is assumed.[5,6]

Only in certain special cases, when slow shock waves are absent, can the piston problem be solved without account of the evolution condition. (This is connected with the fact that fast shock waves are always evolutionary.) Precisely this case has been considered by Bazer.[1]

The impossibility of the existence of nonevolutionary shock waves in magnetohydrodynamics* was shown by Akhiezer, Lyubarskiĭ and Polovin,[9,10] by Kontorovich,[11] and by Syrovatskiĭ.[12]

There are three types of evolutionary discontinuities that move relative to the fluid — fast and slow shock waves and Alfvén discontinuities. Moreover, there are two types of continuous solutions — fast and slow self-similar magnetoacoustic waves.[13,14]

Shock waves are compression waves,[15-17] while self-similar waves are rarefaction waves.[14,18,19] The velocities of these waves are such that only the fast waves (shock or self-similar) can go ahead, followed by the Alfvén discontinuity, and, finally, by the slow waves (shock or self-similar). If it is also taken into account that several of these

*In ordinary hydrodynamics, these ideas were first advanced by Landau and Lifshitz[7] and by Courant and Friedrichs.[8]

waves can exist at the same time, then a large number of qualitatively different motions of the medium are realizable for various piston velocities.

The following boundary conditions are satisfied on the surface of an ideally conducting piston: $v_x = u_x$, $v_y = u_y$, $v_z = u_z$, where **v** is the velocity of medium and **u** is the velocity of piston.

For sufficiently large amplitude of the rarefaction wave, the density of the medium in back of the wave vanishes — cavitation sets in. In this case the boundary conditions

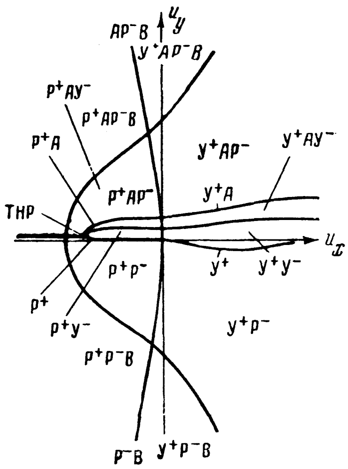
$$\rho = 0,\ H_x(u_y - v_y) - H_y'(u_x - v_x) = 0,$$
$$H_x(u_z - v_z) - H_z(u_x - v_x) = 0 \qquad (1.2)$$

are satisfied on the boundary with the vacuum.

We shall limit ourselves to the most interesting case, in which the magnetic field, the velocity of the piston, and the normal to its surface lie in a single plane (the xy plane). The values of v_z and H_z will be equal to zero in this case not only in the undisturbed medium, but also in all the resultant waves. Therefore, the Alfvén discontinuity can rotate the magnetic field only by 180°. We shall set the undisturbed velocity of the medium v_0 equal to zero. For definiteness, we shall assume that the components of the undisturbed magnetic field H_{0x} and H_{0y} are positive. We shall neglect dissipative processes.

The types of waves that are produced as the piston moves depend on its velocity (u_x, u_y). This dependence is shown in the drawing (the abscissa and ordinate are the longitudinal and transverse components of the velocity of the piston, u_x and u_y, respectively). The letters Y^+ Y^-, P^+, P^-, and A denote respectively the presence of a fast and a slow shock wave, a fast and a slow rarefaction wave (self-similar), and an Alfvén discontinuity. For a sufficiently large amplitude of the slow rarefaction wave, the density of the medium in back of the wave vanishes — cavitation begins. In comparison with ordinary hydrodynamics, in which cavitation sets in when the piston is moved out with a velocity exceeding $2c_0/(\gamma - 1)$ (c_0 is the velocity of sound in the undisturbed medium), in magnetohydrodynamics cavitation sets in at lower piston velocities, provided the velocity of motion of the piston in the transverse direction is sufficiently large.

If the piston moves only in the transverse direction, then cavitation sets in when the piston velocity is 3.67 times the velocity of sound in the undisturbed medium (for $\gamma = 5/3$; this result was obtained earlier by Bazer[1]). Cavitation begins also in the case when the piston moves into the



Waves due to the motion of a piston. The longitudinal component of the piston velocity u_x is plotted along the abscissa, and the transverse component u_y is plotted along the ordinate. The letters Y^+, Y^-, P^+, P^-, A, and B denote the presence of fast and slow shock waves, fast and slow rarefaction waves, an Alfvén discontinuity, and the formation of a vacuum. THP is the point of maximum rarefaction realizable in the fast self-similar wave.

medium and simultaneously moves in the transverse direction. If the piston moves in with supersonic velocity, cavitation begins when the angle between the vector velocity of the piston and its normal to the surface reaches 70° ($\gamma = 5/3$). (We note that in this case the difference between the velocity of displacement of the medium — vacuum boundary and the velocity of the piston is very small.) The presence of cavitation is marked in the drawing by the letter B.

In contrast with the slow rarefaction wave, cavitation is impossible in the fast rarefaction wave. If the Alfvén velocity in the undisturbed medium is much less than the velocity of sound, then at the maximum amplitude of the fast rarefaction wave the Alfvén velocity behind it comes close to the velocity of sound. The point of maximum rarefaction achieved in the fast self-similar wave is denoted by the letters THP in the drawing. It should be noted that upon satisfaction of the condition (1.1), the density of the medium at the point of maximum rarefaction will be very small.

At supersonic transverse piston velocity u_y, generation of a magnetic field takes place, i.e., the growth of the magnetic field from an infinitesimally small to finite values; in this case the magnetic pressure becomes comparable with the hydrostatic pressure or exceeds it. At supersonic velocities of insertion (u_x) and sliding (u_y) of the piston, the magnetic field generated is directly proportional to u_x.

An increase in u_x generally leads to an increase in the amplitude of the shock wave and to a decrease in the amplitude of the self-similar wave. An exception to this rule occurs in the region $Y^+ Y^-$ for supersonic piston velocities. Upon increase in the value of u_x, a redistribution of the amplitudes of the fast and slow shock waves takes place; the increase of the amplitude of the fast shock wave is accompanied by a certain decrease in the amplitude of the slow shock wave.

2. SELF-SIMILAR WAVES

The change in the magnetohydrodynamic quantities in self-similar waves is determined by the differential equation first obtained by Friedrichs (see reference 1):

$$\frac{dq_\pm}{ds} = \frac{1}{\theta}\frac{(1-q_\pm)q_\pm^2}{1-sq_\pm^2}, \qquad \theta = \gamma/(2-\gamma),\ q_\pm = U_\pm^2/c^2,$$

$$U_\pm = \{U^2 + c^2 \pm \sqrt{(U^2+c^2)^2 - 4U_x^2c^2}\}^{1/2}/\sqrt{2}, \quad (2.1)$$

c is the speed of sound, $\mathbf{U}$ is the Alfvén velocity, and $s = c^2/U_x^2 = 4\pi\gamma p/H_x^2$ is the dimensionless pressure; the plus and minus signs in the equations for $q_\pm$ and $U_\pm$ correspond to the fast and slow self-similar waves, respectively.

The transverse magnetic field H_y is determined by the formula

$$H_y = H_x\{(1-q_\pm)(1-sq_\pm)/q_\pm\}^{1/2}\,\mathrm{sign}\,H_{1y}, \quad (2.2)$$

where H_{1y} is the transverse magnetic field ahead of the wave.

The longitudinal and transverse components of the velocity of the liquid are determined by the differential equations

$$dv_x/ds = c\sqrt{q_\pm}/\gamma s, \quad (2.3)$$

$$\frac{dv_y}{ds} = \mp\frac{c}{\gamma s}\sqrt{\frac{1-q_\pm}{1-sq_\pm}}\,\mathrm{sign}\,H_{1y}. \quad (2.4)$$

Equations (2.3) and (2.4) are valid only when the wave is propagated in the positive direction of the x axis and $H_x > 0$. The upper sign in Eqs. (2.4) corresponds to the fast wave, the lower to the slow wave.

In the fast waves, the inequalities

$$sq_+ \geqslant 1,\ q_+ > 1,\ sq_\pm^2 > 1, \quad (2.5)$$

are satisfied, and in the slow waves, the inequalities

$$sq_- \leqslant 1,\ q_- < 1,\ sq_-^2 < 1. \quad (2.6)$$

It follows from (2.1), (2.5), (2.6) that $q_\pm$ always decreases in self-similar waves.

If the Alfven velocity is much less than the velocity of sound, then q_- and q_+ satisfy the relations:

$$q_- = 1/s \ll 1, \quad (2.7)$$

$$q_+ - 1 \equiv \xi_+ = U_y^2/c^2 \ll 1. \quad (2.8)$$

Since q_- decreases, satisfaction of the inequality (2.7) ahead of the slow wave brings about satisfaction of this inequality over the entire slow wave, although in this case the inequality (1.1) can be violated (because of the increase of the transverse magnetic field and the decrease of the density in the slow self-similar wave[18,19]). In exactly the same way, fulfillment of the inequality (2.8) in front of the fast wave brings about satisfaction of this inequality over the entire fast wave.

Let us now consider some limiting cases of self-similar waves, which we shall need in what follows.

1) The slow self-similar wave in the case in which the inequality (2.7) is satisfied. It follows from Eq. (2.1) that

$$q_- = \theta/[(\theta+1)s_1 - s], \quad (2.9)$$

$$H_{2y} = \sqrt{\frac{2}{\gamma}}\left\{1 - \frac{s_2}{s_1}\right\}^{1/2} c_1\sqrt{4\pi\rho_1}\,\mathrm{sign}\,H_{1y}, \quad (2.10)$$

$$\Delta_- v_x = -U_{1x}(2\gamma)^{-1/2}\int_{s_2/s_1}^{1}\sigma^{-(\gamma+1)/2\gamma}d\sigma\Big/\sqrt{1-\sigma/(\theta+1)}, \quad (2.11)$$

$$\Delta_- v_y = -\frac{c_1}{\gamma}\int_{s_2/s_1}^{1}\sigma^{-(\gamma+1)/2\gamma}\sqrt{\frac{1-\sigma/(\theta+1)}{1-\sigma}}\,d\sigma\,\mathrm{sign}\,H_{1y}. \quad (2.12)$$

Here $\Delta_- \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$. The index 1 refers to the region ahead of the wave, the index 2 to the region behind the wave.

2) The slow self-similar wave;

$$1 - q_{2-} \equiv \xi_{2-} \ll 1;\ s_1 \approx 1. \quad (2.12a)$$

In this case the inequality $\xi_- \equiv 1 - q_- \ll 1$ is satisfied over the entire wave. [Violation of (2.7), and consequently of (1.1) takes place as a consequence of the change of magnetohydrodynamic quantities in the fast wave moving ahead of the slow wave.] In this case the values of ξ_-, s, $\Delta_- v_x$, $\Delta_- v_y$ are determined by the expressions

$$\xi_- \equiv 1 - q_- \quad (2.13)$$

$$s = 1 - [\gamma/(\gamma-1)]\,\xi_-^\theta/\xi_{1-}^{\theta-1} + \xi_-\gamma/(\gamma-1), \quad (2.14)$$

$$\Delta_- v_x = -2c_1[1 - (s_2/s_1)^{(\gamma-1)/2\gamma}]/(\gamma-1), \quad (2.15)$$

$$\Delta_- v_y = -\frac{c_1}{\gamma s_1^{(\gamma-1)/2\gamma}}\int_{\xi_{2-}}^{\xi_{1-}} s^{-(\gamma+1)/2\gamma}\sqrt{\frac{1-q_-}{1-sq_-}}\frac{ds}{d\xi_-}d\xi_-\,\mathrm{sign}\,H_{1y}. \quad (2.16)$$

The index 1 refers to the region ahead of the wave, the index 2 to the region behind the wave.

3) The fast self-similar wave.

$$\xi_+ \equiv q_+ - 1 \ll 1, \quad (2.17)$$

$$s = 1 + s_0\xi_+^0 / \xi_{0+}^0 - \gamma\xi_+ / (\gamma - 1), \quad (2.18)$$

$$\Delta_+ v_x = -2c_0 [1 - (s_1/s_2)^{(\gamma-1)/2\gamma}] / (\gamma - 1), \quad (2.19)$$

$$\Delta_+ v_y = \frac{c_0}{\gamma s_0^{(\gamma-1)/2\gamma}} \int_{\xi_{1+}}^{\xi_{0+}} s^{-(\gamma+1)/2\gamma} \sqrt{\frac{1-q_+}{1-sq_+}\frac{ds}{d\xi_+}}\, d\xi_+. \quad (2.20)$$

The index 0 refers to the region ahead of the wave, the index 1 to the region behind the wave.

It follows from Eq. (2.18) that cavitation ($s = 0$) is impossible on the fast self-similar wave. The maximum rarefaction possible in this wave takes place for $sq_+ = 1$; the corresponding value of ξ_{1+} is

$$\xi_{1+} = \xi_{0+} [\xi_{0+} / (\gamma - 1)s_0]^{1/(\theta-1)}, \quad s_1 \approx 1. \quad (2.21)$$

The value of the integral (2.20) depends weakly on the behavior of the function under the integral when $\xi_+ \approx \xi_{1+}$. Therefore, the expression for $\Delta_+ v_y$ can be simplified:

$$\Delta_+ v_y = U_{0y}\left(\frac{U_{0x}}{c_0}\right)^{1/\gamma} \frac{1}{\gamma}\int_{s_1}^{s_2} s^{-(\gamma+1)/2\gamma}(s-1)^{-(\gamma-1)/\gamma} ds. \quad (2.22)$$

However, Eq. (2.22) is not suitable for calculation of the derivative dv_{1y}/dv_{1x} in the vicinity of the point of maximum rarefaction (2.21).

3. SHOCK WAVES

1) 180° Alfvén discontinuity.

The jumps in the velocity and the magnetic field are determined by the expression

$$\Delta_A v_y = 2U_{1y}, \quad \Delta_A H_y = -2H_{1y}. \quad (3.1)$$

The index 1 refers to the region ahead of the discontinuity. The jumps of the other quantities are equal to zero.

2) Fast shock wave.

Under condition (1.1) discontinuities will be the same as without a magnetic field:

$$c_1 = \{c_0^2 + (\gamma - 1)^2 v_{1x}^2 / 4 + [(c_0^2 + (\gamma - 1)^2 v_{1x}^2 / 4)^2 + (\gamma - 1)^2(\gamma v_{1x}^4 / 4 + c_0^2 v_{1x}^2 / 2) - c_0^4]^{1/2}\}^{1/2}, \quad (3.2)$$

$$\rho_1 = \rho_0 \{[(c_1^2 - c_0^2) / (\gamma - 1)] + v_{1x}^2 / 2\} / \{[(c_1^2 - c_0^2) / (\gamma - 1)] - v_{1x}^2 / 2\}. \quad (3.3)$$

The index 0 refers to the region in front of the wave, the index 1 to the region behind the wave.

For high-intensity shock waves, $v_{1x} \gg c_0$, Eqs. (3.2), (3.3) take the form

$$c_1 = v_{1x}\sqrt{\gamma(\gamma - 1)/2}, \quad \rho_1 = \rho_0(\gamma + 1)/(\gamma - 1). \quad (3.4)$$

3) Slow shock wave.

With the satisfaction of the inequality (1.1), the discontinuities of the magnetohydrodynamic quantities on a slow shock wave of low intensity, $\Delta_-\rho \ll \rho$, are determined by the relations:[3]

$$\Delta_- v_x = U_{1x}\Delta_-\rho / \rho_1, \quad \Delta_- p = c_1^2 \Delta_-\rho,$$
$$\Delta_- v_y = U_{1y}(1 \mp \sqrt{1 - 2c_1^2\Delta_-\rho / U_{1y}^2 \rho_1}),$$
$$\Delta_- H_y = H_{1y}(-1 \pm \sqrt{1 - 2c_1^2 \Delta_-\rho / U_{1y}^2 \rho_1}). \quad (3.5)$$

The index 1 refers to the region in front of the wave. The same formulas are obtained also for a slow self-similar wave of low intensity, with the positive sign in front of the radical corresponding to the self-similar wave. In evolutionary waves, the transverse magnetic field H_y does not change sign;[17] therefore, in Eqs. (3.5) the upper sign corresponds to evolutionary waves, and the lower sign corresponds to non-evolutionary waves. This means that only in evolutionary shock waves will the relations between the discontinuities of the magnetohydrodynamical quantities (in first approximation in $\Delta_-\rho$) be the same as in self-similar waves. We shall henceforth write the upper sign in front of the radical for the formulas (3.5).

4. SINGLE RAREFACTION WAVE

1) Slow wave (see the lines P^- and AP^- in the drawing).

The parametric equation of the lines P^- and AP^- has the form (s_1 is the parameter)

$$u_x = -U_{0x}(2\gamma)^{-1/2} \int_{s_1/s_0}^{1} \sigma^{-(\gamma+1)/2\gamma} d\sigma / \sqrt{1 - \sigma(\theta + 1)^{-1}}, \quad (4.1)$$

$$u_y = \mp c_0 \gamma^{-1} \int_{s_1/s_0}^{1} \sigma^{-(\gamma+1)/2\gamma} d\sigma\, [[1 - \sigma/(\theta + 1)] / (1 - \sigma)]^{1/2}. \quad (4.2)$$

The line P^- corresponds to the upper sign in Eq. (4.2); the line AP^- corresponds to the lower sign. For $s_1 = 0$, cavitation sets in; in this case the velocity of the piston is

$$u_x = -U_{0x} f(\gamma), \quad u_y = \mp c_0 g(\gamma), \quad (4.3)$$

$$f(\gamma) = (2\gamma)^{-1/2} \int_0^1 \sigma^{-(\gamma+1)/2\gamma} d\sigma / \sqrt{1 - \sigma(\theta + 1)^{-1}}, \quad (4.4)$$

$$g(\gamma) = \gamma^{-1} \int_0^1 \sigma^{-(\gamma+1)/2\gamma} d\sigma\, [[1 - \sigma/(\theta + 1)] / (1 - \sigma)]^{1/2}. \quad (4.5)$$

To calculate $f(\gamma)$ and $g(\gamma)$, the radicals in Eqs. (4.4), (4.5) must be expanded in series:

$$f(\gamma) = \sqrt{2\gamma}\left[\frac{1}{\gamma - 1} + \frac{1}{2(\theta + 1)(3\gamma - 1)} + \frac{1}{8(\theta + 1)^2(5\gamma - 1)} + \cdots\right], \quad (4.6)$$

$$g(\gamma)=\frac{\sqrt{\pi}\Gamma((\gamma-1)/2\gamma)}{\gamma\Gamma((2\gamma-1)/2\gamma)}\Big[1-\frac{1}{2(\theta+1)}\frac{\gamma-1}{2\gamma-1}-\frac{1}{8(\theta+1)^2}\frac{\gamma-1}{2\gamma-1}\frac{3\gamma-1}{4\gamma-1}-\ldots\Big], \quad (4.7)$$

where $\Gamma(p)$ is the gamma function. For $\gamma = 5/3$, we get

$$f(5/3)=2.78, \quad g(5/3)=3.67. \quad (4.8)$$

The value of q_{2-} at the point where $s_2 = 0$ is given by

$$q_{2-}=\gamma q_{1-}/2. \quad (4.9)$$

The magnetic field is determined by the formula

$$H_{2y}=\pm\sqrt{2/\gamma}c_0\sqrt{4\pi\rho_0}. \quad (4.10)$$

2) Slow wave with cavitation (the line of separation between the regions P^+P^-B and Y^+P^-B or P^+AP^-B and Y^+AP^-B in the drawing).

The equation of the line of separation has the form

$$u_x\mp\sqrt{\gamma/2}U_{0x}u_y/c_0+[f(\gamma)-\sqrt{\gamma/2}\,g(\gamma)]\,U_{0x}=0, \quad (4.11)$$

The upper and lower signs correspond to the absence and presence of an Alfvén discontinuity. The line in the u_x, u_y plane corresponding to the slow rarefaction wave has a kink at the point (4.3), where cavitation begins;

before cavitation: $du_y/du_x=\mp c_0/U_{0x}$;

with cavitation: $du_y/du_x=\mp\sqrt{2/\gamma}\,c_0/U_{0x}$. (4.12)

3) Fast rarefaction wave (the line P^+ in the drawing).

The parametric equation of the line P^+ is determined by Eqs. (2.19), (2.22), in which $\Delta_+v_x = u_x$; $\Delta_+v_y = u_y$; the parameter is the quantity s_1. At the point $u_x = u_y = 0$ we have $du_y/du_x = -U_{0x}U_{0y}/c_0^2$, in accord with reference 3. At the point of maximum rarefaction (2.21)

$$u_x=-\frac{2c_0}{\gamma-1}(1-s_0^{-(\gamma-1)/2\gamma}),$$

$$u_y=U_{0y}(U_{0x}/c_0)^{1/\gamma}h(\gamma), \quad (4.13)$$

$$h(\gamma)=\Gamma\left(\frac{\gamma-1}{2\gamma}\right)\Gamma\left(\frac{1}{\gamma}\right)\Big/\gamma\Gamma\left(\frac{\gamma+1}{2\gamma}\right). \quad (4.14)$$

For $\gamma = 5/3$ we have $h(5/3) = 3.52$.

The value of du_y/du_x in the vicinity of the point of maximum rarefaction (2.21) cannot be determined from (2.22); use must be made of the more exact expression (2.20), from which it follows that

$$du_y/du_x=-\sqrt{(1-q_+)/(1-sq_+)}. \quad (4.14a)$$

At the point of maximum rarefaction we obtain $du_y/du_x = -\infty$.

4) Fast rarefaction wave with an Alfvén discontinuity (the line P^+A in the drawing).

The line P^+A in the plane u_xu_y is determined by parametric equations expressed in terms of the parameter s_1, with $u_x = \Delta_+v_x$ determined by (2.19), and u_y by

$$u_y=2U_{0x}\xi_{1+}\left\{\frac{s_0}{\xi_{0+}}\left(\frac{\xi_{1+}}{\xi_{0+}}\right)^{\theta-1}-\frac{1}{\gamma-1}\right\}^{1/2}\left(\frac{s_0}{s_1}\right)^{1/2\gamma}+\Delta_+v_y, \quad (4.15)$$

where Δ_+v_y is in turn determined by Eq. (2.22), or, more exactly, by Eq. (2.20). The point of maximum rarefaction, as before, is determined by Eqs. (2.21) and (4.13). At the point $u_x = 0$, $u_y = 2U_{0y}$, we have $du_y/du_x = U_{0y}/c_0$, which coincides with reference 3; at the point of maximum rarefaction, $du_y/du_x = +\infty$.

5. A SINGLE SHOCK WAVE

1) Slow shock wave (the lines Y^- and AY^- in the drawing).

In the u_xu_y plane the lines Y^- and AY^- are described by the equation[3]

$$u_x+U_{0x}\,u_y(u_y-2U_{0y})/2c_0^2=0. \quad (5.1)$$

2) Fast shock wave (the line Y^+ in the drawing).

The equation of the line Y^+ in the u_xu_y plane has the form

$$u_y=-U_{0x}U_{0y}u_x^3/\{[(c_1^2-c_0^2)/(\gamma-1)]^2-u_x^4/4\}, \quad (5.2)$$

where c_1 is determined by (3.2), in which v_{1x} must be replaced by u_x. For $u_x \gg c_0$, Eq. (5.2) is greatly simplified:

$$u_y=-4U_{0x}U_{0y}/(\gamma^2-1)\,u_x. \quad (5.3)$$

3) Fast shock wave with Alfvén discontinuity (the line Y^+A in the drawing).

In the u_xu_y plane the line Y^+A is determined by the equation

$$u_y+U_{0x}U_{0y}u_x^3/\{[(c_1^2-c_0^2)/(\gamma-1)]^2-u_x^4/4\}=2U_{0y}(\rho_1/\rho_0)^{1/2}, \quad (5.4)$$

where c_1 and ρ_1 are determined by (3.2) and (3.3) ($v_{1x} = u_x$). In the limiting case $u_x \gg c_0$, Eq. (5.4) takes the form

$$u_y+4U_{0x}U_{0y}/(\gamma^2-1)\,u_x=2U_{0y}[(\gamma+1)/(\gamma-1)]^{1/2}. \quad (5.5)$$

6. TWO RAREFACTION WAVES

1) The cavitation line P^+P^- or P^+AP^- (the line of separation between the regions P^+P^- and P^+P^-B or P^+AP^- and P^+AP^-B in the figure).

The equation of the cavitation line for u_y: $|u_y|/c_0 \gg g(\gamma)(U_{0x}/c_0)^{4/(5-\gamma)}$ has the form

$$u_x = -\frac{2c_0}{\gamma-1} + \frac{2}{\gamma-1}\frac{|u_y|}{g(\gamma)} - U_{0x} f(\gamma)\left[\frac{c_0 g(\gamma)}{|u_y|}\right]^{1/(\gamma-1)}. \quad (6.1)$$

For $|u_y|$ close to $h(\gamma)U_{0y}(U_{0x}/c_0)^{1/\gamma}$: $|u_y - U_{0y}(U_{0x}/c_0)^{1/\gamma}h(\gamma)| \ll U_{0x}(U_{0x}/c_0)^{-1/\gamma}$, the equation of the cavitation lines has the form

$$u_x = -2c_0/(\gamma-1). \quad (6.1a)$$

2) Line of separation between the regions P^+P^- and P^+AP^-.

From qualitative considerations of the topological structure of the regions in the drawing, given in reference 3, it follows that the quantity u_y is determined on the line of separation between the regions P^+P^- and P^+AP^- as the maximum velocity of the medium, v_{2y}, obtained for a given value of u_x by means of two rarefaction waves (P^+ and P^-). In this case the wave P^+ passes through the point of maximum rarefaction (2.21).

The equation of the line of separation has the form

$$u_y = U_{0y}(U_{0x}/c_0)^{1/\gamma}h(\gamma). \quad (6.2)$$

The transverse magnetic field H_{2y} vanishes on the line of separation. At the point of intersection of the lines of separation of regions P^+P^- and P^+AP^- with the cavitation lines, s_2 vanishes; at this point $u_x = -2c_0/(\gamma - 1)$.

3) The line of separation between the regions P^+P^-B and P^+AP^-B.

The equation of the line of separation follows from (1.2):

$$u_y = U_{0y}(U_{0x}/c_0)^{1/\gamma}h(\gamma). \quad (6.3)$$

7. TWO SHOCK WAVES

The value of u_y on the line of separation between the regions Y^+Y^- and Y^+AY^- is determined as the maximum value of v_{2y} obtained in the waves Y^+Y^- (for a given value of u_x), or as the minimum value of v_{2y} obtained in the waves Y^+AY^-. The problem of the motion of the magnetohydrodynamic medium has a unique solution only when $\max v_{2y}|_{Y^+Y^-} = \min v_{2y}|_{Y^+AY^-}$. In satisfying these relations it is necessary to exclude from consideration non-evolutionary shock waves. Since the distance between the lines Y^+ and Y^+A is small in comparison with the velocity of sound, the slow shock wave will have a small amplitude. We can therefore make use of (3.5). The equation of the line of separation between the regions Y^+Y^- and Y^+AY^- has the form

$$u_y + U_{0x}U_{0y}u_x^3/\{[(c_1^2-c_0^2)/(\gamma-1)]^2 - (u_x^4/4)\} = U_{0y}(\rho_1/\rho_0)^{1/2}, \quad (7.1)$$

where the values of c_1 and ρ_1 are determined by Eqs. (3.2) and (3.3). For $u_x \gg c_0$, Eq. (7.1) is simplified:

$$u + \frac{4U_{0x}U_{0y}}{(\gamma^2-1)u_x} = U_{0y}\left(\frac{\gamma+1}{\gamma-1}\right)^{1/2}. \quad (7.1a)$$

The region Y^+Y^- is bounded below in the drawing by the line Y^+, determined for $u_x \gg c_0$ by Eq. (5.3), on which the amplitude of the slow shock wave Y^- vanishes. Since the value of u_y determined by Eq. (5.3) is an increasing function of u_x, an increase in the longitudinal velocity of the piston u_x for fixed transverse velocity u_y brings about a decrease in the amplitude of a slow shock wave. In this case, of course, the amplitude of the fast shock wave increases, for as the piston velocity increases the magnetic field begins to play a smaller role, and in the limiting case $u_x/c_0 \to \infty$ the shock waves become the same as in the absence of a magnetic field. In this case the amplitude of the slow wave for $U_0 \ll c_0$ tends to zero.

8. COMBINATION OF SHOCK WAVES AND RAREFACTION WAVES

1) Fast shock wave and slow self-similar wave without cavitation (regions Y^+P^- and Y^+AP^- in in the drawing).

The value of s_2 on the surface of the piston, for given u_x and u_y, is determined from the relation

$$u_y = \mp c_1\gamma^{-1}\int_{s_2/s_1}^{1} \sigma^{-(\gamma+1)/2\gamma}\{[1-\sigma/(\theta+1)]/(1-\sigma)\}^{1/2}d\sigma, \quad (8.1)$$

where c_1 is determined by Eq. (3.2), in which we set $v_{1x} = u_x$; ρ_1 and H_{2y} are determined by Eqs. (3.3) and (2.10); $s_1 = s_0(c_1/c_0)^{2\gamma/(\gamma-1)}$. The upper sign in Eq. (8.1) corresponds to the region Y^+P^-, the lower to the region Y^+AP^-.

2) Fast shock wave and self-similar wave on the cavitation line (the line separating the regions Y^+P^- and Y^+P^-B, and also Y^+AP^- and Y^+AP^-B in the drawing).

The transverse velocity of the piston is determined by the relation $u_y = \mp g(\gamma)c_1$, where c_1 is determined by Eq. (3.2) ($v_{1x} = u_x$). For $u_x = 0$, we obtain the formula of Bazer:[1] $u_y = \mp g(\gamma)c_0 \approx \mp 3.67\,c_0$ for $\gamma = 5/3$. The slope of the cavitation line at $u_x = 0$ is determined by $du_y/du_x = \mp(\gamma-1)/g(\gamma)/2 \approx \mp 1.2$ for $\gamma = 5/3$, and for $u_x \gg c_0$ by the expression $du_y/du_x = \pm g(\gamma)[\gamma(\gamma$

$-1)/2]^{1/2} \approx \mp 2.7 \approx \pm \tan 70°$ for $\gamma = 5/3$. The transverse magnetic field on the boundary with the piston is equal to $H_{2y} = \pm(2/\gamma)^{1/2}(4\pi\rho_0)^{1/2} c_0$.

The upper sign in the above formulas corresponds to the absence of an Alfvén discontinuity, the lower sign corresponds to the combination Y^+AP^-.

3) Strong shock wave and weak self-similar wave with cavitation (the regions Y^+P^-B and Y^+AP^-B in the drawing).

For a given u_x and u_y, the velocity of the medium v_{2x} on the voundary with the piston or with the vacuum (in the case of cavitation) is determined by the relation

$$v_2 = u_x + (\gamma/2)^{1/2}(U_{1x}/c_1)[|u_y| - c_1 g(\gamma)], \quad (8.1a)$$

where $U_{1x} = H_x(4\pi\rho_1)^{-1/2}$, $\rho_1 = \rho_0(c_1/c_0)^{2/(\gamma-1)}$, and c_1 is expressed in terms of v_{1x} with the aid of Eq. (3.2); in turn, v_{1x} and v_{2x} are related by $v_{1x} = v_{2x} + U_{1x} f(\gamma)$. Solving the resultant equation for v_{2x}, we find all the remaining magnetohydrodynamical quantities. In the particular case of shock waves of large amplitude ($c_1 \gg c_0$) in the presence of cavitation, we find

$$\begin{aligned} v_{2x} = \tfrac{1}{2}\{&u_x - U_{0x} f(\gamma)[(\gamma-1)/(\gamma+1)]^{1/2} \\ &- U_{0x} g(\gamma)[\gamma(\gamma-1)/2(\gamma+1)]^{1/2}\} \\ &+ \{\tfrac{1}{4}u_x^2 + \tfrac{1}{2}U_{0x}u_x f(\gamma)[(\gamma-1)/(\gamma+1)]^{1/2} \\ &- U_{0x}u_x g(\gamma) - [\gamma(\gamma-1)/2(\gamma+1)]^{1/2} \\ &+ U_{0x}|u_y|(\gamma-1)^{-1/2}\}^{1/2}, \end{aligned} \quad (8.2)$$

$$H_{2y} = -v_{2x}[4\pi\rho_0(\gamma+1)]^{1/2}\operatorname{sign} u. \quad (8.3)$$

For $u_x = 0$, Eqs. (8.2) and (8.3) go over into the formulas obtained by Bazer.[1]

If the longitudinal velocity of the piston is not small, $U_{0x}|u_y| \ll u_x^2$, it follows from Eq. (8.2) that the velocities v_{2x} and u_x are close to one another:

$$v_{2x} - u_x = U_{0x}(\gamma+1)^{-1/2}u_x^{-1}\{|u_y| - [\gamma(\gamma-1)/2]^{1/2} g(\gamma)u_x\}. \quad (8.3a)$$

In this case the generated magnetic field is proportional to the longitudinal velocity of the piston

$$H_{2y} = -u_x[4\pi\rho_0(\gamma+1)]^{1/2}\operatorname{sign} u_y. \quad (8.3b)$$

9. RESISTANCE FORCE

The relations obtained make it possible to determine the resistance force F produced when the piston moves uniformly with velocity (u_x, u_y). This force consists of two components: a longitudinal component F_x, the drag, and a transverse component F_y, the lifting force. We limit ourselves to the case of a fast shock wave of high amplitude, accompanied by a slow rarefaction wave.

The force acting on a certain element of area is none other than the momentum flux through this element. Making use of Eq. (51.8) of Landau and Lifshitz[20] for the momentum flux density tensor, we obtain for the most interesting case, $u \gg c_0$ and $U_{0x}|u_y| \gg u_x^2$, an expression for the drag,

$$-F_x = \frac{\gamma+1}{2}\rho_0 u_x^2\left(\frac{2\sigma_2^{1/\gamma}}{\gamma-1}+1\right), \quad (9.1)$$

where the quantity σ_2 is determined from the equality

$$\int_{\sigma_2}^{1}\sigma^{-(\gamma+1)/2\gamma}\left\{\left[1-\frac{\sigma}{\theta+1}\right]\Big/(1-\sigma)\right\}^{1/2}d\sigma = \left(\frac{2\gamma}{\gamma-1}\right)^{1/2}\frac{|u_y|}{u_x}, \quad (9.1a)$$

in the absence of cavitation; $\sigma_2 \equiv 0$ in the presence of cavitation.

The lifting force has the form

$$\begin{aligned} -F_y = {}&(\gamma+1)\rho_0 u_x u_y \sigma_2^{1/\gamma}/(\gamma-1) \\ &+ \sqrt{\gamma+1}\sqrt{1-\sigma_2}\,\rho_0 U_{0x}u_x\operatorname{sign} u_y. \end{aligned} \quad (9.2)$$

It follows from Eq. (9.1) that for a fixed value of u_x, the drag decreases with increasing $|u_y|$. The value of the drag on the cavitation line is $(\gamma+1)/(\gamma-1)$ times smaller than for $|u_y| \ll u_x$. Upon further increases $|u_y|$, the value of the drag does not change.

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ERRATA TO VOLUME 10

page	reads	should read
Article by A. S. Khaĭkin		
1044, title	. . . resonance in lead	. . . resonance in tin
6th line of article	~ 1000 oe	~ 1 oe
Article by V. L. Lyuboshitz		
1223, Eq. (13), second line	$\ldots-\mathrm{Sp}_{1,2}\,\mathscr{E}(\mathbf{e}_1)$	$\ldots-\mathrm{Sp}_{1,2}\,\mathscr{E}(\mathbf{e}_2)\ldots$
1226, Eq. (26), 12th line	$\cdots\{(\mathbf{p}+\mathbf{q},\ \mathbf{p}$	$\cdots\{(\mathbf{p}+\mathbf{q},\mathbf{p})-(\mathbf{p}+\mathbf{q},\ \mathbf{n})\cdot\cdot(\mathbf{pn})\}$
1227, Eqs. (38), (41), (41a) numerators and denominators	$(\mathbf{p}^2-\mathbf{q})$	$(\mathbf{p}^2-\mathbf{q}^2)^2$
1228, top line	$\mathbf{m}_2=\begin{matrix}\mathbf{q}_1-\mathbf{p}_1\\ \mathbf{q}_1-\mathbf{p}_1\end{matrix}$	$\mathbf{m}_2=[\mathbf{m}_3\ \mathbf{m}_1]$

ERRATA TO VOLUME 12

page	reads	should read
Article by Dzhelepov et al.		
205, figure caption	54	5.4
Article by M. Gavrila		
225, Eq. (2), last line	$-2\gamma\Theta^{-4}\,{}^1\!/_8$	$-2\gamma\Theta^{-4}-{}^1\!/_8$
Article by Dolgov-Savel'ev et al.		
291, caption of Fig. 5, 4th line	$p_0 = 50\times10^{-4}$ mm Hg	$p_0 = 5\times10^{-4}$ mm Hg.
Article by Belov et al.		
396, Eq. (24) second line	$\ldots-(4-2\eta)\,\sigma_1+\ldots$	$\ldots+(4-2\eta)\,\sigma_1+\ldots$
396, 17th line (r) from top	. . less than 0.7	. . less than 0.07
Article by Kovrizhnykh and Rukhadze		
615, 1st line after Eq. (1)	$\omega_{0e}^2=2\pi e^2n_e/m_e,$	$\omega_{0e}^2=4\pi e^2n_e/m_e,$
Article by Belyaev et al.		
686, Eq. (1), 4th line	$\ldots b_{\rho_2' m_2'}(s_2')+\cdots$	$\ldots b_{\rho_1' m_1'}(s_1')+\cdots$
Article by Zinov and Korenchenko		
798, Table X, heading of last column	$\sigma_{\pi^-\to\pi^+}=$	$\sigma_{\pi^-\to\pi^-}=$
Article by V. M. Shekhter		
967, 3d line after Eq. (3)	$\varepsilon\equiv 2m_pE+m_p^2$	$\varepsilon\equiv(2m_pE+m_p^2)^{1/2}$
967, Eq. (5), line 2	$+(B_V^2+B_A^2)\ldots$	$+(B_V^2+B_A^2)\,Q\ldots$
968, Eq. (7)	$\ldots(C_V^2+C_A^2).$	$\ldots C_V^2+C_A^2-Q^2(B_V^2+B_A^2).$
968, line after Eq. (7)	for $C_V^2+C_A^2\equiv\ldots$	for $C_V^2+C_A^2$ $-Q^2(B_V^2+B_A^2)\equiv\ldots$
Article by Dovzhenko et al.		
983, 11th line (r)	$\gamma=1.8\pm$	$\gamma=1.8\pm0.2$
Article by Zinov et al.		
1021, Table XI, col. 4	−1,22	1,22
Article by V. I. Ritus		
1079, line 27 (l)	$-\Lambda_{\pm}(t),$	$\Lambda_{\pm}(t),$
1079, first line after Eq. (33)	$\frac{1}{2}(1+\beta).$	$\frac{1}{2}(1\pm\beta).$
1079, 3d line (l) from bottom	$\ldots\mathfrak{N}(q'p;\ pq')\ldots$	$\ldots\mathfrak{N}(p'q;\ pq')\ldots$
Article by R. V. Polovin		
1119, Eq. (8.2), fourth line	$U_{0x}u_xg(\gamma)-[\gamma\cdots$	$-U_{0x}u_xg(\gamma)\,[\gamma\cdots$
1119, Eq. (8.3)	$\cdots$sign $u.$	$\cdots$sign $u_g.$
Article by V. P. Silin		
1138, Eq. (18)	$\cdots+\frac{4}{5}\,c^2k^2$	$\cdots+\frac{6}{5}\,c^2k^2.$