ON THE ELECTROMAGNETIC PROPERTIES OF A RELATIVISTIC PLASMA

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We considered the dielectric constant for a relativistic electron-ion plasma, taking spatial dispersion into account. We obtained expressions for the screening radius and the skin-depth. Both undamped and weakly damped plasma oscillations were considered.

1. Relativistic plasma has recently drawn more and more the attention of physicists, as witness the number of papers devoted to the theory of a relativistic electron plasma.[1-7] The present communication is devoted to a consideration of the electromagnetic properties of an electron-ion plasma characterized by a complex dielectric constant that takes spatial dispersion into account. We shall in the following not take into account the dissipation caused by the collisions between particles.

2. In a medium with spatial dispersion — the simplest example of which is a plasma — the connection between the induction and the electrical field strength is well known to be (see, for instance, reference 8) nonlocal both in time and in space. We have, namely, for an infinite uniform medium

$$D_i(\mathbf{r}, t) = \int d\mathbf{r}' \int_{-\infty}^{t} dt' \hat{\varepsilon}_{ij}(\mathbf{r}-\mathbf{r}', t-t') E_j(\mathbf{r}', t'). \quad (1)$$

We take for the dielectric constant tensor the quantity

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \int d\mathbf{r} e^{-i\mathbf{k}\mathbf{r}} \int_0^{\infty} dt e^{i\omega t} \hat{\varepsilon}_{ij}(\mathbf{r}, t). \quad (2)$$

It is necessary also to define the induction of the electric field. We shall not use in the following the concept of the magnetic field strength, so that we take for $\mathbf{D}$ the definition

$$\mathbf{D}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) + 4\pi \int_{-\infty}^{t} dt' \mathbf{j}(\mathbf{r}, t').$$

These relations define a dielectric constant that takes spatial dispersion into account.

In the case of an isotropic system, the dielectric-constant tensor can be written in the form[2]

$$\varepsilon_{ij}(\omega, \mathbf{k}) = (\delta_{ij} - k_i k_j / k^2)\, \varepsilon^{tr}(\omega, k) + (k_i k_j / k^2)\, \varepsilon^{l}(\omega, k).$$

Such a tensor enables us to write down many electromagnetic properties of the medium. In particular, the transverse field oscillations are defined by the condition

$$c^2 k^2 - \omega^2 \varepsilon^{tr}(\omega, k) = 0, \quad (3)$$

and the longitudinal oscillations by the condition

$$\varepsilon^{l}(\omega, k) = 0. \quad (4)$$

The conventional dielectric constant corresponds to the limit of the longitudinal and transverse dielectric constants for $k = 0$

$$\varepsilon(\omega) = \lim_{k=0} \varepsilon^{l}(\omega, k) = \lim_{k=0} \varepsilon^{tr}(\omega, k).$$

The concept of a screening radius for the interaction is of great importance to the description of the interaction between particles. Such a concept has a simple physical meaning only in the case of a static field, and is thus defined through the limit of ϵ_{ij} for $\omega = 0$. The limit of the longitudinal dielectric constant is then finite. The screening radius for the longitudinal interaction is therefore defined by the equation

$$\lim_{k=0} \lim_{\omega/k=0} k^2(\varepsilon^{l} - 1) = r_{\mathrm{scr}}^{-2}.$$

It is necessary to let ω tend to zero first, and afterwards take the limit $k \to 0$. This means that one can only speak about a screening radius at large distances, where the decrease of the field is characterized by an exponential law.*

We must finally discuss still one more characteristic of the electromagnetic properties, also determined by the dielectric constant. In the limit $\omega/k \to 0$, the skin depth is a characteristic distance in the variation of the transverse field. When the mean free path can be assumed to be infinite, the anomalous skin-effect

*In the language of the theory that uses Green functions to describe many-particle systems, the different limits of ε when $k/\omega \to 0$ and when $\omega/k \to 0$ correspond to different limits of the field Green functions in the time and space regions for small k and ω. A similar situation occurs also for the vertex part and also for the two-particle Green function, as was noted by Landau.[9]

occurs. The transverse dielectric constant can then be written in the form*

$$\varepsilon^{tr} = (4\pi i/\omega)\,\sigma^{tr}(k), \qquad \sigma^{tr} = C/k,$$

where C is a constant which depends on the actual properties of the plasma. One easily sees from this and from Eq. (3) that the skin depth of the transverse field is equal to

$$\delta = (2c^2/\pi C\omega)^{1/3}.$$

We go now over from these general considerations to a discussion of the dielectric-constant tensor and of the quantities that determine this tensor for a relativistic plasma.

3. We shall use Vlasov's self-consistent field approximation to obtain the dielectric-constant tensor of a relativistic plasma. We have then for a description of the electron and ion distributions (see, for instance, references 2, 3, 5, and 7)

$$\frac{\partial f_{el}}{\partial t} + \mathbf{v}_{el}\frac{\partial f_{el}}{\partial \mathbf{r}} + e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_{el}\times\mathbf{B}]\right)\frac{\partial f_{el}}{\partial \mathbf{p}} = 0,$$
$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i\frac{\partial f_i}{\partial \mathbf{r}} - Ze\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_i\times\mathbf{B}]\right)\frac{\partial f_i}{\partial \mathbf{p}} = 0. \qquad (5)$$

f_{el} and f_i are here the electron and ion distribution functions, $\mathbf{v}_{el}$ and $\mathbf{v}_i$ the velocities of the corresponding particles, and Z the ionic charge. The electrical charge and current densities are equal to

$$\rho = e\int d\mathbf{p}\,(f_e - Zf_i), \qquad \mathbf{j} = e\int d\mathbf{p}\,(\mathbf{v}_{el}f_{el} - Z\mathbf{v}_i f_i).$$

We can obtain the dielectric constant from the linearized transport equations, in which we retained in the last term only the equilibrium distribution function, as we assume that there is no constant magnetic field. Moreover, it is necessary to take retardation into account when we determine the dielectric constant. In that case, if we have at time t_0 a given non-equilibrium addition $\delta f(\mathbf{r}, \mathbf{p}, t_0)$ to the distribution function, we have at time t

$$\delta f_{el}(\mathbf{r}, \mathbf{p}, t) = \delta f_{el}(\mathbf{r} - \mathbf{v}_{el}(t - t_0), \mathbf{p}, t_0)$$
$$- \frac{\partial f_{el}^0}{\partial \mathbf{p}} e\int_{t_0}^{t} dt'\mathbf{E}(\mathbf{r} - \mathbf{v}_{el}(t - t'), t'). \qquad (6)$$

Here f_{el}^0 is the electron equilibrium distribution function. The solution of the transport equation for the ions has a similar form.

Assuming the interaction to be switched on adiabatically infinitely far back in time, we have from (6)

$$\delta f_{el}(\mathbf{r}, \mathbf{p}, t) = -\frac{\partial f_{el}^0}{\partial \mathbf{p}} e\int_{-\infty}^{t} dt'\mathbf{E}(\mathbf{r} - \mathbf{v}_{el}(t - t'), t'). \qquad (7)$$

Using (3) and (7) we get

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \delta_{ij} + \frac{4\pi e^2}{\omega}\int d\mathbf{p}\left\{\frac{v_{eli}}{\omega - \mathbf{k}\mathbf{v}_{el}}\frac{\partial f_{el}^0}{\partial p_j} + Z^2\frac{v_{ii}}{\omega - \mathbf{k}\mathbf{v}_i}\frac{\partial f_i^0}{\partial p_j}\right\}. \qquad (8)$$

The dielectric constant tensor (8) was obtained by us as a function of the real arguments ω and k. In accordance with the fact that we are dealing with a retardation, the contour for integration over the momenta must be arranged such that the pole $\omega = (\mathbf{k}\cdot\mathbf{v})$ of the integrand on the right hand side of (8) is encircled from below.[10]

From (8) we have

$$\varepsilon(\omega) = 1 - \frac{4\pi e^2}{3\omega^2}\int d\mathbf{p}\left\{f_{el}^0\frac{\partial \mathbf{v}_{el}}{\partial \mathbf{p}} + Z^2 f_i^0\frac{\partial \mathbf{v}_i}{\partial \mathbf{p}}\right\},$$
$$r_{scr}^{-2} = -4\pi e^2\int d\mathbf{p}\,\{f_{el}^{0'} + Z^2 f_i^{0'}\},$$
$$C = -\frac{\pi e^2}{2}\int d\mathbf{p}\left\{f_{el}^{0'}v_{el}\delta\left(\frac{\mathbf{k}\mathbf{v}_{el}}{kv_{el}}\right) + Z^2 f_i^{0'}v_i\delta\left(\frac{\mathbf{k}\mathbf{v}_i}{kv_i}\right)\right\}. \qquad (9)$$

We have taken it into account here that the equilibrium distribution functions are functions of the energy; the prime indicates a derivative with respect to the energy.

4. Let the electrons be ultrarelativistic. Their contribution to the dielectric constant will then be of the form

$$\delta\varepsilon_{el\,ij}(\omega, \mathbf{k}) = \frac{4\pi e^2 c}{\omega}\int d\mathbf{p}\,\frac{\partial f_{el}^0}{\partial p}\frac{p_i p_j}{p^2}\frac{1}{\omega - c\mathbf{k}\mathbf{p}/p}. \qquad (10)$$

We have then for the longitudinal and transverse dielectric constants

$$\delta\varepsilon_{el}^{l}(\omega, k) = \frac{4\pi e^2}{ck^2}A\left\{1 + \frac{\omega}{2ck}\ln\left|\frac{\omega - ck}{\omega + ck}\right| + \frac{i\pi\omega}{4ck}\left(\frac{ck - \omega}{|ck - \omega|} + 1\right)\right\}, \qquad (11)$$

$$\delta\varepsilon_{el}^{tr}(\omega, k) = \frac{\pi e^2}{ck\omega}A\left\{-\frac{2\omega}{ck} + \left[1 - \left(\frac{\omega}{ck}\right)^2\right]\ln\left|\frac{\omega - ck}{\omega + ck}\right| + \right.$$
$$\left. + \frac{i\pi}{2}\left[1 - \left(\frac{\omega}{ck}\right)^2\right]\left(\frac{ck - \omega}{|ck - \omega|} + 1\right)\right\},$$
$$A \equiv c\,(\partial N_{el}/\partial\mu_{el}) \equiv -\int d\mathbf{p}\,\partial f_{el}^0/\partial p. \qquad (12)$$

In the case of a Boltzmann electron distribution $[(f \sim \exp(-cp/\kappa T_{el})]$ we get

$$A = cN_{el}/\kappa T_{el}, \qquad (13)$$

where N_{el} is the number of electrons per unit volume,* κ is Boltzmann's constant, and T_{el}

*In the case of a quantum-mechanical system, the limit of the dielectric constant as $\omega/k \to 0$ contains terms $\sim(k/\omega)^2$, which lead to diamagnetism. One can easily understand this from the definition of the magnetic permeability[2] $\mu(\omega, \mathbf{k})$:

$$1 - \mu^{-1}(\omega, k) = (\omega^2/c^2k^2)(\varepsilon^{tr} - \varepsilon^{l}).$$

*In the case where there is an equilibrium number of electron-positron pairs, $N_{el} = 0.183\,(\kappa T_{el}/\hbar c)^3$.

the electron temperature. For the case where the electron gas is degenerate we have $A = 3N_{el}/p_0$, where $p_0 = (3\pi^2)^{1/3}\hbar N_{el}^{1/3}$ is the limiting momentum of the Fermi distribution.* Equations (11) and (12) are similar to the ones occurring in the theory of a degenerate nonrelativistic electron gas. The reason for this is that, both in the case of the ultrarelativistic gas and in the case of the nonrelativistic degenerate electron gas, the particles determining the dielectric constant have all the same velocity, the velocity of light in our case and the velocity on the Fermi surface in the case of a degenerate Fermi gas.

The limiting characteristics of $\epsilon(\omega)$, r_{scr}^{-2}, and C following from Eqs. (11) and (12) are of the form

$$\delta\varepsilon_{el}(\omega) = -\frac{4\pi e^2 c}{3\omega^2}A, \quad \delta(r_{scr}^{-2})_{el} = \frac{4\pi e^2}{c}A, \quad \delta C_{el} = \frac{\pi e^2}{4}A. \quad (14)$$

For nonrelativistic particles, and we consider the ions to be such particles, the contribution to the dielectric constant is well known to be

$$\delta\varepsilon_{i\,ij}(\omega, \mathbf{k}) = \frac{4\pi e^2 Z^2}{\omega M}\int dp\frac{p_i}{\omega - \mathbf{kp}/M}\frac{\partial f_i^0}{\partial p_j},$$

where M is the ionic mass. In particular we have for a Maxwell distribution of the ions

$$\delta\varepsilon_i^l(\omega, k) = \frac{\omega_{0i}^2}{k^2}\frac{M}{\varkappa T_i}\left\{1 + \frac{\omega}{k}\sqrt{\frac{M}{2\varkappa T_i}}\frac{1}{\sqrt{\pi}}\int\frac{dx e^{-x^2}}{x - (\omega/k)\sqrt{M/2\varkappa T_i}}\right\}, \quad (15)$$

$$\delta\varepsilon_i^{tr}(\omega, k) = \frac{\omega_{0i}^2}{k\omega}\sqrt{\frac{M}{2\varkappa T_i}}\frac{1}{\sqrt{\pi}}\int\frac{dx e^{-x^2}}{x - (\omega/k)\sqrt{M/2\varkappa T_i}}, \quad (16)$$

where $\omega_{0i}^2 = 4\pi e^2 Z^2 N_i/M$, and where the pole of the integrand is encircled from below.

From (15) and (16) we have the following equations which are analogous to Eqs. (14)

$$\delta\varepsilon_i(\omega) = -\frac{\omega_{0i}^2}{\omega^2}, \quad \delta(r_{scr}^{-2})_i = \frac{4\pi e^2 Z^2 N_i}{\varkappa T_i},$$

$$\delta C_i = \frac{\omega_{0i}^2}{4}\sqrt{\frac{M}{2\pi\varkappa T_i}}. \quad (17)$$

Comparing Eqs. (14) and (17) we see that if the electrons have a Boltzmann distribution, the extra contribution to $\epsilon(\omega)$ is determined by the electrons in each case where the condition $\kappa T_{el} \ll Mc^2$ is satisfied. In order that the same be true for the skin depth, it is necessary that the condition $\kappa T_{el} \ll \sqrt{Mc^2\kappa T_i}$ be satisfied. It is evident that this condition is violated when $T_{el} \gtrsim 10^3 T_i$. Finally, for the screening radius the situation is the same as in the nonrelativistic case, namely

$$r_{scr}^{-2} = (4\pi e^2/\varkappa T_{el})[1 + ZT_{el}/T_i].$$

This is due to the fact that in the nonrelativistic limit the screening is independent of the mass of the particles, but the difference in the relativistic case lies essentially in the dependence of the mass on the velocity. We note that the conditions under which the electromagnetic properties of the plasma are determined by the electrons or by the ions are completely similar for ultrarelativistic Fermi-Dirac electrons; we must only use $cp_0/3$ instead of κT_{el}.

5. We turn now to a consideration of the propagation of electromagnetic waves in a relativistic plasma.

In a nonrelativistic isotropic plasma the motion of the particles and accordingly the change, reflecting this motion, of the spatial dispersion of the dielectric constant influences the propagation of transverse waves only weakly. In our case the particle velocity is very large and the part played by the spatial dispersion therefore increases. This is clear from the following relations for the transverse waves, obtained from (3) by assuming that the electron energy is small compared to the rest mass energy of the ions.*

$$\omega^2 = \frac{4}{3}\pi e^2 cA + \frac{4}{5}c^2k^2 \quad \text{if } \omega \gg ck, \quad (18)$$

$$\omega^2 = 2\pi e^2 cA + c^2k^2 \quad \text{if } \omega \to ck. \quad (19)$$

Indeed, in the case of Eq. (18), taking the motion of the particles into account changes the term proportional to k^2 by 20%. In the case of Eq. (19), however, the constant term is changed by a factor of one and a half by taking into account the spatial dispersion. Because the phase velocity of the transverse waves is greater than the velocity of light and is equal to it in the limiting case of infinitely short waves, the transverse waves are not damped. We can thus drop the imaginary parts when substituting (12) and (16) into (3).

We turn now to longitudinal waves. Two modes of natural vibrations, the so-called plasma and acoustical branches are possible, as in the case of a nonrelativistic plasma.[11] We consider first the plasma oscillations. In this case one can easily see from (4), (14), and (17) that in the long-wave case the frequency tends to a limit equal to†

$$\omega_0 = (4\pi e^2 A/3 + \omega_{0i}^2)^{1/2}. \quad (20)$$

*The dielectric constant of a degenerate relativistic electron gas was considered in a paper by Lindhard.[2]

*In the nonrelativistic case the equation $\omega^2 = \omega_{0\,el}^2 + c^2k^2$, where $\omega_{0\,el}^2 = 4\pi e^2 N_{el}/m$ corresponds to Eqs. (18) and (19).

†Assuming that the electron energy is small compared with the rest-mass energy of the ions, we have $\omega^2 = (4/3)\pi e^2 cA + (3/5)c^2k^2$ in the region $ck \ll \omega_0$.

It follows from this that as $k \rightarrow 0$ the phase velocity increases without limit.

It is well known that in the long-wave region, where the phase velocity is greater than the velocity of light, there is no damping.[4] As long as we can speak about ultrarelativistic electrons, i.e., as long as we can neglect the difference between their velocity and the velocity of light, the minimum phase velocity of the longitudinal plasma waves is also the same as the velocity of light. Indeed, we have in the short-wave limit ($kc \gg \omega_0$) from (4), (11), and (15);

$$\omega = ck\left\{1 + 2\exp\left[-\frac{ck^2}{2\pi e^2 A} - 2 + \frac{\omega_{oi}^2}{2\pi e^2 cA}\right]\right\}. \quad (21)$$

The damping is then, of course, also equal to zero. However, to be able to apply Eq. (21) to the case of a Boltzmann plasma, the electron temperature must satisfy the condition

$$\varkappa T_{el} \gg mc^2 \ln (ck/\omega_0).$$

It is thus necessary for short wavelengths, where this condition is violated, to take into account the fact that the particle velocity is different from the velocity of light. In other words it is no longer possible to use Eq. (10), obtained for ultrarelativistic electrons.

Since, however, the average thermal velocity of the particles in an ultrarelativistic gas differs only very little from the velocity of light, the minimum phase velocity will also differ little from the light velocity. One can, namely, use (8) to verify that this difference is of the order of magnitude of $c(mc^2/\kappa T_{el})^2$. It is also important that the number of electrons with velocities appreciably different from the average thermal velocity be very small, i.e., that there be a small spread in electron velocities. This leads to the fact that when the phase velocity of the plasma wave tends to its minimum value the damping turns out to be relatively small, namely $\gamma/\omega \sim (mc^2/\kappa T_{el})^2$.

The second branch of longitudinal vibrations — acoustical waves — is only possible in a plasma because of the spatial dispersion of the dielectric constant. This should already be evident from the fact that the velocity of such waves can be small compared with the electron velocity. This last fact leads to the occurrence of damping caused by the electrons. The Cerenkov mechanism of damping of the oscillations in a plasma occurs when there is some group of particles for which the condition $(\mathbf{k}\cdot\mathbf{v}) = \omega$ is satisfied. In our case the electron velocity differs little from the light velocity; if, however, the sound velocity is small compared to the light velocity, only a small part of the electrons, moving nearly perpendicularly to the direction of the propagation of sound, will take part in the absorption of the vibrations. Similarly, if the phase velocity of the wave is appreciably larger than the average thermal velocity of the ions, only a small group of ions which have velocities much higher than the thermal ones will be involved in the absorption of sound. Under such circumstances where a small part of the particles in the plasma is involved in the absorption, one can expect the damping of sound to turn out to be relatively small.

If the electron velocities have a Boltzmann distribution the velocity of sound turns out to be small compared with the average velocity and at the same time large compared with the thermal velocity of the ions, when $Mc^2 \gg Z\kappa T_{el} \gg \kappa T_i$. We get then from (4), (11), and (15) for the velocity of sound

$$v_s = \omega/k = \sqrt{Z\varkappa T_{el}/M}. \quad (22)$$

Equation (22) is the same as the corresponding formula for the sound velocity obtained in the nonrelativistic plasma theory (see reference 12). This is understandable, as the sound velocity in that case is independent of the electron mass.

If the electron distribution is a Fermi-Dirac one, the condition similar to those under which Eq. (22) is valid in the Boltzmann case is determined by the inequalities $Mc^2 \gg Zp_0c \gg \kappa T_i$. We have then

$$v_s = \sqrt{Zp_0 c/3M}. \quad (23)$$

The contributions from the electrons and the ions add up for the absorption of sound waves of a velocity determined by Eqs. (22) and (23). The electron part of the logarithmic decrement is connected with the sound frequency by the relation

$$\gamma_{el} = \frac{\pi}{4}\frac{v_s}{c}\omega. \quad (24)$$

We note that such an absorption of sound in a plasma is very similar to the absorption of sound by electrons in a metal in the low-temperature region, which has been extensively studied recently both theoretically and experimentally.

The ionic part of the logarithmic decrement is determined by the expression

$$\gamma_i = \omega\sqrt{\pi}\left(\frac{Mv_s^2}{2\varkappa T_i}\right)^{3/2}\exp\left(-\frac{Mv_s^2}{2\varkappa T_i}\right), \quad (25)$$

the structure of which shows that for this part we can essentially use the results of the nonrelativistic theory.[13]

We emphasize that to be able to apply Eqs. (24) and (25) it is also necessary that the Cerenkov dissipation of the sound wave be the basic one, as the entire foregoing analysis does not take into account the usual dissipation mechanism, which is connected with collisions.

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Translated by D. ter Haar
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ERRATA TO VOLUME 10

page	reads	should read
Article by A. S. Khaĭkin		
1044, title	. . . resonance in lead	. . . resonance in tin
6th line of article	~ 1000 oe	~ 1 oe
Article by V. L. Lyuboshitz		
1223, Eq. (13), second line	$\ldots - Sp_{1,2}\mathscr{E}(e_1)$	$\ldots - Sp_{1,2}\mathscr{E}(e_2)\ldots$
1226, Eq. (26), 12th line	$\cdots\{(p+q, p$	$\cdots\{(p+q, p)-(p+q, n)\cdot(pn)\}$
1227, Eqs. (38), (41), (41a) numerators and denominators	(p^2-q)	$(p^2-q^2)^2$
1228, top line	$m_2 = \frac{q_1 - p_1}{q_1 - p_1}$	$m_2 = [m_3\, m_1]$

ERRATA TO VOLUME 12

page	reads	should read
Article by Dzhelepov et al.		
205, figure caption	54	5.4
Article by M. Gavrila		
225, Eq. (2), last line	$-2\gamma\Theta^{-4}\,1/8$	$-2\gamma\Theta^{-4} - 1/8$
Article by Dolgov-Savel'ev et al.		
291, caption of Fig. 5, 4th line	$p_0 = 50 \times 10^{-4}$ mm Hg	$p_0 = 5 \times 10^{-4}$ mm Hg.
Article by Belov et al.		
396, Eq. (24) second line	$\ldots - (4-2\eta)\sigma_1 + \ldots$	$\ldots + (4-2\eta)\sigma_1 + \ldots$
396, 17th line (r) from top	. . less than 0.7	. . less than 0.07
Article by Kovrizhnykh and Rukhadze		
615, 1st line after Eq. (1)	$\omega_{0e}^2 = 2\pi e^2 n_e/m_e$,	$\omega_{0e}^2 = 4\pi e^2 n_e/m_e$,
Article by Belyaev et al.		
686, Eq. (1), 4th line	$\cdots b_{\rho_2 m_2}(s_2') + \cdots$	$\cdots b_{\rho_1 m_1}(s_1') + \cdots$
Article by Zinov and Korenchenko		
798, Table X, heading of last column	$\sigma_{\pi^- \to \pi^+} =$	$\sigma_{\pi^- \to \pi^-} =$
Article by V. M. Shekhter		
967, 3d line after Eq. (3)	$\varepsilon \equiv 2m_pE + m_p^2$	$\varepsilon \equiv (2m_pE + m_p^2)^{1/2}$
967, Eq. (5), line 2	$+ (B_V^2 + B_A^2)\ldots$	$+ (B_V^2 + B_A^2)\, Q \ldots$
968, Eq. (7)	$\ldots (C_V^2 + C_A^2)$.	$\ldots C_V^2 + C_A^2 - Q^2(B_V^2 + B_A^2)$.
968, line after Eq. (7)	for $C_V^2 + C_A^2 \equiv \ldots$	for $C_V^2 + C_A^2 - Q^2(B_V^2 + B_A^2) \equiv \ldots$
Article by Dovzhenko et al.		
983, 11th line (r)	$\gamma = 1.8 \pm$	$\gamma = 1.8 \pm 0.2$
Article by Zinov et al.		
1021, Table XI, col. 4	−1,22	1,22
Article by V. I. Ritus		
1079, line 27 (1)	$-\Lambda_{\pm}(t)$,	$\Lambda_{\pm}(t)$,
1079, first line after Eq. (33)	$\frac{1}{2}(1+\beta)$.	$\frac{1}{2}(1\pm\beta)$.
1079, 3d line (1) from bottom	$\ldots\mathfrak{N}(q'p;\, pq')\ldots$	$\ldots\mathfrak{N}(p'q;\, pq')\ldots$
Article by R. V. Polovin		
1119, Eq. (8.2), fourth line	$U_{0x}u_xg(\gamma) - [\gamma\cdots$	$-U_{0x}u_xg(\gamma)\,[\gamma\cdots$
1119, Eq. (8.3)	$\cdots$sign u.	$\cdots$sign u_g.
Article by V. P. Silin		
1138, Eq. (18)	$\cdots + \frac{4}{5}c^2k^2$	$\cdots + \frac{6}{5}c^2k^2$.