

ON THE TEMPERATURE OF LIGHTNING AND FORCE OF THUNDER

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The temperature in the lightning channel was measured by a spectroscopic method with the help of the N II and O II lines and a value $T \sim 20\,000^\circ$ was obtained. This value is in good agreement with the results of a calculation based on the hydrodynamic theory of development of the spark channel. The hydrodynamic model was further used to calculate the force of the thunder, i.e., the pressure on the front of the shock wave. Excess pressures are obtained which in a number of cases may lead to destruction of objects located a few meters from the lightning.

WE report in this paper preliminary results of spectroscopic measurements of the temperature of a lightning channel in the high-current stage, and of calculations of the pressure on the front of the shock wave produced by the lightning.

Many papers dealing with the spectrum of lightning have been published;¹ so far, however, its temperature has not been measured nor have the pressures produced by the thunder been calculated.

The conditions under which the lightning channel is produced and the state of the plasma in it are similar to the those in the channel of a condensed spark discharge in air, which have been thoroughly investigated in our laboratory. This enables us to identify the thunder with the shock wave produced by the rapid expansion of the channel,² and to calculate the pressure on the front of the shock wave (thunder) in analogy with the procedure used for the spark discharge.³

In the ordinary laboratory spark discharge, produced at atmospheric pressure, the concentration of the electrons is $N_e \geq 10^{17} \text{ cm}^{-3}$. At these values of N_e , the excited levels of the atoms and ions have a Boltzmann distribution which sets in within $\sim 10^{-7}$ sec. As a result, the channel plasma emits Kirchhoff-law⁴ radiation practically during its entire lifetime. The gas temperature reaches the electron temperature $T_e = T_i = T$ within $\sim 10^{-6}$ sec (reference 5), and can therefore be determined spectroscopically from the atomic lines. The same conclusions apply to lightning; the concentration of the electrons in the lightning channel, as follows from estimates based on the widths of the H_α and H_β lines in the lightning spectra which we investigated, is of the same order of magnitude.

Let us assume that the temperature is constant over the channel section. Then, by Kirchhoff's law, the radiation intensity at a frequency ω is given within the limits of the spectral line by the expression

$$I(\omega) = I_0(\omega)(1 - e^{-k_\omega l}), \quad (1)$$

where $I_0(\omega)$ is the intensity of black-body radiation at the channel temperature, l is the channel diameter, and k_ω is the plasma absorption coefficient for the frequency ω .

In the case of a line broadened by collisions and by the Doppler effect, we have

$$k_\omega = k_0 \frac{a}{\pi} \int_{-\infty}^{+\infty} \frac{\exp(-y^2)}{a^2 + (v-y)^2} dy, \quad (2)$$

$$k_0 = (2\pi^{1/2}/mc) (N_f/\Delta\omega_0), \quad v = 2(\omega - \omega_0)/\Delta\omega_d.$$

Here k_0 is the coefficient of absorption at the center of the line in the absence of broadening by collisions, $a = \Delta\omega_c/\Delta\omega_d$ where ω_d is the Doppler width of the line and ω_c is the line width due to the collisions.*

Let us estimate the optical thickness of the plasma for different portions of the line. The estimates given below pertain to the 3995-Å line N II.

Assuming $T \sim 2 \times 10^4$ deg K and $\lambda \sim 4000$ Å, we obtain for the nitrogen ions $\Delta\lambda_d = 0.06$ Å. Estimates and measurements of a laboratory spark show that $\Delta\lambda_c$ ranges from several tenths of an angstrom to several angstroms. Assuming $N_0 \sim 10^{17} \text{ cm}^{-3}$ and a channel diameter $l \sim 10$ cm,

*When $N_e = N_i \sim 10^{17} \text{ cm}^{-3}$, the statistical line skirt can be neglected for a line with a quadratic Stark effect.

Table I.

Distance from the center of the line, $\Delta\lambda, \text{A}$	$k_{\omega}l$				
	$\Delta\lambda_c (\text{A}) = 0.05$	0.1	0.15	0.5	2
0.00	104	79.5	65	31.5	6.2
0.08	55	51	45	27	6.2
0.16	11.8	17	19.6	18.4	6.1
0.23	8.8	9.2	10	11.8	5.9
0.31	2.7	2.9	5.2	8.4	5.7
0.38	1.0	2.1	3.5	6.1	5.5
0.45	0.9	1.3	2.1	4.4	5.2

Table II.

Line pair $\lambda (\text{A})$	Channel temperature ($^{\circ}\text{K}$) determined from the spectrograms				
	№ 1	№ 2	№ 3	№ 4	Average
NII 5045	20000	31000	23000	—	25000
NII 3995	—	—	—	—	—
NII 5045	16000	25000	17000	22000	20000
NII 4447	—	—	—	—	—
NII 4447	14000	21000	—	—	18000
OII 3857	—	—	—	—	—
Average					21000

we obtain the values of $k_{\omega}l$ listed in Table I. It follows from this table that, at least within $\pm 0.5 \text{ A}$ and perhaps also at greater distances from the center of the line, the optical thickness of the plasma is in practice appreciably greater than unity. Consequently, the lightning channel radiates as a black body in the central portion of the line. By photographing the spectrum with a spectrograph having an apparatus function width $\leq 1 \text{ A}$ and measuring the relative intensity at the center of the image of each of the lines with different λ , we can determine the temperature of the channel from Eq. (1).

We have photographed the lightning spectrum with the spectrographs SP-48 (operating range 3800 — 6800 A, dispersion 80 — 95 A/mm) and SP-49 (operating range 2600 — 3800 A, dispersion 76 — 80 A/mm). The slit was 0.1 mm wide. These instruments yield a strong reduction, and the width of the apparatus function, measured against the neon and mercury lines and the lines of the lightning spectrum, amounted to $\sim 1 \text{ A}$. The lightning spectrum was photographed at night. At the beginning of the thunderstorm the spectrographs were aimed at the center of the storm area and were then made to follow the motion of the storm. Each spectrogram is thus the result of the superposition of light from several lightnings and corresponds to the entire duration of each discharge. Almost all the photographic density was produced by the direct light from the lightning channel, and not by the light scattered in the clouds; this was verified by aiming the spectrograph on a portion of the sky illuminated only by the scattered light from the lightning.

The photographs were produced with Pankhrom X and type "D" films, of sensitivity 1200 — 1500 GOST units. The film was pre-exposed prior to use to a density $S = 0.2$ to increase the sensitivity (by a factor 3 — 4). The density markers used were photographs of a 9-step wedge, obtained with the aid of a blue filter. To determine the spectral sensitivity of the film, the spectrum of a lumino-phor of known intensity distribution was photographed. The density and spectral-sensitivity

markers were photographed on a section adjacent to the working section of the film and subjected to the same exposure as the working section; both sections were developed simultaneously.*

The density of the film was measured with an MF-4 recording microphotometer. The background was measured on both sides of the line and was allowed for in the usual manner. The spectrograms were calibrated against the mercury and neon spectra; the lines were identified with the aid of the Moore tables.

A total of nine spectrograms was processed. We identified and measured the positions of some 100 lines of which approximately one-half were not previously observed. The majority of the lines belongs to neutral and singly-ionized atoms of oxygen and nitrogen. The observed lines with wavelengths 5600, 5471, 4542, and 4058 A possibly belong to the ions of greater multiplicity (O III, O V, and N III). However, this identification is very unreliable. Several edges of the N_2^+ bands are also observed; the continuous spectrum is relatively intense.

Table III.

Current growth rate	Relative pressure on the front of the shock wave, kg/cm^2		
	$r(\text{cm})=5$	50	500
30kA/10 μsec	9.4	9.2	0.9
30kA/100 μsec	9.5	0.25	0.02
30kA/100 μsec	2.8	Shock wave turns into sound wave	Shock wave turns into sound wave

To determine the channel temperature we measured the lines N II 3995 A, N II 4447 A, O II 3857 A, and N II 5045 A. The measurement results are listed in Table II. The difference in the numbers should probably be attributed to the

*The use of the average contrast factor for the entire 4000–5000 A region does not, as shown by special estimates, introduce any appreciable error in the temperature measurement. Likewise, with the types of film employed, the error due to the difference in the exposure of the lightning spectrum and the density markers, resulting from violation of the reciprocity law, is relatively small.

measurement error, which is very high and amounts to $\sim 5000^\circ$. The value $T \sim 20\,000^\circ$ obtained for the channel temperature apparently characterizes a certain average stage of the discharge. The final stages and the peripheral portions of the discharge have obviously a lower temperature, as evidenced by the presence of O I, N I, and N_2^+ bands in the spectrum. The initial stages of the discharge have probably a somewhat higher temperature.*

The temperature obtained is in good agreement with that estimated by the Braginskii formula,⁷ which is based on the hydrodynamic theory of development of the channel:

$$T = 4.35\rho_0^{1/4}(Jt^{-3/4})^{2/3}.$$

Here ρ_0 is in units of 1.29×10^{-3} g/cm³, J is in kiloamp, t is in μ sec and T in ev. Taking as typical values $J \sim 30$ kiloamp and $t = 100 - 1000$ μ sec,⁸ we obtain $T = 16,000 - 25,000^\circ$.

We can now apply the hydrodynamic theory of development of the spark-discharge channel to the calculation of the other parameters of the lightning channel, particularly the radius and the pressure of the shock wave. We assume, following Braginskii's calculations,⁷ that the current in the lightning varies as $J \sim t^{3/4}$, and that the conductivity of the channel is practically constant at $\sigma = 2 \times 10^{14}$ cgs esu. Then, assuming no loss to radiation, the Drabkina formulas³ yield the following expression for the radius and velocity of the shock-wave front:

$$r = 1.11\eta^{1/4}\rho_0^{-3/16}J^{3/4}t^{13/16}, \quad a = 5.55 \cdot 10^6 \eta^{1/4}\rho_0^{3/16}J^{5/4}t^{-9/16}.$$

Here r is in cm, a in cm/sec, J in kiloamp, t in μ sec, and ρ_0 in units of 1.29×10^{-3} g/cm³. The coefficient η defines the ratio of the energy required for the translational motion of the gas to the total energy released in the channel. According to Taylor's calculations for a spherical shock wave,⁹ $\eta = 0.1$. Gegechkori² calculated $\eta = 0.2$ for a cylindrical wave from a laboratory spark discharge.

The pressure on the front of the shock wave is connected with the velocity of the front by the relation

*The temperature of the channel of the laboratory spark discharge in air amounts to $35\,000^\circ$ at the initial stages of the discharge.⁵ Mak's measurements⁶ have shown that when the wave front of the current of the laboratory discharge is varied over a wide range, the discharge temperature remains practically constant. One can therefore assume that in spite of the considerable differences in the parameters of the discharge circuits of the lightning and of the laboratory spark, the lightning temperature in the initial stage also amounts to $30\,000 - 35\,000^\circ$. At the present time we are setting up experiments on the measurement of the lighting temperature with time sweep.

$$p_f = \frac{2\rho_0 a^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1} \rho_0,$$

where p_0 is the atmospheric pressure and γ the adiabatic exponent. Carrying out the calculations for a lightning with $J \sim 30$ kiloamp, and assuming that this value is reached within $t = 10, 100,$ and 1000 μ sec, respectively, we obtain the values of p_f listed in Table III.

Thus, at an average current growth of 30 kiloamp/100 μ sec, considerable pressures are developed on the front of the shock wave, and consequently the thunder can seriously damage objects located at a distance up to several meters. By way of an example we indicate that, in accordance with the handbook data,¹⁰ an excess pressure of 0.07 kg/cm² acting on the entire surface will shatter a window pane, 0.14 kg/cm² will damage frame buildings and telegraph poles, 0.365 kg/cm² produces cracks in 9-inch brick walls, etc.

The lightning spectra were obtained in the summer of 1958 at the Zvenigorod Station of the Institute of Atmospheric Physics, Academy of Sciences, U.S.S.R.

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