BREMSSTRAHLUNG FROM A LONGITUDINALLY POLARIZED ELECTRON WITH ACCOUNT OF THE FINITE SIZE OF THE NUCLEUS

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The effect of the finite size of the nucleus on the angular distribution of circularly-polarized bremsstrahlung emitted by a longitudinally polarized high-energy electron is considered. An expression for the angular distribution of external bremsstrahlung is derived by taking into account the mean square radius $\langle r^2 \rangle$ of the nuclear charge distribution and the longitudinal spin correlation between the initial electron state and emitted γ quantum. A formula is deduced for the effect of the finite nuclear size on the angular dependence of the degree of circular polarization of the bremsstrahlung.

1. INTRODUCTION

THE polarization correlation between the electron and the γ quantum in the bremsstrahlung of an electron in the Coulomb field of a point nucleus was considered in several papers.[1-7] It was shown that the bremsstrahlung quanta from a high-energy electron polarized in (or opposite to) the direction of motion have circular polarization, the degree of which may reach almost 100 percent near the upper limit of the spectrum.[1-5] It has been established experimentally that for a longitudinally polarized electron with kinetic energy ~ 2 Mev the degree of circular polarization of the bremsstrahlung reaches a maximum value ~ 95 percent in the upper end of the spectrum.[8-11] Thus, the theoretical results[1-5] on the polarization properties of the bremsstrahlung have proved to be in satisfactory agreement with the experimental data.

As shown by Olsen and Maximon,[5] the degree of circular polarization of bremsstrahlung quanta from a polarized high-energy electron is practically independent of the screening of the field of the nucleus and of the Coulomb corrections. Biel and Burhop[12] considered the bremsstrahlung of a relativistic electron in the Coulomb field of an extended nucleus without allowance for the polarized states of the incident electron and the emitted γ quantum, in the Born approximation. They have shown that when the electron has high energy ($E \gg m_0c^2$) the influence of the density of nuclear charge distribution on the angular distribution of the bremsstrahlung becomes appreciable in the region of γ-quantum emission angles $\theta > 10°$. An analysis of the influence of the finite dimensions of the nucleus on the polarization of the electrons in double scattering is the subject of a paper by Kerimov and Arutyunyan.[13]

In the present article we calculate in the Born approximation the angular dependence of the external bremsstrahlung of a relativistic electron with allowance for the finite dimensions of the nucleus and for the polarized states of the initial electron and of the emitted γ quantum. Formulas are derived for the dependence of the ratio of the cross section of circularly polarized bremsstrahlung from an extended nucleus to the cross section for a point nucleus on the angle of emission of the γ quantum and the mean square radius of the nucleus for a given value of energy of the longitudinally polarized electron. It can be expected that the experimental investigation of the formulas for the cross section of bremsstrahlung from polarized electrons in the region of high energies and angles can also yield definite information on the form factor and on the mean square radius of the distribution of the nuclear charge.

2. ANGULAR DISTRIBUTION OF POLARIZED RADIATION WITH ALLOWANCE FOR THE DIMENSION OF THE NUCLEUS

To take account of the finite dimensions of the nucleus, we shall use the form factor $F(q)$ of the charge density distribution, for which we have, for not too high energies,

$$F(q) = \int_0^\infty \rho(r) \frac{\sin qr}{qr} 4\pi r^2 \, dr \approx 1 - \frac{1}{6} q^2 \langle r^2 \rangle. \quad (1)$$

Here $\rho(\mathbf{r})$ is the density, $\langle r^2 \rangle$ is the mean square radius of the nuclear charge distribution, and $\hbar\mathbf{q}$ is the momentum transferred to the nucleus ($\mathbf{q} = \mathbf{k} - \mathbf{k}' - \boldsymbol{\kappa}$, $\boldsymbol{\kappa}$ is the photon wave vector). At electron energies from 20 to 100 Mev we can confine ourselves to the first two terms of the expansion of $F(q)$ (see reference 14) and obtain a bremsstrahlung cross section that depends only on the mean square of the nuclear radius. At very high electron energies it is necessary to take into account the higher moments of the nuclear charge density.

The longitudinal polarization of an electron with momentum $\mathbf{p} = \hbar\mathbf{k}$ is described by the eigenvalue $s = \pm 1$ of the projection operator $(\boldsymbol{\sigma}\cdot\mathbf{p}/p)\psi = s\psi$ (see references 15 and 1). When $s = 1$ we have a right-hand polarized electron (the spin is in the direction of the motion), and when $s = -1$ the polarization is left hand (spin opposite to the direction of motion). Circular polarization of the bremsstrahlung is characterized by a photon field polarization vector (see references 15 and 1)*

$$\mathbf{a} = \frac{1}{\sqrt{2}} \sum_{l=\pm 1} (\boldsymbol{\beta} + il\,[\boldsymbol{\varkappa}^0\boldsymbol{\beta}]).$$

Here

$$\boldsymbol{\beta} = [\boldsymbol{\varkappa}^0\mathbf{j}]/\sqrt{1-(\boldsymbol{\varkappa}^0\mathbf{j})^2},$$

$\boldsymbol{\kappa}^0 = \boldsymbol{\kappa}/\kappa$ is a unit vector in the direction of emission of the bremsstrahlung photon, $\mathbf{j}$ is a unit vector of arbitrary direction, $l = \pm 1$ describes the circular polarization of the radiated γ quantum; $l = 1$ for a γ quantum with right-hand polarization and -1 for left-hand polarization.

In the Born approximation we obtain from reference 1, with the aid of formulas (2) — (10), the following expression for the differential cross section of the external bremsstrahlung of the electron, with allowance for the form factor of the nucleus and the longitudinal polarizations of the electron (in the initial and final states) and of the emitted γ quantum

$$d\sigma_{lss'}(\theta,\theta')\,d\Omega\,d\Omega' = |F(q)|^2 \Big\{ \frac{1}{4}\, d\sigma_{\mathrm{B-H}}(\theta,\theta') + \frac{a_T}{8q^4}\,[ss' f_1(\theta,\theta') + ls f_2(\theta,\theta') + ls' f_3(\theta,\theta')] \Big\}\, d\Omega\, d\Omega'. \tag{2}$$

Here

$$f_1(\theta,\theta') = \frac{1}{2kk'} \Big\{ [2(k_0^2 + KK')(4K'^2 - q^2) - 4k_0^2(3K'^2 + K^2)] \frac{k^2\sin^2\theta}{\Delta^2} + [2(k_0^2+KK')(4K^2 - q^2) - 4k_0^2(3K^2 + K'^2)] \frac{k'^2\sin^2\theta'}{\Delta'^2}$$
$$- 2[2(k_0^2 + KK')(4KK' + 2\varkappa^2 - q^2) - 4k_0^2(K+K')^2] \frac{kk'\sin\theta\sin\theta'\cos(\varphi'-\varphi)}{\Delta\Delta'}$$
$$+ 16k_0^2\varkappa^2\frac{KK'}{\Delta\Delta'} + 4\varkappa^2KK'\frac{k^2\sin^2\theta + k'^2\sin^2\theta'}{\Delta\Delta'}$$
$$+ 4\varkappa k_0^2(K+K')\Big(\frac{1}{\Delta^2} - \frac{1}{\Delta'^2}\Big) kk'\sin\theta\sin\theta'\cos(\varphi'-\varphi)$$
$$+ 4\varkappa k_0^2\frac{K+K'}{\Delta\Delta'}(k^2\sin^2\theta - k'^2\sin^2\theta') - 4\varkappa^2k_0^2\Big(\frac{K'}{\Delta'} + \frac{K}{\Delta}\Big)$$
$$- 4\varkappa^2k_0^2\Big[\frac{K'(K' + k'\cos\theta')}{\Delta^2} + \frac{K(K + k\cos\theta)}{\Delta'^2}\Big]\Big\},$$

$$f_2(\theta,\theta') = \frac{2}{k}\Big\{[\varkappa K(k\cos\theta + k'\cos\theta') - \varkappa^2K + \varkappa k_0^2]\frac{k^2\sin^2\theta}{\Delta^2} + [\varkappa K(k\cos\theta + k'\cos\theta') + \varkappa^2K - \varkappa k_0^2]\frac{k'^2\sin^2\theta'}{\Delta'^2}$$
$$- 2\varkappa K(k\cos\theta + k'\cos\theta')\frac{kk'\sin\theta\sin\theta'\cos(\varphi'-\varphi)}{\Delta\Delta'}$$
$$+ \varkappa(\varkappa K - k_0^2)\frac{k^2\sin^2\theta - k'^2\sin^2\theta'}{\Delta\Delta'}$$
$$- \varkappa^2k_0^2\frac{k^2\sin^2\theta - k'^2\sin^2\theta'}{\Delta^2\Delta'}$$
$$- \varkappa k_0^2\Big(\frac{1}{\Delta^2} - \frac{1}{\Delta'^2}\Big)kk'\sin\theta\sin\theta'\cos(\varphi'-\varphi)\Big\},$$

$$f_3(\theta,\theta') = \frac{2}{k'}\Big\{[\varkappa K'(k'\cos\theta' + k\cos\theta) - \varkappa^2K' - \varkappa k_0^2] \times \frac{k^2\sin^2\theta}{\Delta^2} + [\varkappa K'(k'\cos\theta' + k\cos\theta) + \varkappa^2K' + \varkappa k_0^2]\frac{k'^2\sin^2\theta'}{\Delta'^2}$$
$$- 2\varkappa K'\frac{(k\cos\theta + k'\cos\theta')}{\Delta\Delta'}kk'\sin\theta\sin\theta'\cos(\varphi'-\varphi)$$
$$+ \varkappa k_0^2\Big(\frac{1}{\Delta^2} - \frac{1}{\Delta'^2}\Big)kk'\sin\theta\sin\theta'\cos(\varphi'-\varphi)$$
$$+ \varkappa(\varkappa K' + k_0^2)\frac{k^2\sin^2\theta - k'^2\sin^2\theta'}{\Delta\Delta'}$$
$$- \varkappa^2k_0^2\frac{k^2\sin^2\theta - k'^2\sin^2\theta'}{\Delta\Delta'^2}\Big\}; \tag{3}$$

$$\Delta = K - k\cos\theta, \qquad \Delta' = K' - k'\cos\theta',$$
$$K = \sqrt{k^2 + k_0^2}, \qquad K' = \sqrt{k'^2 + k_0^2},$$

$$q^2 = k^2 + k'^2 + \varkappa^2 - 2k\varkappa\cos\theta + 2k'\varkappa\cos\theta' - 2kk'(\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi'-\varphi)),$$

$$\cos\theta = \frac{\mathbf{k}\boldsymbol{\varkappa}}{k\varkappa}, \qquad \cos\theta' = \frac{\mathbf{k}'\boldsymbol{\varkappa}}{k'\varkappa}, \qquad a_T = Z^2\alpha^3\frac{k'd\varkappa}{2\pi^2k\varkappa}. \tag{4}$$

Here $E = c\hbar K$, $\mathbf{p} = \hbar\mathbf{k}$ and $E' = c\hbar K'$, $\mathbf{p}' = \hbar\mathbf{k}'$ are the total energy and momentum of the electron before and after radiation, respectively; $\epsilon_\gamma = c\hbar\kappa = c\hbar(K - K')$ and $\mathbf{p}_\gamma = \hbar\boldsymbol{\kappa}$ are the energy and momentum of the radiated γ quantum; $k_0 = m_0c/\hbar$ is the rest mass of the electron; α is the fine-structure constant; $d\Omega$ and $d\Omega'$ are the solid angles of the direction of the electron momentum before and

*$[\boldsymbol{\varkappa}^0\boldsymbol{\beta}] = \boldsymbol{\varkappa}^0 \times \boldsymbol{\beta}$.

after radiation; $s' = \pm 1$ is the eigenvalue of the projection operator $\boldsymbol{\sigma}\cdot\mathbf{p}'/p'$, which characterizes the longitudinal polarization of the electron in the final state.

In Eq. (2), $d\sigma_{\mathrm{B-H}}(\theta, \theta')$ is the known cross section of the unpolarized bremsstrahlung, given by formula (25.13) of Heitler's book,[16] the terms proportional to ls and ls' define the longitudinal spin correlation between the radiated γ quantum and the electron in the initial and final states, respectively, and the term proportional to ss' characterizes the correlation between the longitudinal polarizations of the electron in the initial and final states.

Summing (2) over the final spin states of the electron, we obtain a formula for the differential bremsstrahlung cross section with allowance for the form factor and the spin correlation between the electron and the γ quantum $(\sim ls)$:

$$d\sigma_{ls}(\theta, \theta')\, d\Omega\, d\Omega' = |F(q)|^2 \Big\{\frac{1}{2}\, d\sigma_{\mathbf{B-H}}(\theta, \theta') + ls\, \frac{a_T}{4q^4} f_2(\theta, \theta')\Big\} d\Omega\, d\Omega'. \tag{5}$$

Integrating (5) over the solid angle $d\Omega'$ of the scattered electron with allowance of (1), we obtain the following expression for the angular distribution of the circularly-polarized bremsstrahlung from the longitudinally-polarized electron, with allowance for the finite dimensions of the nucleus:

$$d\sigma_{ls}(\theta, \langle r^2\rangle)\, d\Omega = d\sigma_{ls}^T(\theta)\, d\Omega - 2\pi a_T \{\langle r^2\rangle [\Phi_2(\theta) + ls\,\Phi_3(\theta)] - \langle r^2\rangle^2 [\Phi_4(\theta) + ls\,\Phi_5(\theta)]\}\, d\Omega. \tag{6}$$

Here

$$\Phi_2(\theta) = \frac{1}{3}\Big\{\frac{k_0^2(k^2+2K^2)}{2k^2\Delta^2} - [\varkappa^2 k^2 K' + 2T^2 K(k^2+K^2)]\frac{1}{2k^2T^2\Delta} + \frac{3T^2+\varkappa^2}{2T^2} - \frac{\varepsilon^T}{2k'T}\Big(\frac{2k_0^2K'^2}{\Delta^2} + \frac{\varkappa^3\Delta}{T^2} + \frac{\varkappa^2}{T^2}(\varkappa K' + k'^2)\Big) + \frac{\varepsilon}{2k'}\Big(\frac{K^2+K'^2}{\Delta} - K'\Big) + \frac{L_0}{2kk'}\Big[\frac{k_0^2K}{k^2\Delta^2}(\varkappa k_0^2 + k^2K') - \frac{\varkappa k_0^2K^2}{k^2\Delta} - \frac{\varkappa^2}{2}\Big]\Big\},$$

$$\Phi_3(\theta) = \frac{1}{6k}\Big\{\frac{\varkappa^2K}{T^2} - \frac{\varkappa^2KK'}{T^2\Delta} - \frac{4\varkappa K^3}{k^2\Delta} + \varkappa k_0^2\frac{2K^2+k^2}{k^2\Delta^2} + \frac{\varkappa(K^2+k_0^2)}{k^2} + \frac{\varkappa\varepsilon^T}{k'T}\Big[\frac{2k_0^2K'}{\Delta} - \frac{2k_0^2KK'}{\Delta^2} - \frac{\varkappa^2K\Delta}{T^2} - \varkappa K(\varkappa K' + k'^2)\frac{1}{T^2}\Big] + \frac{L_0}{2kk'}\Big[\frac{2k_0^2K(\varkappa k_0^2 + k^2K')\varkappa}{k^2\Delta^2} - \frac{2k_0^2(2\varkappa K^2 + k^2K)\varkappa}{k^2\Delta} + \frac{\varkappa^2K(K^2+k_0^2)}{k^2}\Big] + \frac{\varepsilon\varkappa}{k'}\Big[\frac{K}{\Delta}(K+K') - K\Big]\Big\},$$

$$\Phi_4(\theta) = \frac{1}{36}\Big\{k_0^2\Big[\frac{1}{2}(T^2+k'^2) - 2K'^2\Big]\frac{1}{\Delta^2} + [KK'(K+2K') - \varkappa k'^2 - T^2(\varkappa + K'\sin^2\theta)]\frac{1}{\Delta} + (K+K')\Delta - (k_0^2 + 2(2K^2+K'^2)) + \frac{\varepsilon}{k'}[k_0^2(2KK' - K^2 - K'^2 + T^2\sin^2\theta)\frac{1}{\Delta} - \frac{1}{2}(K^2+K'^2+k_0^2)\Delta + K(K^2+K'^2) + \frac{3}{2}k_0^2K']\Big\},$$

$$\Phi_5(\theta) = \frac{1}{72k}\Big\{-\varkappa k_0^2(4k_0^2 + k'^2 + T^2)\frac{1}{\Delta^2} + 2\varkappa K[4k_0^2 + K'(K+K')]\frac{1}{\Delta} + 6\varkappa\Delta K - 2\varkappa(2KK' + 6K^2 - k_0^2) + \frac{\varepsilon}{k'}\Big[-\frac{2k_0^2\varkappa^2K}{\Delta} - \varkappa(k^2+KK')\Delta + \varkappa(2K^3 + K'(2k^2 - k_0^2))\Big]\Big\}; \tag{7}$$

$$d\sigma_{ls}^T(\theta) = \tfrac{1}{2}\, d\sigma_{\mathbf{B-H}}(\theta) + ls\cdot 2\pi a_T\Phi_1(\theta),$$

$$d\sigma_{\mathbf{B-H}}(\theta) = 4\pi a_T\Phi_0(\theta);$$

$$T = |\mathbf{k} - \boldsymbol{\varkappa}|, \qquad \varepsilon = \ln\frac{K'+k'}{K'-k'}, \qquad \varepsilon^T = \ln\frac{T+k'}{T-k'},$$

$$L_0 = \ln\frac{KK' + kk' - k_0^2}{KK' - kk' - k_0^2}. \tag{8}$$

$d\sigma_{ls}^T(\theta)$ is the cross section of the circularly-polarized bremsstrahlung, emitted by the longitudinally-polarized electron in the field of the point nucleus, and given by formula (17) of reference 1. The functions $\Phi_0(\theta)$ and $\Phi_1(\theta)$, which are included in (8), are determined by formulas (18) and (19) of reference 1; $d\sigma_{\mathrm{B-H}}(\theta)$ is the Bethe-Heitler bremsstrahlung cross section integrated over the angles of the scattered electron [see reference 1, formula (18)]. The terms proportional to $\langle r^2\rangle$ and $\langle r^2\rangle^2$ represent the corrections to the cross section of the bremsstrahlung from the longitudinally-polarized electron, due to the finite dimensions of the nucleus. These corrections account for the influence of the finite dimensions of the nucleus on the polarization correlation $(\sim ls)$ and the usual angular correlation $[\sim \Phi_2(\theta)$ and $\Phi_4(\theta)]$ between the electron and the γ quantum in bremsstrahlung. If only one correction term, corresponding to the region of not very large energies $(qr < 1)$ is taken into account in the form factor, it is possible to neglect the Coulomb effect in the circular polarization of the bremsstrahlung.

After averaging over the polarizations of the incident electron and summing over the polarizations of the emitted γ quantum, we obtain from (6)

$$d\sigma(\theta, \langle r^2\rangle)\, d\Omega = d\sigma_{\mathbf{B-H}}(\theta)\, d\Omega - 4\pi a_T\{\langle r^2\rangle\,\Phi_2(\theta) - \langle r^2\rangle^2\,\Phi_4(\theta)\}\, d\Omega. \tag{9}$$

Formula (9) describes the angular distribution of the unpolarized bremsstrahlung with allowance for the mean square radius $\langle r^2 \rangle$ of the distribution of the nuclear charge. As $\langle r^2 \rangle \to 0$ (Coulomb center concentrated in a point), formula (6) goes into $d\sigma_{B-H}(\theta)$. Thus, the cross section (6) obtained is a generalization of the known bremsstrahlung of an electron on a point nucleus with allowance for the longitudinal polarization of the spins of the electron and of the emitted γ quantum for finite nuclear dimensions.

3. DEGREE OF POLARIZATION OF RADIATION WITH ALLOWANCE FOR THE DIMENSIONS OF THE NUCLEUS

According to (6) and (8), the cross section of the circularly polarized bremsstrahlung emitted by a longitudinally-polarized electron in the field of a nucleus of finite dimensions is connected with the cross section in the field of a point nucleus $d\sigma_{ls}^T(\theta)$ by the relation

$$d\sigma_{ls}(\theta, \langle r^2\rangle) = d\sigma_{ls}^T(\theta)\left\{1 - \langle r^2\rangle \frac{\Phi_2(\theta) + ls\,\Phi_3(\theta)}{\Phi_0(\theta) + ls\,\Phi_1(\theta)} + \langle r^2\rangle^2 \frac{\Phi_4(\theta) + ls\,\Phi_5(\theta)}{\Phi_0(\theta) + ls\,\Phi_1(\theta)}\right\}. \quad (10)$$

In the case of unpolarized bremsstrahlung we have in lieu of (10)

$$d\sigma(\theta, \langle r^2\rangle) = d\sigma_{B-H}(\theta)\left\{1 - \langle r^2\rangle \frac{\Phi_2(\theta)}{\Phi_0(\theta)} + \langle r^2\rangle^2 \frac{\Phi_4(\theta)}{\Phi_0(\theta)}\right\}. \quad (11)$$

If the longitudinal polarization of the electron is fixed ($s = 1$ or -1), the degree of polarization of the bremsstrahlung, corresponding to the two types of circular polarization ($l = \pm 1$) of the radiated quantum, can be determined from the formula

$$P(\theta, \langle r^2\rangle) = (\{d\sigma_{ls}\}_{l=1} - \{d\sigma_{ls}\}_{l=-1}) / (\{d\sigma_{ls}\}_{l=1} + \{d\sigma_{ls}\}_{l=-1}). \quad (12)$$

From this, using (10), we obtain for the degree of circular polarization of the bremsstrahlung on a nucleus of finite dimensions

$$P(\theta, \langle r^2\rangle) = sP^T(\theta)\frac{1 - \langle r^2\rangle d_3(\theta) + \langle r^2\rangle^2 d_5(\theta)}{1 - \langle r^2\rangle d_2(\theta) + \langle r^2\rangle^2 d_4(\theta)}, \quad (13)$$

where

$$d_3(\theta) = \Phi_3(\theta)/\Phi_1(\theta), \quad d_5(\theta) = \Phi_5(\theta)/\Phi_1(\theta),$$
$$d_2(\theta) = \Phi_2(\theta)/\Phi_0(\theta), \quad d_4(\theta) = \Phi_4(\theta)/\Phi_0(\theta),$$
$$P^T(\theta) = \Phi_1(\theta)/\Phi_0(\theta). \quad (14)$$

Here $P^T(\theta)$ is the degree of circular polarization of the bremsstrahlung produced by a longitudinally-polarized electron in the Coulomb field of a point nucleus [see reference 1, formula (20)].

Formulas (10), (11), and (13) describe, at high energies, the influence of the finite dimensions of the nucleus on the angular dependence of the cross section, and also the influence of the degree of circular polarization of the bremsstrahlung from a longitudinally-polarized electron in the Born approximation. We can use here for the mean square radius $\langle r^2\rangle$ of the nucleus the value obtained from experiments on the scattering of high-energy electrons by nuclei.[14] In the extreme relativistic case (e.r.) of high energies, when $E, E' \gg m_0c^2$, the functions $\Phi_1(\theta)$, which determine the angular dependence of the cross section and of the polarization, become much simpler and we obtain for them the following expressions

$$\Phi_2^{\text{e.r.}}(\theta) = \frac{1}{6}\left\{3 + \frac{a^2b^2k_0^2}{T^2} - \left[4 + a^2b^2(1-a)\frac{k_0^2}{T^2}\right]\frac{1}{\Delta_0} - \frac{\varepsilon^T a^2b^3k_0^3}{T^3(1-a)}(1 - a\cos\theta) - \frac{a^2\varepsilon'}{2(1-a)} + \varepsilon\left(\frac{1}{\Delta_0} - \frac{1}{2}\right)\frac{2-2a+a^2}{1-a}\right\},$$

$$\Phi_3^{\text{e.r.}}(\theta) = \frac{a}{6}\left\{1 + \frac{ab^2k_0^2}{T^2} - \left[4 + a(1-a)\frac{b^2k_0^2}{T^2}\right]\frac{1}{\Delta_0} - \frac{\varepsilon^T ab^3k_0^3}{T^3(1-a)}(1 - a\cos\theta) + \frac{a\varepsilon'}{2(1-a)} + \frac{\varepsilon}{1-a}\left[\frac{2-a}{\Delta_0} - \left(1 - \frac{a}{2}\right)\right]\right\},$$

$$\Phi_4^{\text{e.r.}}(\theta) = \frac{1}{36}\Big\{-2k_0^2b^2(3-2a+a^2) + b^2k_0^2(2-a)\Delta_0 + [b^2k_0^2(1-a)(3-3a+a^2) - T^2(a + (1-a)\sin^2\theta)]\frac{1}{\Delta_0} - k_0^2\frac{(1-a)^2}{\Delta_0^2} + \frac{\varepsilon}{b^2(1-a)}\Big[(T^2\sin^2\theta - a^2b^2k_0^2)\frac{1}{\Delta_0} - k_0^2b^4(2-2a+a^2)\left(\frac{\Delta_0}{2} - 1\right)\Big]\Big\},$$

$$\Phi_5^{\text{e.r.}}(\theta) = \frac{abk_0}{36}\Big\{-2bk_0(4-a) + 3bk_0\Delta_0 + bk_0(2-3a+a^2)\frac{1}{\Delta_0} - \frac{k_0(1-a)}{b\Delta_0^2} - \frac{\varepsilon k_0}{2(1-a)}\Big[\frac{2a}{b\Delta_0} - (2-a)(1+\cos\theta)\Big]\Big\};$$

$$a = \frac{\varepsilon_\gamma}{E}, \quad b = \frac{E}{m_0c^2}, \quad \varepsilon^T = \ln\frac{T + (1-a)}{T - (1-a)},$$
$$\varepsilon = 2\ln[2b(1-a)], \quad \varepsilon' = -2\ln a, \quad \Delta_0 = 1 - \cos\theta,$$
$$T = k_0b\sqrt{1 + a^2 - 2a\cos\theta},$$
$$1/k_0 = \hbar/m_0c = 3.86\cdot 10^{-11}\ \text{cm}.$$

Here $E = cp$ is the kinetic energy of the extreme-relativistic electron, ϵ_γ is the energy of the bremsstrahlung quantum. We do not give here the functions $\Phi_0^{\text{e.r.}}(\theta)$ and $\Phi_1^{\text{e.r.}}(\theta)$ for a point-like center, since these are readily derived from (18) and (19) of reference 1 by assuming ϵ_γ, E, and $E' \gg m_0c^2$.

To illustrate the derived formulas (10), (11), and (13), let us estimate the role of finite nuclear dimensions in the case of bremsstrahlung of both a polarized and an unpolarized electron on the nucleus of Ag^{108}. The mean-square radius of Ag^{108}, determined from electron-scattering experiments, is (see reference 14)

$$\langle r^2 \rangle = 19.594 \cdot 10^{-26} \text{ cm}^2.$$

It was observed experimentally that the left-polarized electron of high energy ($s = -1$) from β emitters radiates predominantly bremsstrahlung quanta with left-hand circular polarization ($l = -1$, quantum spin opposite to quantum direction).[8-11] For a right-polarized high-energy electron ($s = 1$) the emitted bremsstrahlung quanta should have right-hand circular polarization ($l = 1$, quantum spin parallel to the direction of motion).

Figure 1 shows the dependence of the cross section ratios

$$I_1 = d\sigma_{ls}(\theta, \langle r^2 \rangle)/d\sigma_{ls}^T(\theta),$$
$$I_2 = d\sigma(\theta, \langle r^2 \rangle)/d\sigma_{\text{B-H}}(\theta) \qquad (15)$$

on the angle of emission θ of the bremsstrahlung quanta for the incident-electron and radiated-photon energies $E = 50$ Mev and $\epsilon_\gamma = 25$ Mev, and for $s = 1$ and $l = 1$ (or $s = -1$, $l = -1$). Figure 2 shows the dependence of the degree of circular polarization of the bremsstrahlung for an extended nucleus, $P(\theta, \langle r^2 \rangle)$ and for a point nucleus

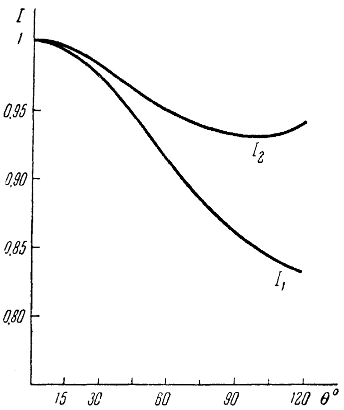

FIG. 1. Dependence of the cross-section ratios (15) on the angle of emission θ of the bremsstrahlung quanta, for Ag^{108} with $E = 50$ Mev, $\varepsilon_\gamma = 25$ Mev, $s = 1$, and $l = 1$.

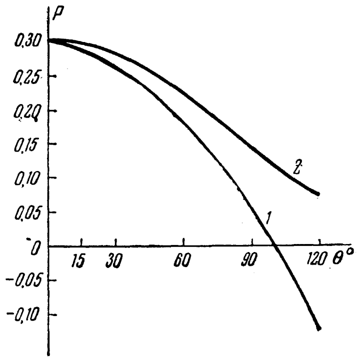

FIG. 2. Dependence of the degree of circular polarization of radiation, P, on the angle of emission θ of bremsstrahlung quanta with $E = 50$ Mev, $\varepsilon_\gamma = 25$ Mev, and $s = 1$: curve $1 - P(\theta, r^2)$ for Ag^{108}, curve $2 - P^T(\theta)$.

$P^T(\theta)$ on the angle of emission θ of the bremsstrahlung quanta with $E = 50$ Mev, $\epsilon_\gamma = 25$ Mev, and $s = 1$.

It is seen from Fig. 1 that allowance for the finite dimensions of the nucleus exerts a greater influence on the angular distribution of the circularly-polarized bremsstrahlung (curve I_1) than on the angular distribution of unpolarized bremsstrahlung (curve I_2). For small values of the angle, $\theta < 10°$, the cross section of the bremsstrahlung on an extended nucleus, $d\sigma_{ls}(\theta, \langle r^2 \rangle)$, coincides with the cross section for a point-like center $d\sigma_{ls}^T(\theta)$. When the angle θ exceeds 10°, the cross section ratio for the extended and point-like centers is greatly reduced.

As can be seen from Fig. 2, allowance for the finite dimensions of the nucleus leads to a noticeable reduction in the degree of circular polarization of the bremsstrahlung. In the range of larger angles of emission of the bremsstrahlung quanta ($\theta > 10°$) the deviations of the polarization $P(\theta, \langle r^2 \rangle)$ for an extended nucleus from the polarization $P^T(\theta)$ for a point-like nucleus are quite noticeable. In view of the lack of experimental data on bremsstrahlung of polarized high-energy electrons, we cannot as yet estimate the accuracy of the resultant formulas. However, the result obtained can serve as an indication that the effects of the finite dimensions of the nucleus may contribute greatly to the cross section and to the circular polarization of the bremsstrahlung produced by longitudinally-polarized high-energy electrons.

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