AN INVESTIGATION OF THE PROPERTIES OF TRANSURANIUM ELEMENTS BASED ON THE SUPERFLUID MODEL OF THE NUCLEUS

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The properties of strongly deformed transuranic elements are investigated on the basis of the superfluid model of the nucleus. Some insignificant modifications are introduced in Nilsson's schemes by employing the experimental data and taking into account the effect of superfluidity. The pairing energies are computed and the following values for the coupling constants are obtained: $G_N = 0.020\hbar\omega_0$, $G_P = 0.022\hbar\omega_0$, where $\hbar\omega_0 = 41A^{-1/3}$ Mev. Single-particle excitation spectra are calculated for odd-mass nuclei, the calculated level density being about twice as large as that predicted by the Nilsson scheme. Single-particle excitations in even-even nuclei are computed, and in all the calculated spectra for the even-even nuclei (Th^{232}, U^{234}, Pu^{238}, Pu^{240}, Pu^{242}, Cm^{246}, Cf^{248}) the lowest levels are found to be the 1^- levels, which lie below 1 Mev. Corrections due to superfluidity of the ground and excited states are computed for the β and γ transitions. The results obtained are self-consistent: correct values for pairing energies and levels of even and odd nuclei are obtained for the same values of G, whereas variation of G by 30 — 40 percent leads to a pronounced deviation from the experimental data.

THE mathematical methods developed by Bogolyubov for the theory of superfluidity and superconductivity[1] have made it possible to account for the residual nucleon interactions leading to pair correlations in the independent-particle model. This was used by the present author as a basis to formulate[2] a superfluid model of the nucleus.

The present article is devoted to a study, based on the superfluid model, of the properties of strongly deformed transuranic elements. We use as the self-consistent field the Nilsson potential, the energy levels of which are slightly corrected in accordance with the experimental data. Under the assumption of the adiabatic approximation, we calculate the single-particle levels of both odd and even-even nuclei, the pairing energies, and also the corrections to the probabilities of the β and γ transitions.

1. FUNDAMENTAL EQUATIONS

The superfluid model of the nucleus, formulated by the author earlier,[2] being based on the self-consistent potential assumed in the shell or unified model, takes into account the interaction of the nucleons under the following assumptions.

1) The residual interactions, both between neutrons and between protons, are described by a Hamiltonian of the form

$$H = \sum_{s\sigma} \{E(s) - \lambda\} a^+_{s\sigma} a_{s\sigma} - G \sum_{s,s'} a^+_{s+} a^+_{s-} a_{s'-} a_{s'+}. \tag{1}$$

2) The calculations are carried out for each definite nucleus, and we neglect a certain averaging connected with the conservation of the number of particles in the mean.

The states of the nucleus are described by a set of quantum numbers $s\sigma$, determined by the form of the self-consistent field; $\sigma = \pm 1$, characterizes, for example, the sign of Ω, the projection of the nucleon momentum on the symmetry axis of the nucleus; E(s) are the energy levels in the self-consistent field. A certain simplification in the physical picture is the assumption that the interaction G is constant. The chemical potential λ is determined from the condition

$$n = \sum_{s\sigma} \langle a^+_{s\sigma} a_{s\sigma} \rangle, \tag{2}$$

where n is the number of particles and $<\ldots>$ denotes averaging over a certain state. When the variational principle is used, the chemical potential plays the role of the Lagrange multiplier. We note that the Hamiltonian (1) should be considered as part of the total Hamiltonian containing, for example, collective interactions.

To solve our problem we use the variational principle proposed by Bogolyubov.[3] Operating as in reference 4, we obtain for the case of the ground state of a system consisting of an even number of particles the following equations[5,6]

$$2/G = \sum [C^2 + \{E(s) - \lambda\}^2]^{-1/2}, \quad (3)$$

$$n = \sum_s \{1 - (E(s) - \lambda)\,[C^2 + \{E(s) - \lambda\}^2]^{-1/2}\} \quad (4)$$

for the determination of C and λ. The energy of the ground state is found in the form

$$\mathscr{E} = \sum E(s)\left\{1 - \frac{E(s) - \lambda}{[C^2 + \{E(s) - \lambda\}^2]^{1/2}}\right\} - \frac{C^2}{G}, \quad (5)$$

and the wave function is

$$\Psi = \prod_s \{u_s + v_s a^+_{s+} a^+_{s-}\}\,\Psi_0, \quad (6)$$

with $a_s\Psi_0 = 0$ and

$$u_s^2 = \frac{1}{2}\left\{1 + \frac{E(s) - \lambda}{[C^2 + \{E(s) - \lambda\}^2]^{1/2}}\right\},$$

$$v_s^2 = \frac{1}{2}\left\{1 - \frac{E(s) - \lambda}{[C^2 + \{E(s) - \lambda\}^2]^{1/2}}\right\}. \quad (7)$$

The wave functions of the excited states are written in the following manner:[5]

$$\Psi(s_1, s_2) = \prod_{s \neq s_1, s_2} (u_s + v_s a^+_{s+} a^+_{s-})\, a^+_{s_1\sigma_1} a^+_{s_2\sigma_2}\Psi_0, \quad s_1 \neq s_2, \quad (8)$$

$$\Psi(s_1, s_1) = \prod_{s \neq s_1} (u_s + v_s a^+_{s+} a^+_{s-})\,(u_{s_1} a^+_{s_1+} a^+_{s_1-} - v_{s_1})\Psi_0, \quad (8')$$

while the energy and the fundamental equations are obtained in the form

$$\mathscr{E}(s_1, s_2) = E(s_1) + E(s_2) + \frac{1}{2} G (v^2_{s_1} + v^2_{s_2})$$

$$+ \sum_{s \neq s_1, s_2} E(s)\left\{1 - \frac{E(s) - \lambda}{[C^2 + \{E(s) - \lambda\}^2]^{1/2}}\right\} - \frac{C^2}{G}, \quad (9)$$

$$\frac{2}{G} = \sum_{s \neq s_1, s_2} [C^2 + \{E(s) - \lambda\}^2]^{-1/2},$$

$$n = 2 + \sum_{s \neq s_1, s_2}\left\{1 - \frac{E(s) - \lambda}{[C^2 + \{E(s) - \lambda\}^2]^{1/2}}\right\}. \quad (10)$$

In the case of an odd shell, if the odd nucleon is in the state s_i, then the energy of the system and the equations for C and λ are found in the form

$$\mathscr{E}(s_i) = E(s_i) + \frac{G}{2} v^2_{s_i} + \sum_{s \neq s_i} E(s)\left\{1 - \frac{E(s) - \lambda}{[C^2 + \{E(s) - \lambda\}^2]^{1/2}}\right\}$$

$$- \frac{C^2}{G}, \quad (11)$$

$$\frac{2}{G} = \sum_{s \neq s_i} [C^2 + \{E(s) - \lambda\}^2]^{-1/2},$$

$$n = 1 + \sum_{s \neq s_i}\left\{1 - \frac{E(s) - \lambda}{[C^2 + \{E(s) - \lambda\}^2]^{1/2}}\right\} \quad (12)$$

These equations describe the ground state, for the most part with $E(s_i) = E_F$, as well as the excited states of a system with an odd number of particles, the wave function of which is written in the form

$$\Psi(s_i) = \prod_{s \neq s_i} (u_s + v_s a^+_{s+} a^+_{s-})\, a^+_{s_i\sigma_i}\Psi_0. \quad (13)$$

Thus, to determine C and λ of both the ground states and the superfluid excited states it is necessary to solve the corresponding system of equations. This approach differs substantially from the approach used by many authors.[7] Whereas in the case of the superfluid model of the nucleus the quantities C and λ are determined by solving suitable equations, and the interaction constant G is determined from the experimental values of the paired energy, in reference 7 the values of C are determined from the pairing energy, and λ is set equal to the energy of the Fermi surface E_F.

The advantage of the approach based on the superfluid model of the nucleus over the approach used in reference 7 lies, first, in the fact that it becomes possible to determine C and λ for excited states, to take account of the variation of C and λ with change in deformation of the nucleus, etc., something that cannot be done in the case used in reference 7. Secondly, by calculating the change of λ on going from nucleus to nucleus and from the ground state to the excited state, we take account at the same time of the change of the properties of the nucleus as a many-body system. Thirdly, our calculations are more single-valued and reliable, since we have at our disposal a single interaction constant G, which changes slowly and monotonically from nucleus to nucleus, whereas C changes abruptly, depending on the specific variation of the energy levels of the self-consistent field.

In the superfluid model of the nucleus we come upon the question of the orthogonality of the ground and excited states. It is easy to show that all the states of the odd shell are orthogoanl to each other, and that the ground and excited states given by Eq. (8) for an even shell with $s_1 \neq s_2$ are also orthogonal to each other. However, the ground and excited states given by (8′) are not orthogonal to each other when $s_1 = s_2$, namely

$$(\Psi^*(s_1, s_1)\,\Psi(s_2, s_2)) = (u'_{s_1} v''_{s_1} - u''_{s_1} v'_{s_1})\,(u'_{s_2} v''_{s_2} - u''_{s_2} v'_{s_2}) \prod_{s \neq s_1, s_2} (u'_s u''_s + v'_s v''_s), \quad (14)$$

where u'_s and v'_s pertain to the excited state $\Psi(s_1, s_2)$ while u''_s and v''_s pertain to $\Psi(s_2, s_2)$.

To estimate the error connected with the conservation of the number of particles in the mean, we calculate the mean square fluctuation Δn of the number of particles n. For the ground state of the even shell we obtain

$$(\Delta n)^2 = \sum_s C^2/(C^2 + \{E(s) - \lambda^2\}). \quad (15)$$

In the case of the excited state $\Psi(s_1, s_2)$ the sum should not contain terms with $s = s_1$ and $s = s_2$. In the case of the state $\Psi(s_i)$ of the odd shell, the sum (15) should not contain the term with $s = s_i$.

2. PAIRING ENERGY AND SINGLE-PARTICLE LEVELS OF ODD NUCLEI

It is known that the Nilsson potential[8] yields neither the necessary sequence of energy levels nor the correct values of the energy differences between levels. This is due, first, to the deficiency of the Nilsson scheme itself, for although it gives the correct basic laws, it does not take sufficient account of the totality of the nuclear phenomena and, second, to the need of accounting for the residual interactions, which lead to pairing correlations. Using the experimental data[9,10] on single-particle levels of odd nuclei, let us analyze the course of the levels in the Nilsson schemes for $Z > 82$ and $N > 126$, with allowance for the influence of the superfluidity of the ground and excited states.

In reference 2 we obtain, on the basis of calculations with the superfluid model, the influence of superfluidity on the behavior of single-particle levels of odd nuclei; this influence reduces to the following: 1) superfluidity, as a rule, does not cause a change in the ground state of the nucleus as given by the Nilsson scheme; 2) the excitation energies decrease with increasing interaction constant G; 3) the hole and particle levels behave differently with increasing G, but the relative sequence of hole (particle) levels does not change.

From analysis of the experimental data on single-particle levels and equilibrium deformations, as well as from the allowance for superfluidity, we reach the conclusion that the Nilsson level schemes given in reference 9 must be modified as follows:

1. In the level scheme for nuclei with odd Z, $Z > 82$, the level $11/2^-[505]$, which does not appears in a single nucleus, is dropped by $0.4\hbar\omega_0$ ($\hbar\omega_0 = 41 A^{-1/3}$ Mev).

2. In the level scheme for nuclei with odd N, $N > 126$, we make the following changes: a) since the level $13/2^+[606]$, does not appear in any nucleus, it is dropped by $0.25\hbar\omega_0$, which is equivalent to dropping the subshell $i_{13/2}$; b) the subshell $j_{15/2}$ is raised by $0.02\hbar\omega_0$; c) the level $1/2^+[631]$ is dropped by $0.017\hbar\omega_0$, which is equivalent to dropping the subshell $d_{5/2}$.

Table I. Neutron pairing energies

N	δ	Pairing energy P_N, Mev			
		$G = 0.016\ \hbar\omega_0$	$0.020\ \hbar\omega_0$	$0.024\ \hbar\omega_0$	Experiment[11]
150	0.26	0.43	0.94	1.58	0.7—1.1
148	0.26	0.23	0.88	1.57	0.8—1.2
146	0.26	—	0.82	—	0.7—0.9
144	0.24	0.27	0.97	1.69	0.8—1.0

Using the corrected Nilsson schemes, we calculate the pairing energies to find G for both proton and neutron interactions. The numerical solutions of (3), (4), and (12) were obtained with an electronic computer. After calculating $\mathcal{E}(Z, N)$ by formulas (5) and (11), we obtain the pairing energies

$$P_N(Z, N) = 2\,\mathcal{E}(Z, N-1) - \mathcal{E}(Z, N) - \mathcal{E}(Z, N-2) \quad (16)$$

for $G = 0.016$, 0.020, and $0.024\hbar\omega_0$. The results of the calculations are summarized in Tables I and II, from which it is seen that the pairing energies depend very strongly on G.

Comparing the calculated pairing energies with the experimental values, we find that for neutron interactions $G_N \approx 0.020\hbar\omega_0$ (more accurately, $0.018\hbar\omega_0 < G_N < 0.022\hbar\omega_0$), and for proton interactions $G_p \approx 0.022\hbar\omega_0$ (more accurately, $0.020\hbar\omega_0 < G_p < 0.024\hbar\omega_0$). From a comparison of the values of the interaction constants G, obtained above, with their values in the region $150 < A < 190$, it is seen that G decreases insignificantly on going from the rare-earth region to the transuranic region.

We now calculate the excitation spectrum of odd nuclei, solving (12) and finding the energy difference (11) for nuclei with odd N from $N = 141$ to $N = 149$, and for nuclei with odd Z from $Z = 91$ to $Z = 95$, for several deformations δ. In order to describe the behavior of the basic quantities, we list in Table III the values of C, λ and Δn for the case $Z = 93$ and $\delta = 0.26$ for the ground and two excited states, and as a comparison for the ground state with $Z = 94$. It is seen from Table III that C and λ change strongly both on going from the even to the odd nucleus and on going from the ground to the excited state. In odd-Z nuclei with $G_p = 0.016$

Table II. Proton pairing energies

Z	δ	Pairing energies P_Z, Mev			
		$G = 0.016\ \hbar\omega_0$	$0.020\ \hbar\omega_0$	$0.024\ \hbar\omega_0$	Experiment[11]
96	0.25	0.14	0.74	—	0.8—1.3
90	0.24	0.55	1.17	1.91	~1.4
94	0.25	0.30	0.93	—	0.8—1.4
92	0.24	0.45	1.12	1.90	0.8—1.1

Table III. Dependence of C, λ, and Δn on G ($\delta = 0.26$, $E_F = 5.628\hbar\omega_0$)

$G/\hbar\omega_0$	C	$\lambda - E_F$	Δn	$G/\hbar\omega_0$	C	$\lambda - E_F$	Δn
$Z = 94$, **ground state**				$Z = 93$, **excited state, particle, $5/2^-[523]$**			
0.016	0.027	0.005	1.2	0.016	0	−0.038	0
0.020	0.052	0.005	1.6	0.020	0.030	−0.054	1.0
0.024	0.085	0.006	2.0	0.024	0.062	−0.049	1.6
$Z = 93$, **ground state**				$Z = 93$, **excited state, hole, $1/2^-[530]$**			
0.016	0	0.009	0	0.016	0.020	0.004	1.0
0.020	0	−0.005	0	0.020	0.041	0.003	1.4
0.024	0.043	−0.031	1.1	0.024	0.069	0.001	1.7

$\hbar\omega_0$, and in individual cases with $G_p = 0.020\hbar\omega_0$, there are no pairing correlations in the ground states, and when $G_p = 0.016\hbar\omega_0$ the vanishing of superfluidity is observed in several excited states. In nuclei with odd N, no vanishing of superfluidity is observed for $G_N = 0.016\hbar\omega_0$, although C is quite small for the ground states of several nuclei. It is seen from Table III that the chemical potential fluctuates about the Fermi surface energy E_F, the value of which is $5.628\hbar\omega_0$ for $Z = 93$. We note that these changes in λ are smaller in the region $150 < A < 190$ than the changes given in reference 2.

The second assumption of the superfluid model, neglect of the conservation of the number of particles in the mean, can severely restrict the accuracy of the calculations and therefore calls for a numerical estimate. After calculating the mean square fluctuation Δn of the number of particles, it is necessary to compare it with twice the number of levels over which the summation is carried out, in our case with the number 48. From the calculated values of Δn, some of which are listed in Table III, it can be concluded that the error due to the fluctuation of the number of particles is on the order of 5 percent.

Comparing the calculated single-particle levels of the odd nuclei with the experimental data, we see that the agreement is quite crude; the best agreement occurs when $G_p = 0.024\hbar\omega_0$ for nuclei with odd Z and when $G_N = 0.020\hbar\omega_0$ for nuclei with odd N.

It must be noted that the density of the calculated low-energy levels is in good agreement with the experimental data[10] and is approximately twice the level density given by the Nilsson scheme. In Table IV we list by way of an example the low-lying energy levels of several nuclei. To obtain a more detailed agreement between the calculated and measured levels it is necessary to improve the level scheme of the self-consistent field.

We note that the effect of superfluidity on the spectra of odd nuclei is somewhat weaker in the transuranic region than the rare-earth region, although G has decreased insignificantly. Furthermore, for identical values of G the effect of superfluidity on a nucleus with odd Z is somewhat weaker in the transuranic region than in the region $150 < A < 190$, apparently owing to the change in the level density.[10]

3. EXCITATION SPECTRA OF EVEN-EVEN NUCLEI

Let us calculate the single-particle levels of the even-even nuclei on the basis of the superfluid

Table IV. Energies of excited states of odd nuclei* (Mev)

	$N = 143$		$N = 147$				$Z = 93$	
	$\delta = 0.24$		$\delta = 0.26$		$\delta = 0.27$		$\delta = 0.25$	
Ground state	0	$7/2^-$	0	$5/2^+$	0	$5/2^+$	0	$5/2^+$
Particle levels	0.18	$1/2^+$	0.18	$7/2^+$	0.26	$7/2^+$	0.08	$5/2^-$
	0.34	$5/2^+$	0.25	$9/2^-$	0.31	$9/2^-$	0.74	$3/2^-$
	0.71	$7/2^+$	1.07	$1/2^-$	0.96	$1/2^-$	0.77	$7/2^+$
Hole levels	0.26	$5/2^+$	0.20	$1/2^+$	0.30	$1/2^+$	0.27	$1/2^-$
	0.56	$3/2^+$	0.47	$7/2^-$	0.56	$7/2^-$	0.51	$3/2^+$
	0.60	$5/2^-$	0.88	$5/2^+$	0.99	$5/2^+$	0.78	$3/2^-$

*$G = 0.020\hbar\omega_0$ for odd-N nuclei and $0.024\hbar\omega_0$ for odd-Z nuclei.

Table V. Single-particle levels of U^{234} (in Mev) (for $\delta = 0.24$)

Proton levels: Ω and parity	G in $\hbar\omega_0$: 0.020	G in $\hbar\omega_0$: 0.024	Neutron levels: Ω and parity	G in $\hbar\omega_0$: 0.018	G in $\hbar\omega_0$: 0.022
$2^-, 3^-$	0.75	1.09	$1^-, 6^-$	0.78	1.32
$2^+, 3^+$	0.87	1.19	$2^+, 3^+$	1.09	1.54
$1^+, 4^+$	0.98	1.33	0^+	1.10	1.40
0^+	1.07	1.29	$2^-, 5^-$	1.15	1.57
0^+	1.15	1.37	$1^+, 6^+$	1.19	1.65
$1^-, 4^-$	1.16	1.39	0^+	1.20	1.57
$0^-, 5^-$	1.19	1.45	$0^+, 5^+$	1.31	1.75
0^+	1.31	1.51	$3^-, 4^-$	1.40	1.68
$1^-, 4^-$	1.37	1.69	$1^-, 2^-$	1.44	1.80
$1^-, 2^-$	1.38	1.59	$1^+, 4^+$	1.52	1.84
0^+	1.61	1.80	$0^-, 5^-$	1.57	1.88
$1^+, 2^+$	1.64	1.91	$1^-, 6^-$	1.52	1.85
$1^+, 2^+$	1.74	1.94	0^+	1.74	1.96
$1^-, 4^-$	1.94	2.12	0^+	1.86	2.13
$0^-, 3^-$	1.97	2.15	$1^-, 4^-$	1.91	2.18

model of the nucleus. For this purpose we solve Eqs. (3), (4), and (10) and calculate the energy of the ground and excited states. We note that the excited states given by Eq. (8) are doubly degenerate, unlike the states given by (8′). The excitation spectrum calculated on the basis of the superfluid model of the nucleus shows, by way of a minimum, the most probable spins and parities of the lower excited states, i.e., the calculated spectra of the even-even nucleus contribute to the analysis of the experimental data at least as much as the Nilsson scheme does to the analysis of the odd nuclei.

The influence of superfluidity on the excitation spectra of the even-even nuclei is quite strong. Furthermore, whereas in the case of odd nuclei the single-particle levels condense about the ground state with increasing G, in even-even nuclei the single-particle levels shift away from the ground state with increasing G, i.e., the gap increases. The change in the behavior of the levels with increasing G is demonstrated in Table V for U^{234}.

Tables VI and VII list the values of C, λ, and Δn for Pu^{238}, for both proton and neutron interaction. In the tables the Fermi-surface level is numbered n_F, the next level is denoted $n_F + 1$, etc. Calculations show that C diminishes strongly on going from the ground state to the lowest lying excited states, vanishing in individual cases even when $G_N = 0.022\hbar\omega_0$ and $G_p = 0.024\hbar\omega_0$. The vanishing of superfluidity in the excited states with the lowest energies leads to a reduction in the gap compared with $2[C^2 + (E_F - \lambda)^2]^{1/2}$. The values of the chemical potential λ fluctuate about E_F, and the deviations of λ from E_F are greater than the deviations in odd nuclei, reaching 0.5 Mev.

Calculating in this case, too, the mean square fluctuation Δn of the number of particles and comparing it with the number 48, i.e., with twice the

Table VI. Principal characteristics of the interaction of the neutrons of Pu^{238} ($\delta = 0.26$)

	$G/\hbar\omega_0 = 0.018$: C	$\lambda - E_F$	Δn	$G/\hbar\omega_0 = 0.022$: C	$\lambda - E_F$	Δn
	Ground state					
—	0.063	0.029	1.87	0.108	0.030	2.42
	Excited states given by (8′)					
(n_F, n_F)	0.028	0,[illegible]76	1.07	0.057	0.070	1.50
(n_F-1, n_F-1)	0.042	0,0[illegible]	1.44	0.076	0.076	1.93
(n_F+1, n_F+1)	0.028	−0.023	1.04	0.059	−0.011	1.53
	Excited states given by (8)					
(n_F, n_F+1)	0	0.84	0	0	0.051	0
(n_F, n_F+2)	0.0002	0.024	0	0.051	0.005	1.30
(n_F-1, n_F)	0.035	0.080	1.26	0.067	0.073	1.73
(n_F+1, n_F+2)	0.035	−0.023	1.24	0.069	−0.013	1.76

Table VII. Principal characteristics of the interaction of the protons of Pu^{238} ($\delta = 0.26$)

	$G/\hbar\omega_0=0,020$			$G/\hbar\omega_0=0,024$		
	C	$\lambda - E_F$	Δn	C	$\lambda - E_F$	Δn
Ground state						
	0.052	0	1,60	0,085	0,01	1,96
Excited states given by (8′)						
(n_F, n_F)	0.0004	0,09	0,01	0,027	0,07	0.78
(n_F-1, n_F-1)	0,001	0,08	0,02	0,053	0,07	1,37
(n_F+1, n_F+1)	0	−0,06	0	0,036	−0,05	1,06
Excited states given by (8)						
(n_F, n_F+1)	0	0,10	0	0	0,08	0
(n_F, n_F+2)	0,0002	0	0,01	0,01	−0,03	0,25
(n_F-1, n_F)	0	0,09	0	0,04	0,07	1,11
(n_F+1, n_F+2)	0,018	−0,06	0,67	0,049	−0,05	1,35

number of levels, we find that $\Delta n/48$ is on the order of 5 percent. Thus, the changes in C and λ on going from the ground states to the excited states, both in even and in odd nuclei, are considerably greater than the errors due to the conservation of the number of particles in the mean.

As shown above, the excited states (8) are orthogonal neither to each other nor to the ground states (which we denote by $|0\rangle$). Let us estimate the non-orthogonality, say for Pu^{238}. For the proton interactions at $G_p = 0.024\hbar\omega_0$, we obtain

$$\langle n_F-1,\ n_F-1\,|\,0\rangle = 0,15,$$
$$\langle n_F,\ n_F\,|\,n_F-1,\ n_F-1\rangle = 0,01, \qquad \langle n_F,\ n_F\,|\,0\rangle = 0,42,$$
$$\langle n_F+1,\ n_F+1\,|\,n_F-1,\ n_F-1\rangle = 0,07,$$
$$\langle n_F+1,\ n_F+1\,|\,0\rangle = 0,40,$$
$$\langle n_F+1,\ n_F+1\,|\,n_F,\ n_F\rangle = 0,67. \qquad (17)$$

In the case of neutron interactions we obtain for $G_N = 0.022\hbar\omega_0$.

$$\langle n_F-1,\ n_F-1\,|\,0\rangle = 0,17,$$
$$\langle n_F,\ n_F\,|\,n_F-1,\ n_F-1\rangle = 0,001, \qquad \langle n_F,\ n_F\,|\,0\rangle = 0,44,$$
$$\langle n_F+1,\ n_F+1\,|\,n_F-1,\ n_F-1\rangle = 0,08,$$
$$\langle n_F+1,\ n_F+1\,|\,0\rangle = 0,31,$$
$$\langle n_F+1,\ n_F+1\,|\,n_F,\ n_F\rangle = 0.22. \qquad (17')$$

It is seen from this that the admixture of excited states (8′) is essentially large, and therefore the energy difference between the states (8) is calculated with greater accuracy than their energy relative to the ground state. The energy of the states 0^+ (8′) is calculated with considerably lower accuracy.

The calculated energies of several excited states of U^{234} with deformation $\delta = 0.24$ and of $Pu^{238,240}$ and Cm^{246} with $\delta = 0.26$ are listed in Tables V, VIII, and IX. Among the low-lying levels there are states 0^+, which enable us to treat, for example, the states 0^+ with energies 1.15 and 1.62 Mev of U^{234} as single-particle states. The values given for the level energies enable us to analyze the experimental data.

The most interesting result of the calculations is that the lowest level in all the spectra of even-even nuclei which we have calculated is the level 1^-, inasmuch as the level $\Omega = 0$ with negative parities appears as $I = 1^-$. This is the result of

Table VIII. Single-particle levels of Pu^{238} and Pu^{240} (Mev) ($\delta = 0.26$)

Pu^{238}, neutron levels		Pu^{238}, proton levels		Pu^{240}, neutron levels	
Ω and parity	$G=0,020\ \hbar\omega_0$	Ω and parity	$G=0,024\ \hbar\omega_0$	Ω and parity	$G=0,020\ \hbar\omega_0$
$3^-,4^-$	0,89	$0^-,5^-$	0,85	$2^+,3^+$	0,90
0^+	1,03	0^+	0,97	0^+	1,11
0^+	1,08	0^+	1,13	0^+	1,14
$1^-,6^-$	1,18	$2^+,3^+$	1,39	$1^-,6^-$	1,16
$2^+,3^+$	1,32	$1^-,4^-$	1,44	$3^+,4^+$	1,20
$2^+,3^+$	1,40	$1^+,6^+$	1,51	$4^-,5^-$	1,28
$1^-,6^-$	1,55	$1^+,4^+$	1,65	$3^-,4^-$	1,44
$0^+,5^+$	1,56	$2^-,3^-$	1,67	$0^-,7^-$	1,44
$0^-,7^-$	1,60	$1^-,4^-$	1,70	$1^+,6^+$	1,51
$1^+,2^+$	1,64	$1^-,6^-$	1,75	$2^-,7^-$	1,51
0^+	1,73	$1^+,2^+$	1,88	$0^+,5^+$	1,56
$3^+,4^+$	1,77	$1^+,4^+$	1,97	0^+	1,62

Table IX. Single-particle levels of Cm^{246} (Mev) ($\delta = 0.26$)

Proton levels		Neutron levels		Proton levels		Neutron levels	
Ω and parity	$G_p = 0.024\hbar\omega_0$	Ω and parity	$G_N = 0.022\hbar\omega_0$	Ω and parity	$G_p = 0.024\hbar\omega_0$	Ω and parity	$G_N = 0.022\hbar\omega_0$
$1^+, 4^+$	1,00	$1^-, 8^-$	0.90	$2^-, 5^-$	1,57	$3^-, 4^-$	1,63
$1^-, 6^-$	1.10	0^+	0.91	$0^-, 5^-$	1,62	$4^+, 5^+$	1.64
$1^-, 4^-$	1.30	0^+	0.93	0^+	1.80	$3^+, 4^+$	1.70
$1^+, 6^+$	1.40	$2^-, 7^-$	1.25	$1^+, 2^+$	1.83	0^+	1.76
0^+	1.49	$1^+, 6^+$	1,34	$1^+, 6^+$	1.93	$2^-, 3^-$	1.80
0^+	1,47	$4^-, 5^-$	1,62	0^+	2.05	$0^+, 7^+$	1,96

a fortuitous combination of levels in the Nilsson scheme. It follows from our calculations that the single-particle levels 1^-, which lie lower than 1 Mev, should be observed in the nuclei Th^{232}, U^{234}, $Pu^{238,240,242,244}$, Cm^{246}, and Cf^{248}. It is possible that analogous calculations can confirm the appearance of 1^- levels in other transuranic elements, particularly in the lighter isotopes of thorium.

Actually, using the Nilsson scheme for nuclei with odd N, it can be seen that, starting with $N = 130$, the sequence of levels is $1/2^-$, $1/2^+$, $3/2^-$, $3/2^+$, $5/2^-$, $5/2^+$, etc., i.e., the adjacent levels have opposite parities and $|\Omega_i - \Omega_{i-1}| = 1$ or 0, which is quite favorable for the appearance of the 1^- levels. A similar picture is observed in the level scheme of nuclei with odd Z.

Thus, the calculations performed indicate that the low-lying levels with $I = 1$ and negative parity, which appear in many transuranic elements, can be single-particle but not collective levels.

4. CORRECTIONS TO THE PROBABILITIES OF THE BETA AND GAMMA TRANSITIONS

Let us calculate the corrections to the β and γ transitions, connected with the superfluidity of the ground and excited states. Knowing C and λ, it is easy to obtain the wave functions of the ground and excited states (6), (8), (8′), and (13). It is easy to show that the corrections to the probabilities are separated in the form of factors smaller than unity.

Calculating the corrections to the probabilities of the β decay of the odd nucleus, when the neutron from state s_1 goes into a proton in state s_2, i.e.,

$$(N = 2n_N + 1;\ Z = 2n_Z) \to (N = 2n_N;\ Z = 2n_Z + 1),$$

we obtain

$$R = (u_{s_1}^{2n_N} u_{s_2}^{2n_Z})^2 \prod_{s \neq s_1} (u_s^{2n_N} u_s^{(s_1)(2n_N+1)} + v_s^{2n_N} v_s^{(s_1)(2n_N+1)})^2$$
$$\times \prod_{s' \neq s_2} (u_{s'}^{2n_Z} u_{s'}^{(s_2)(2n_Z+1)} + v_{s'}^{2n_Z} v_{s'}^{(s_2)(2n_Z+1)})^2. \qquad (18)$$

The correction to the probability of the β decay of the odd-odd nucleus, i.e., of the decay

$$(N = 2n_N + 1,\ Z = 2n_Z - 1) \to (N = 2n_N,\ Z = 2n_Z),$$

is determined in the form

$$R_0 = (u_{s_1}^{2n_N} v_{s_2}^{2n_Z})^2 \prod_{s \neq s_1} (u_s^{2n_N} u_s^{(s_1)(2n_N+1)} + v_s^{2n_N} v_s^{(s_1)(2n_N+1)})^2$$
$$\times \prod_{s' \neq s_2} (u_{s'}^{2n_Z} u_{s'}^{(s_2)(2n_Z-1)} + v_{s'}^{2n_Z} \cdot v_{s'}^{(s_2)(2n_Z-1)})^2, \qquad (19)$$

where in u_s^{2n} and v_s^{2n} it is necessary to substitute C and λ for the ground state of the even shell, and in $u_s^{(s_i)(2n\pm1)}$, $v_s^{(s_i)(2n\pm1)}$ it is necessary to substitute C and λ of the ground or excited states of the odd shell, if the odd particle is in the state s_i.

In Table X we list the calculated corrections to the probabilities of the β decay of $Pu^{237,243}$ and

Table X. Correction R to the probabilities of β decay of odd nuclei for $G_p = 0.024\hbar\omega_0$ and $G_N = 0.020\hbar\omega_0$

Parent		Daughter		R
nucleus	state	nucleus	state	
Pu^{237}	$7/2\ 7/2^-$ [743]	Np^{237}	$5/2\ 5/2^+$ [642]	0.106
	$7/2\ 7/2^-$ [743]		$5/2\ 5/2^-$ [523]	0.090
Np^{239}	$5/2\ 5/2^+$ [642]	Pu^{239}	$1/2\ 1/2^+$ [631]	0.195
	$5/2\ 5/2^+$ [642]		$5/2\ 5/2^+$ [622]	0.120
	$5/2\ 5/2^+$ [642]		$7/2\ 7/2^-$ [743]	0,282
	$5/2\ 5/2^+$ [642]		$5/2\ 5/2^+$ [633]	0.326
Pu^{243}	$7/2\ 7/2^+$ [624]	Am^{243}	$5/2\ 5/2^+$ [642]	0,238
	$7/2\ 7/2^+$ [624]		$7/2\ 7/2^+$ [633]	0.613

Table XI. Corrections $R\gamma$ to the probabilities of electromagnetic transitions in odd nuclei for $G_p = 0.024\hbar\omega_0$ and $G_N = 0.020\hbar\omega_0$

Nucleus	Initial state	Final state	Energy (Mev)	Multipolarity	R_γ
Np^{237}	5/2 5/2−[523]	7/2 5/2+[642]	0.06	$E1$	0.46
	3/2 3/2−[521]	7/2 5/2−[523]	0.16	$E2$	0.65
	3/2 3/2−[521]	5/2 5/2−[523]	0.21	$M1$	0.93
Pu^{237}	1/2 1/2+[631]	7/2 7/2−[743]	0.14	$E3$	0.26
Pu^{239}	7/2 7/2−[743]	5/2 5/2+[642]	0.11	$E1$	$0.58\cdot10^{-3}$
	5/2 5/2+[622]	3/2 1/2+[631]	0.29	$E2$	0.207

Np^{239}. It is seen from the table that the corrections can be quite important in individual cases.

The corrections to the electromagnetic transitions are of more complicated form. For the case of an odd nucleus we obtain

$$R_\gamma = (u_{s_2}^{(s_1)} u_{s_1}^{(s_2)} - \eta v_{s_1}^{(s_2)} v_{s_2}^{(s_1)})^2 \prod_{s \neq s_1, s_2} (u_s^{(s_1)} u_s^{(s_2)} + v_s^{(s_1)} v_s^{(s_2)})^2, \quad (20)$$

where $\eta = 1$ for electric transitions and $\eta = -1$ for magnetic ones.

In Table XI we list the corrections to the γ transitions. We note that these corrections fluctuate within greater limits than the corrections for β decay.

Thus, our calculations show that in the calculation of the probabilities of β and γ transitions in strongly deformed transuranic elements it is necessary to take into account the superfluidity of the ground and excited states.

CONCLUSION

The superfluid model of the nucleus is based on the shell and unified models and is a further development of these models. At the first stage of the investigations, carried out on the basis of the superfluid model of the nucleus, we disregarded the long-range residual interactions, responsible for the main collective properties of the nucleus.

It must be noted that the results of the calculations on the basis of the superfluid model of the nucleus for specified values of G and for specified energy levels of the effective potential are single valued. In light of this single-valuedness, particular interest attaches to the mutual compatibility of the results obtained, namely: for the same values of the interaction constants G we obtain reasonable values of the pairing energies and of the levels of the even and odd nuclei, and when G is changed, say, by 30 — 40 percent to either side, a sharp discrepancy is observed with experiment, both in the level behavior and in the pairing energies.

It must also be noted that on the basis of the superfluid model of the nucleus it is possible to carry out all-out investigations of the properties of strongly deformed nuclei, for which it is necessary to calculate the moments of inertia of the ground and excited states, the probabilities of β and γ transitions, the magnetic moments, etc. On the other hand, to obtain more detailed results it is necessary to determine more accurately the behavior of the energy levels of the self-consistent field.

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