A BOSON DOUBLET EQUATION

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An equation for a boson doublet is set up. The equation is shown to be invariant to two transformations of the Pauli-Gürsey type.

THE question as to whether bosons are described by wave equations of the Klein-Gordon form or by Kemmer-Duffin equations has not yet received a conclusive answer. It is therefore interesting to investigate the physical content and mathematical properties of both types of equation both for individual particles and for multiplets. The corresponding equation for Fermion doublets has been analyzed in sufficient detail by Gürsey.[1] One of the most interesting properties of this equation is its invariance to the Pauli-Gürsey transformation group. In this work, we examine the question as to whether or not there exists a doublet boson equation which is invariant to transformations of the type of the Pauli-Gürsey group. A doublet boson equation could have immediate practical application, since the K mesons have a manifest doublet structure.

The simultaneous linearization of two Klein-Gordon equations gives the first-order equation

$$(B_\nu \partial/\partial x_\nu + k_0 I)\,\psi = 0, \tag{1}$$

with which we will describe the boson doublet. Here

$$B_\nu = 1 \times \beta_\nu, \quad I = 2\sigma_1 \times 1, \tag{2}$$

where β_ν (and later η_ν) are 5×5 or 10×10 Kemmer-Duffin matrices[2] and σ_k are the 2×2 spin matrices. The sign × designates the direct product of matrices.

Equation (1) can be shown to be invariant under the transformations

$$\psi' = \exp(i\lambda + i\lambda_k T_k)\cdot\psi, \tag{3}$$

where λ and λ_k are four real parameters and

$$T_1 = \sigma_1 \times 1, \quad T_2 = \sigma_2 \times \eta_5, \quad T_3 = \sigma_3 \times \eta_5;$$
$$\eta_5 = \eta_1\eta_2\eta_3\eta_4. \tag{4}$$

The matrices T_k (k = 1, 2, 3) can be regarded as isotopic spin matrices. The transformation (3) gives a conservation law for the number of bosons and conservation laws for the isotopic spin components.

The electric charge operator has the form

$$Q = e\left(T_3 + \tfrac{1}{2}\right). \tag{5}$$

The equation for the boson in an external electromagnetic field, A_ν, will be

$$\left[B_\nu\left(\frac{\partial}{\partial x_\nu} + \frac{i}{\hbar c} QA_\nu\right) + k_0 I\right]\psi = 0. \tag{6}$$

The charge-conjugate wave function is

$$\psi^C = C\overline{\psi}^T, \qquad \overline{\psi} = \psi^*(1 \times \eta_4),$$

where the operator C can be one of the operators

$$C_1 = 2\sigma_3 \times 1, \qquad C_2 = 1 \times \eta_5. \tag{7}$$

Equation (1) is invariant under the canonical transformations

$$\psi' = a\psi + bIC_2\overline{\psi}^T, \quad \psi' = a\psi + bC_2\overline{\psi}^T,$$
$$|a|^2 + |b|^2 = 1, \tag{8}$$

which are analogous to the Pauli transformation.

One possible assumption is that equation (6) describes the K meson doublet (K^+, K^0). If that is the case, it is not necessary to include K^- and $\overline{K}^0$ mesons explicitly in the scheme of elementary particles, since they can be obtained by charge conjugation. The place thus vacated in the scheme can be used for new mesons (for example, for D^- and D^0 mesons). Then the new mesons can be particles with strangeness −1, and not necessarily with strangeness +2, as in the Gell-Mann scheme.

We note further that the equation for the meson doublet can be written in a form which takes into account the mass difference of the mesons:[3]

$$[B_\nu\, \partial/\partial x_\nu + k_0 I \exp(fT_3)]\,\psi = 0. \tag{9}$$

Here f is a real constant. Equation (9) is invariant under the transformation

$$\psi' = \exp(i\lambda + i\lambda_3 T_3)\cdot\psi \tag{10}$$

It is also invariant under two transformations of the type (8), but these transformations are not canonical, i.e., they do not preserve the commutation relations given, for example, in the book by Akhiezer and Berestetskii.[4]

[1] F. Gürsey, Nuovo cimento **7**, 411 (1958).
[2] W. Pauli, Revs. Modern Phys. **13**, 203 (1941).
[3] H. Oiglane, JETP **34**, 1337 (1958), Soviet Phys. JETP **7**, 922 (1958).
[4] A. I. Akhiezer and V. B. Berestetskii, Квантовая электродинамика (Quantum Electrodynamics), 2nd ed., Fizmatizdat.

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