INVESTIGATION OF THE SPECTRUM AND ASYMMETRY OF ELECTRONS FROM THE $\pi - \mu -$ e DECAY IN NUCLEAR EMULSION

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Submitted to JETP editor December 1, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 1042-1049 (April, 1961)

The energy spectrum and spatial asymmetry of positrons from the π^+-μ-e decay in nuclear emulsion placed in a magnetic field have been measured. The values obtained for the Michel parameter $\rho = 0.66 \pm 0.07$ and the asymmetry parameter $\delta = 0.63 \pm 0.12$ are in agreement with the theory of the two-component neutrino.

1. EXPERIMENTAL METHOD AND DATA

In this article, we present the results of the study of the energy spectrum of electrons produced in the π^+-μ-e decay in nuclear emulsion and the dependence of the spatial asymmetry of the electrons on their energy. Part of the data has been published earlier.[1,2]

The basic measurements were carried out on μ^+ mesons from π-μ-e decays occurring in emulsion. A small part of the measurements were made for μ^--e^- decays. The experiment was performed with stacks of 50 — 100 NIKFI-R emulsion pellicles $10 \times 10 \times 0.04$ cm or $10 \times 15 \times 0.04$ cm. The stacks were exposed to beams of π^+ or μ^- mesons from the Joint Institute for Nuclear Research proton synchrotron in Dubna. For the study of the decay asymmetry, the emulsion stacks were placed between the poles of an electromagnet in a field of 15 koe parallel to the plane of the pellicles. In the measurements without a magnetic field, the stacks were placed in a double magnetic shield in which the field was $< 10^{-2}$ oe.[3] The stacks were developed with semi-automatic equipment described by Samoĭlovich et al.[4]

The decay-electron spectrum was measured by the multiple scattering method. The selection criteria for the electron tracks required that the track length be at least 1 mm and that the point of the μ — e decay occur at least 50 μ from the surface of the pellicle. Moreover, all the analyzed decays had to be at least 1 cm from the edge of the pellicle.

During the first phase of the experiment, the measurements were made on a practically "noiseless" microscope stage which had glass guides,[5] a turntable for rapid orientation of the track, and a microscope stage-feed screw with an electronic device for the automatic displacement of the track by an arbitrary cell length. This "noiseless" stage was coupled to a Lumipan microscope. During the second phase of the experiment, the measurements were carried out on a Koristka MS-2 microscope.

We determined the energy of the decay electrons with the aid of a semi-automatic device for scattering measurements, a description of which will be given below. The parameter by which we determined the electron energy with this device was the mean value of the absolute magnitude of the second (D_2) and third (D_3) differences of the track coordinates perpendicular to the direction of displacement of the measuring stage of the microscope. The semi-automatic measuring device "excluded without replacement" second differences whose absolute value exceeded $4\bar{D}_2$ and the related third differences independently of their magnitude.

The measurements were made twice by two observers. The data of both measurements were averaged. We eliminated the noise by means of the formula $D_{2true}^2 = D_2^2 - \Delta_2^2$, where Δ_2 is the mean value of the second differences of the noise measured with great accuracy on electron tracks whose over-all length was ~ 4 cm. The values of the noise for the second differences were 0.19 and 0.24 μ for the measurements on the Koristka and Lumipan microscopes, respectively. The cell length for the measurements was chosen so that the signal-to-noise ratio for the third differences was within the limits of 2.4 — 4.5.

The transition from the second differences to the energy was effected by means of the formula

$$E\,[\mathrm{Mev}] = K/\alpha\,[\mathrm{deg}],$$

where α is the scattering angle, which is equal to $(D_2/t)(180/\pi)$, and K is the scattering constant. This constant depends on the cell length t (in microns) in the following way:[6]

Table I

	Experiment No.			
	1	2	3	4
Sign of μ meson	+	—	—	+
Magnetic field, oe	0	0	11000	17000
Number of particles in the spectrum	1102	302	302	1867

$$K = K_0 (0.272 \log 6.31\, t + 0.090)^{1/2}. \quad (1)$$

We took $K_0 = 26.3 \pm 0.5$ deg-Mev-$(100\,\mu)^{-1}$ (see discussion below as regards the choice of the value of this constant). For steep tracks with a dip angle greater than 6° with respect to the plane of the emulsion, we introduced a correction for the track dip. For a comparison of the measured spectra with the theoretical spectra, the electron energy ϵ was expressed in fractions of the maximum energy which the electron could attain in the $\mu^+ - e^+$ decay: $\epsilon = E/52.8$ (E in Mev).

For the study of the decay asymmetry, it was necessary to measure the electron angular distributions in addition to their energy. This task was simplified by the fact that we measured the energy only for decay electrons emitted "forward" or "backward."[3] In the first case, the μ^+-meson and electron tracks made angles of $\gamma = 0 \pm 45°$ and $\beta = 0 \pm 45°$ or $\gamma = 180 \pm 45°$ and $\beta = 180 \pm 45°$, respectively, relative to the magnetic field direction. In the second case, the directions of flight of the μ^+ meson and electron were opposite to each other, i.e., $\gamma = 0 \pm 45°$ and $\beta = 180 \pm 45°$ or $\gamma = 180 \pm 45°$ and $\beta = 0 \pm 45°$. Such a choice of angles made it possible to use for the analysis of the dependence of the asymmetry on the energy the statistically most significant part of the angular distribution.

A summary of the experiments and the statistics is given in Table I. Experiments 3 and 4 were set up to observe the asymmetry for μ^- decays in emulsion and the possible effect of the magnetic field on the magnitude of this asymmetry.[7] Since the electron spectrum from μ^- decays in emulsion does not differ from the positron spectrum, these data were included in the total statistics.

The obtained data are collected in Table II, where the spectra measured in experiments 1, 2 + 3, and 4 are given for the energy intervals $\Delta\epsilon = 0.1$. The last spectrum was obtained for positrons from 1867 decays in a strong magnetic field of 17 000 oe. The same spectrum was used for the measurement of the asymmetry parameter δ. For this spectrum, the number of positrons emitted "backward" are shown in the parentheses.

Figure 1 shows a histogram for the spectrum of positrons emitted "backward" and "forward."

Table II

Interval ϵ	Spectra		
	1	3+4	2
0—0.1	1	2	5(2)
0.1—0.2	14	10	28(19)
0.2—0.3	44	27	93(46)
0.3—0.4	76	45	142(72)
0.4—0.5	124	69	201(117)
0.5—0.6	150	76	261(144)
0.6—0.7	146	84	293(171)
0.7—0.8	178	114	251(156)
0.8—0.9	132	73	203(139)
0.9—1.0	93	35	152(99)
1.0—1.1	60	37	95(64)
1.1—1.2	37	13	62(48)
1.2—1.3	20	9	35(24)
1.3—1.4	10	6	19(15)
1.4—1.5	10	2	10(8)
1.5	7	2	17(15)
Number of particles	1102	604	1867(1139)

2. COMPARISON OF THE RESULTS WITH THE THEORY

The theory of β decay with account of parity nonconservation gives the following expression for the spectrum and angular distribution of the electrons produced in the decay $\mu \to e + \nu + \bar{\nu}$:[8]

$$N(\varepsilon, \vartheta)\, d\varepsilon d\Omega = \{3(1-\varepsilon) + 2\rho\,(\tfrac{4}{3}\varepsilon - 1)$$
$$\pm\, \xi \cos\vartheta [(1-\varepsilon) + 2\delta\,(\tfrac{4}{3}\varepsilon - 1)]\}\, \varepsilon^2 d\varepsilon\, d\Omega. \quad (2)$$

This expression was obtained by neglecting the radiation effects and the electron mass in comparison with its momentum. The constants ξ, ρ, and δ are related to the interaction constants in a definite way. In particular, it is known from two-component theory that $\rho = \delta = 3/4$ and $|\xi| = |C_V C_A^* + C_A C_V^*| \times (|C_V^2| + |C_A^2|)^{-1}$, while in the theory of the universal Fermi interaction with coupling constants of equal absolute magnitude and opposite sign ($C_V = -C_A$) we have $\rho = \delta = 3/4$ and $|\xi| = 1$.

The aim of the measurements was to determine the parameters ξ, ρ, and δ. The value of the

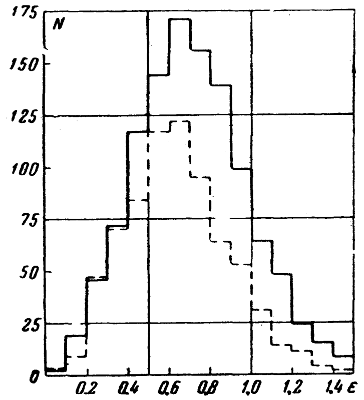


FIG. 1. Energy spectrum of positrons emitted backward (solid line) and forward (dotted line) in $\pi-\mu-e$ decays.

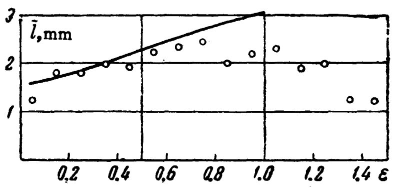


FIG. 2. Dependence of the mean track length in emulsion on the positron energy in the $\mu^+ - e^+$ decay. The solid curve is the calculated distribution of mean lengths; the circles represent the experimental data

parameter ξ for nuclear emulsion and its dependence on the magnetic field has been discussed previously.[3] It was shown that, for the NIKFI-R emulsion employed by us, the limiting value of the coefficient ξ obtained in a field of 17 000 oe is equal to $|\xi| = 0.85 \pm 0.05$.

In the present work, we determined the parameters ρ and δ. For a statistical estimate of these parameters by the χ^2 method, it is necessary to calculate the integrals

$$\Phi(\varepsilon)\,d\varepsilon = d\varepsilon \int \varphi(\varepsilon')\,\Gamma(\varepsilon', \varepsilon)\,d\varepsilon', \quad (3)$$

which result from "convolution" of the initial theoretical function with the "instrument function" $\Gamma(\epsilon', \epsilon)$ giving the probability of obtaining an energy ϵ from a measurement of a true energy ϵ'.

A similar "spreading" of the initial spectra in our measurements resulted from two basic factors: radiative slowing down of electrons in emulsion and dispersion of the scattering measurements.[9] The radiation length in emulsion is close to 29 mm, and although the lengths of a large part of the electron tracks lay within the limits of 1 — 3 mm, the distortion of the spectral shape due to bremsstrahlung proved to be important in the high-energy region, at the end of the electron spectrum. The loss of energy by the electrons due to bremsstrahlung in emulsion is described by the Bethe-Heitler formula[10] giving the probability that an electron with an initial energy ϵ_0 has, after traversing a layer t, an energy in the interval ϵ_t, $\epsilon_t + d\epsilon_t$:

$$\pi(\varepsilon_0, \varepsilon_t, t)\,d\varepsilon_t = \frac{d\varepsilon_t}{\varepsilon_0}(1+a)^{bt}\left(\frac{\varepsilon_t}{\varepsilon_0}\right)^a \frac{[\ln(\varepsilon_0/\varepsilon_t)]^{bt-1}}{(bt-1)!}. \quad (4)$$

Here t is the track length in radiation units; a and b are numerical coefficients equal, respectively, to $2/5$ and $4/3$ for $\epsilon_0 \le 0.57$ and $1/4$ and $4/3$ for $\epsilon_0 > 0.57$.

The "convolution" of this expression with the theoretical spectrum involves finding the function

$$\varphi^*(\varepsilon)\,d\varepsilon = \int_{\varepsilon_0=\varepsilon}^{\varepsilon_0=1} \varphi(\varepsilon_0)\,d\varepsilon_0\,\pi(\varepsilon_0, 2\varepsilon - \varepsilon_0, t). \quad (5)$$

The substitution of $2\epsilon - \epsilon_0$ for ϵ_t in the integration is connected with that fact that in the measurements by the multiple scattering method, we actually measure the mean energy $\epsilon = (\epsilon_t + \epsilon_0)/2$.

In "convolution" of the theoretical spectrum with this expression for the energy of an electron traversing a layer of substance t, one should keep in mind the fact that the mean track length of the electrons, owing to what is known as the "flat stack effect" is different for different parts of the spectrum. This is illustrated in Fig. 2, where the mean length of the electron tracks is laid off along the ordinate axis and the measured value of the energy, along the abscissa axis. On the basis of these data, we divided the region of integration into five intervals ($\Delta\epsilon$ = 0—0.2, 0.2—0.4, 0.4—0.6, 0.6—0.8, and 0.8—1.0) taking the mean value of t for each interval from Fig. 2.

The functions obtained for $\varphi^*(\epsilon)$ are subject to a second "convolution" with distributions characterizing the scattering measurements. It is well known that the second differences in scattering measurements have a Gaussian distribution.[6] The necessity of measuring the entire spectrum with a constant signal-to-noise ratio leads to an increase in the dispersion of the scattering measurements as one goes from the beginning of the spectrum to the end. In this connection, we divided the spectrum into five intervals coinciding with those mentioned above, and we constructed the instrument function for each interval:

$$\Gamma_j(\varepsilon, \bar{\varepsilon}) = \sum w(n_i)\frac{\bar{\varepsilon}}{\varepsilon^2}\frac{\sqrt{n_i}}{\lambda\sqrt{2\pi}}\exp\left[-\frac{n_i}{2\lambda^2}\left(\frac{\varepsilon-\bar{\varepsilon}}{\varepsilon}\right)^2\right]. \quad (6)$$

Here $w(n_i)$ is the relative number of tracks measured with a division into n_i cells; λ is the dispersion parameter, which is equal to the coefficient of $1/\sqrt{n}$ in the expression for the relative error of the measurements: $\Delta D_2/D_2 = \lambda/\sqrt{n}$.

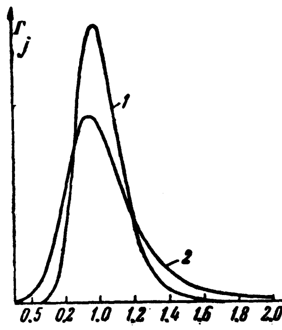


FIG. 3. The functions Γ_j for the beginning (1) and end (2) of the spectrum.

Figure 3 shows examples of the function Γ_j for the intervals $\Delta\epsilon$ = 0—0.2 and 0.8—1.0 with λ = 0.95.

The value of the parameter λ depends on the signal-to-noise ratio, and, under our conditions of measurement, its expected value[6] lies in the limits 0.8—1.0 for the second differences.

Constructing for each interval j the functions Γ_j, we carry out the second "convolution" according to the formula

$$\varphi^{**}(\varepsilon) = \int_0^{\bar{\varepsilon}=1} \varphi_i^*(\bar{\varepsilon})\, d\bar{\varepsilon}\Gamma(\varepsilon, \bar{\varepsilon}), \qquad (7)$$

by using for each interval of integration over the limits $\bar{\epsilon} = 0-1$ the corresponding function Γ_j. The bulk of these calculations was performed on an electronic computer.

As a result of the double "convolution" of the theoretical spectra and the instrument function, the obtained spectra $\varphi^{**}(\epsilon)$, with which the experimental data should be compared for the estimate of the parameters ρ and δ, proved to depend not only on the estimated parameters, but also on the parameters determining the measurements, especially on the quantity λ.

On the other hand, the form of the experimentally obtained spectrum depends on the choice of the scattering constant K. The parameters ρ and δ estimated from the minimum value of the χ^2 sum will therefore depend on the values chosen for λ and K.

In accordance with the general ideas underlying the statistical methods of estimating parameters, the "best fit" to the experimental data is the spectrum which gives the minimum value of χ^2 for the variation of the three parameters λ, K, and ρ. This means that we seek values of these parameters which satisfy the equation

$$\partial\chi^2/\partial K = \partial\chi^2/\partial\lambda = \partial\chi^2/\partial\rho = 0.$$

We estimated the parameters ρ and δ by the following procedure. Considering a small change of the parameters K and λ and varying the parameter ρ in the interval 0.4—0.8 for each pair of values of K and λ, we found the values of K and λ for the absolute minimum $\chi^2_{\min}$. In the analysis, we considered the values $K = K_0(1+\eta)$, where $\eta = -0.03$, 0.00, 0.02, 0.04 and $\lambda = 0.7$, 0.8, 0.9, 1.0, 1.1, 1.2. The exact values of $\chi^2_{\min}$ were found by quadratic extrapolation. Obtaining in this way the optimum values of K and λ, we determined ρ. For the analysis of the parameters K, λ, and ρ, we first used the spectrum for 3581 particles (Table II), where we divided it into nine intervals: $\Delta\epsilon = 0.4-0.5$, 0.5—0.6, 0.6—0.7, 0.7—0.8, 0.8—0.9, 0.9—1.0, 1.0—1.1, 1.1—1.2, 1.2—1.5.

Figure 4 represents the dependence of the value of $\chi^2_{\min}$ on the value of K. In accordance with this estimate, we take for the quantity K_0 in formula (1) the value corresponding to the minimum of K_0, i.e., $K_0 = 26.3 \pm 0.4$, where the error corresponds to the deviation of the function χ^2 to a value exceeding that at the minimum by $\pm\sqrt{2p}$ (p is the number of degrees of freedom), i.e., to the value 9. The obtained value of K_0 is in agreement, within the limits of the measurement error, with the values of K_0 obtained in calibration measurements reported in the literature.[6]

In a similar way, we obtained for the quantity λ the value $\lambda = 0.97 \pm 0.07$. This value is 5—10% higher than that following from the relations usually employed.[6] The difference can be accounted for by the additional dispersion due to the radiation losses and the device for the scattering measurements.

It should be noted that the choice of the quantity λ is not as important as the choice of K; for a given K, the quantity ρ is practically independent of λ, while the dependence of ρ on K is very strong: $\partial\rho/\partial K \approx 0.18$.

Figure 5 shows the results of the statistical analysis of the values of the parameter ρ determined from the χ^2 test for the values of the parameters K and λ indicated above. The analysis was made for the total spectrum obtained with 3580 particles. Since radiative corrections to the spectrum[11,12] and the corrections for the "flat stack effect" are appreciable at the beginning of

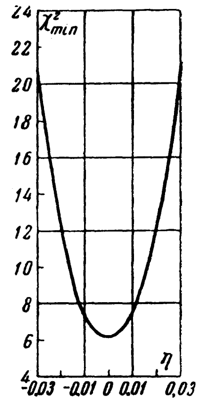


FIG. 4. Dependence of the value of $\chi^2_{\min}$ on the scattering constant $K = K_0(1+\eta)$.

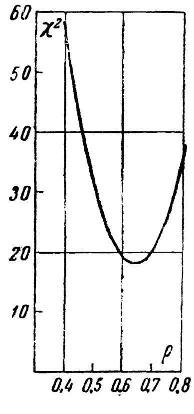


FIG. 5. Dependence of the value of χ^2 on the Michel parameter ρ for the positron spectrum.

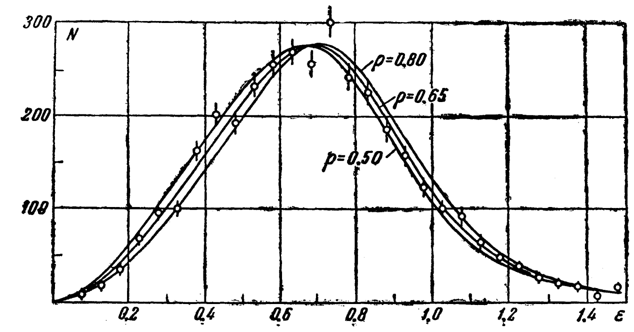


FIG. 6. Positron spectrum (3580 particles).

the spectrum, the analysis was begun with $\epsilon = 0.4$. The spectrum was split up into intervals of width $\Delta\epsilon = 0.05$ and extended to $\epsilon = 1.5$, which corresponds to twenty-two degrees of freedom. The minimum value of χ^2 in this figure corresponds to $\rho = 0.64$, which is also the estimate of this parameter from our data.

For the estimate of the error, we used the error formula

$$\sigma_\rho^2 = \sigma_{st}^2 + (\partial\rho/\partial K)\,\sigma_K^2 + (\partial\rho/\partial\lambda)^2\sigma_\lambda^2,$$

where the first term is the statistical error in the determination of ρ, and the second and third terms are the errors due to the uncertainty in the values of the constants K and λ. Our analysis indicates that the last error can be neglected in comparison with the first two. The statistical error in the determination of ρ was equal to 0.03. The basic error results from the uncertainty in the scattering constant. This was equal to $0.18 \times 0.4 = 0.07$. We finally obtain

$$\rho = 0.64 \pm 0.10.$$

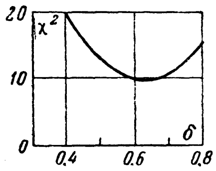
This value of the parameter ρ is in agreement, within the limits of experimental error, with the values of ρ obtained by the emulsion technique[9] and by other methods.[13] The estimate given above of the parameter ρ was made without taking into account the radiative corrections. The introduction of the radiative corrections to the spectrum[11,12] in a manner similar to that used by Rosenson[14] and Dudziak et al.[15] shifts the effective value of ρ from 0.64 to 0.66, without affecting the error estimate.

Figure 6 shows the positron spectrum. The solid curves represent the theoretical spectrum for $\rho = 0.50$, 0.65, and 0.80 broadened by bremsstrahlung and the instrument error.

For an estimate of the parameter δ, we used the spectrum of the "forward-backward" difference shown in Fig. 1. As follows from formula (2), this difference is independent of the quantity ρ.

In order to obtain the theoretical spectral distribution of the difference, we should bear in mind

FIG. 7. Dependence of the quantity χ^2 on the parameter δ for the positron spectrum (from 1867 cases).



the fact that in our case we have an almost "flat" geometry. If in (2) we replace $\cos\vartheta$ by $\cos\gamma\cos\beta$ and $d\Omega$ by $d\gamma\, d\beta$ and integrate over intervals of angles γ and β which were used [see formula (2)], we obtain

$$n(\varepsilon)\,d\varepsilon = \frac{32N\xi}{\pi^2}\left\{(1-\varepsilon) + 2\delta\left(\frac{4}{3}\varepsilon - 1\right)\right\}\varepsilon^2 d\varepsilon\cdot 0.975. \quad (8)$$

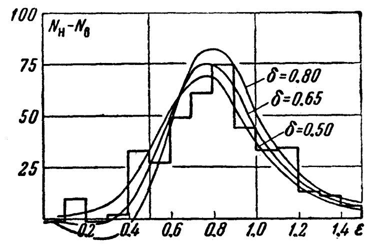
The coefficient 0.975 in this formula takes into account the deviation of our geometry from a "flat" geometry. The difference expected from this formula is 411 particles (for $|\xi| = 0.85$), while the number observed in the spectrum was 417.

Carrying out the above-described "convolution" operation for the spectrum (8), we obtain a family of curves with the parameter δ. The "best value" of the parameter δ was determined by the χ^2 test, where the quantity χ^2 was defined by the formula[16]

$$\chi^2 = \sum (N_{bi} - p_i N_i)^2 / N_i p_i (1 - p_i).$$

In this formula, N_i and N_{bi} are the total number of particles and the number of particles emitted backward, respectively, for the i-th interval and p_i and $(1 - p_i)$ are the theoretical probabilities of the emission of electrons "backward" and "forward," respectively, calculated from the "convolution" spectra for the value $\rho = 0.66$. Owing to the presence of the coefficients p_i and $(1 - p_i)$, the quantity ρ indirectly affects the value of δ.

FIG. 8. Dependence of the asymmetry on the positron energy in the $\mu^+ - e^+$ decay.



A plot of the dependence of χ^2 on δ is shown in Fig. 7. It follows from this plot that the "best" value of δ is $\delta = 0.63$. In Fig. 8, the solid curves represent the energy dependence of the asymmetry corresponding to this value of δ and the values $\delta = 0.8$ and 0.5. To estimate the error in the value $\delta = 0.63$, we start from the formula

$$\sigma^2(\delta) = \sigma_{st}^2 + (\partial\delta/\partial K)^2\sigma_K^2 + (\partial\delta/\partial\rho)^2\sigma_\rho^2 + (\partial\delta/\partial\xi)^2\sigma_\xi^2.$$

The statistical error of this estimate is 0.08. The errors due to the uncertainty in the determination of the constants K and ρ are equal, respectively, to 0.07 and 0.08. The last term can be neglected. We finally obtain

$$\delta = 0.63 \pm 0.13.$$

This value is in agreement with the determinations of δ made by other methods.[13,17] The radiative corrections, which are important at the beginning of the spectrum, have practically no effect on this estimate.[18]

3. CONCLUSIONS

The measurements made in this and in the previous experiments give the following values for the parameters of $\mu - e$ decays in emulsion: $|\xi| = -0.85 \pm 0.05$, $\rho = 0.66 \pm 0.07$, $\delta = 0.63 \pm 0.12$. As was indicated earlier,[3] the deviation of $|\xi|$ from the value $|\xi| = 1$ predicted by the V-A variant of the theory is far beyond the limits of error, but can be attributed to the presence of an additional depolarization mechanism not eliminated by the magnetic field. The values of the Michel parameter ρ and of the asymmetry parameter δ proved to be less than the value 0.75 predicted by the two-component theory without the radiative corrections. It should be borne in mind, however, that the method of measuring the particle energy from its multiple scattering has, in general, a tendency to give a lowered value of the energy. This leads to a decrease in the values of ρ and δ obtained experimentally. Moreover, a source of a systematic shift in the values can be the incomplete correspondence between the "convolution" operations and the actual conditions of measurement. Therefore the results obtained by us should be considered to be in agreement with the two-component neutrino theory.

The authors thank A. I. Alikhanov for his interest in this work. The authors also thank the scanning staff for scanning a large number of pellicles and for making difficult measurements, and O. N. Vasil'ev for performing the calculations on the electronic computer.

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Translated by E. Marquit
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