INTERACTION OF CHARGED-PARTICLE BEAMS WITH LOW-FREQUENCY PLASMA OSCILLATIONS

A. I. AKHIEZER, A. B. KITSENKO, and K. N. STEPANOV

Physico-Technical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor January 27, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 1866-1870 (June, 1961)

The interaction of a charged-particle beam with the low-frequency oscillations of a plasma (magnetoacoustic and Alfvén waves) is analyzed in the absence of collisions. It is shown that if the thermal velocity spread of the particles in the beam is sufficiently small this interaction can cause the plasma-beam system to become unstable.

1. As is well known, a charged-particle beam passing through a plasma can excite electromagnetic oscillations in the plasma.[1,2] These can be high-frequency oscillations, due to the electrons alone, and/or low-frequency oscillations, in which both the electrons and ions participate. In general, excitation of oscillations is associated with a plasma instability. Hence, to clarify the stability conditions in a plasma it is important to investigate the interaction of charged-particle beams with all the waves that can propagate in the plasma. In the present paper we investigate the interaction of a compensated beam of charged particles with low-frequency plasma oscillations (primarily the magnetoacoustic waves) in the presence of a fixed magnetic field parallel to the direction of motion of the beam.*

If the plasma is dilute so that the oscillation frequency ω is much higher than the collision frequency $1/\tau$ the kinetic equation must be used to analyze the plasma oscillations. If, however, $\omega\tau \ll 1$, the plasma can be treated as a hydrodynamic system. We consider the case $\omega\tau \gg 1$, which is of greatest interest.

2. The general dispersion equation for plasma oscillations in an external magnetic field, with an arbitrary particle velocity distribution, can be written in the form

$$An^4 + Bn^2 + C = 0, \tag{1}$$

where $n = kc/\omega$, $\mathbf{k}$ is the wave vector and the quantities A, B and C are expressed in terms of the components of the dielectric tensor ϵ_{ij}.

We assume that the following conditions are satisfied:

$$\omega \ll \omega_{Hi}, \qquad kv_i \ll \omega_{Hi}, \qquad kv_0 \ll \omega_{Hi}, \tag{2}$$

where ω_{Hi} is the ion gyromagnetic frequency, $v_i = (T_i/M)^{1/2}$ is the mean thermal velocity of the ions (T_i is the ion temperature and M is the ion mass), and $\mathbf{v}_0$ is the beam velocity. In this case, as an approximation the general dispersion equation (1) can be separated into two equations

$$(kc/\omega)^2 \cos^2\theta - \varepsilon_{11} = 0, \tag{3}$$

$$(kc/\omega)^2 - \varepsilon_{22} - \varepsilon_{23}^2/\varepsilon_{33} = 0, \tag{4}$$

which describe respectively the Alfvén and magnetoacoustic waves (the 3 axis is along the external magnetic field $\mathbf{H}_0$, the wave vector $\mathbf{k}$ lies in the 1-3 plane and θ is the angle between $\mathbf{k}$ and $\mathbf{H}_0$).

We assume that the equilibrium velocity distributions of the plasma electrons and ions are Maxwellian with characteristic temperatures T_e and T_i; the velocity distribution of the particles in the beam is assumed to be

$$f_{e,i} = n_0' \left(\frac{m_{e,i}}{2\pi T_{e,i}'}\right)^{3/2} \exp\left\{-\frac{m_{e,i}(\mathbf{v}-\mathbf{v}_0)^2}{2T_{e,i}'}\right\} \tag{5}$$

(n_0' is the number density of the beam particles, T_e' and T_i' are the temperatures of the electrons and ions in the beam, $m_e = m$, $m_i = M$). In this case the components of the tensor ϵ_{ij} are given by[9]

*The excitation of acoustic oscillations that result from the motion of the plasma electrons relative to the ions in a highly nonisothermal plasma with no magnetic field has been studied by Gordeev[3] (cf. also references 4 and 5). The same problem, but with a magnetic field, has been investigated by Bernstein and Kulsrud[6] for the case in which the Alfvén velocity is large compared with the acoustic velocity. Analyses of the excitation of magnetohydrodynamic waves in which the thermal motion of the plasma particles is neglected have been carried out by a number of authors.[5,7,8]

$$\varepsilon_{11} = 1 + \sum_\alpha \frac{\Omega_\alpha^2 (\omega - k_\parallel v_{0\alpha})^2}{\omega_{H\alpha}^2 \omega^2},$$

$$\varepsilon_{22} = \varepsilon_{11} + \sum_\alpha \frac{\Omega_\alpha^2 k^2 v_\alpha^2}{\omega_{H\alpha}^2 \omega^2}\, 2i\sqrt{\pi}\sin^2\theta z_\alpha w(z_\alpha),$$

$$\varepsilon_{33} = 1 + \sum_\alpha \frac{\Omega_\alpha^2}{k_\parallel^2 v_\alpha^2}(1 + i\sqrt{\pi} z_\alpha w(z_\alpha)),$$

$$\varepsilon_{23} = -\sum_\alpha \frac{\Omega_\alpha^2}{\omega\omega_{H\alpha}}\sqrt{\pi}\,\mathrm{tg}\,\theta z_\alpha w(z_\alpha), \qquad (6)^*$$

where

$$w(z_\alpha) = e^{-z_\alpha^2}\Big(\pm 1 + \frac{2i}{\sqrt{\pi}}\int_0^{z_\alpha} e^{t^2}dt\Big), \qquad z_\alpha = \frac{\omega - k_\parallel v_{0\alpha}}{\sqrt{2}k_\parallel v_\alpha},$$

$$\Omega_\alpha^2 = 4\pi e^2 n_{0\alpha}/m_\alpha, \qquad v_\alpha^2 = T_\alpha/m_\alpha,$$

$$\omega_{H\alpha} = e_\alpha H_0/m_\alpha c, \qquad k_\parallel = k\cos\theta$$

(the upper sign is taken in the expression for $w(z_\alpha)$ when $k_\parallel > 0$ and the lower sign is taken when $k_\parallel < 0$; the subscript α indicates the kind of particle for both the plasma and beam).

3. Using Eq. (6) and the dispersion equation (3), we find the following expression for the frequency of the Alfvén wave as modified by the presence of the beam:†

$$\omega = k_\parallel \frac{v_0\Omega_i'^2 \pm [(\Omega_i^2 + \Omega_i'^2) c^2\omega_{Hi}^2 - \Omega_i^2\Omega_i'^2 v_0^2]^{1/2}}{\Omega_i^2 + \Omega_i'^2}. \qquad (7)$$

We see that the plasma-beam system is unstable against the excitation of Alfvén waves if the following condition is satisfied‡

$$v_0^2 > V_A^2 + V_A'^2, \qquad (8)$$

where $V_A = H_0/\sqrt{4\pi n_0 M}$, $V_A' = H_0/\sqrt{4\pi n_0' M}$.

It follows from Eq. (8) that Alfvén waves cannot be excited at sufficiently low beam densities. However, this result has been obtained neglecting the coupling between the Alfvén waves and the magnetoacoustic waves. When this coupling is taken into account it is found that an instability also arises at densities for which the relation in (8) is not satisfied. In this case the growth factor for $V_A \gtrsim v_i$ is found to be at least $(\omega_{Hi}/\omega)^2$ times smaller than the growth factor for the magnetoacoustic waves. For this reason we shall not investigate this question in any greater detail.

4. We now consider the interaction of the beam with the magnetoacoustic waves, assuming that the beam density is small compared with the plasma density. If $kv_i \ll \omega \ll kv_e$, in the absence of the beam, the solution of the dispersion equation (4) is[9]

$$\omega_\pm = kV_\pm,$$

$$V_\pm^2 = \tfrac{1}{2}\{V_A^2 + s^2 \pm [(V_A^2 + s^2)^2 - 4V_A^2 s^2\cos^2\theta]^{1/2}\}, \qquad (9)$$

where $s = (T_e/M)^{1/2}$. We note that when $V_A \sim s$ the quantities $V_\pm$ are also of order s. In this case the condition $\omega \gg kv_i$ is satisfied only for a highly nonisothermal plasma ($T_e \gg T_i$); when $T_e \lesssim T_i$ the magnetoacoustic waves are strongly damped.

In addition to assuming $kv_i \ll \omega \ll kv_e$ we assume that the condition $|\omega - k_\parallel v_0| \gg kv_e'$ is satisfied; this means that the thermal spread of the electrons in the beam is small. With these assumptions the solution of the dispersion equation can be written in the form

$$\omega = k_\parallel v_0 + \varepsilon, \qquad (10)$$

where $|\epsilon| \ll |k_\parallel v_0|$. It follows from Eqs. (4) and (6) that

$$\varepsilon = (M/m)^{1/2}\varepsilon_0 = \pm k_\parallel v_0\left[\frac{n_0' M(v_0^2\cos^2\theta - V_A^2)s^2\cos^2\theta}{n_0 m(v_0^2\cos^2\theta - V_+^2)(v_0^2\cos^2\theta - V_-^2)}\right]^{1/2}(1 + i\eta), \qquad (11)$$

$$\eta = \pm\sqrt{\frac{\pi}{8}}\frac{v_0}{v_e}\left(\frac{2s^2}{v_0^2}\sin^2\theta + \frac{V_A^2 - v_0^2\cos^2\theta}{s^2}\right) \qquad (12)$$

(the upper sign in (12) corresponds to $k_\parallel < 0$ and the lower sign to $k_\parallel > 0$).

The quantity ϵ is given by Eq. (11) if the thermal motion of the electrons in the beam can be neglected. If $V_A \sim s$ it is legitimate to neglect this motion if the inequality $n_0 T_e' \ll n_0' T_e$ is satisfied. Furthermore, the condition $|\epsilon| \ll k_\parallel v_0$ must be satisfied. Thus, when $V_A \sim s$, Eq. (11) is valid when $T_e'/T_e \ll n_0'/n_0 \ll m/M$.

We see that the plasma-beam system is unstable against the excitation of magnetoacoustic waves for the conditions being considered here. If the velocity v_0 does not lie in the interval $s < v_0 < V_A$ the instability arises even if the quantity η ($|\eta| \ll 1$) is neglected. Even when v_0 lies within the interval $s < v_0 < V_A$, however, the imaginary part of ϵ is proportional to η.

The frequency increment ϵ, given by (11), becomes infinite when $v_0\cos\theta \to V_\pm$. In this case Eq. (11) does not apply and must be replaced by the expression

$$\frac{\varepsilon}{\omega_\pm} \equiv \left(\frac{M}{m}\right)^{1/3}\frac{\varepsilon_0}{\omega_\pm} = \frac{-1 + i\sqrt{3}}{2^{4/3}}\left|\frac{n_0' M\cos^2\theta s^2(V_\pm^2 - V_A^2)}{n_0 m V_\pm^2(V_\pm^2 - V_\mp^2)}\right|^{1/3}. \qquad (13)$$

Thus, for the strongest interaction (resonance), i.e., $V_\pm = v_0\cos\theta$, the growth factor for the oscillations is proportional to $(n_0'/n_0)^{1/3}$ rather than $(n_0'/n_0)^{1/2}$.

*tg = tan.

†Hereinafter quantities pertaining to the beam are denoted by primes.

‡The condition (8) is given by Dokuchaev[7] (cf. also reference 9).

5. The solutions in (10) — (13) for the dispersion equation (4) apply for a highly nonisothermal plasma. When $|\omega - k_{\|}v_0| \gg kv_e'$, however, Eq. (4) admits of a solution of the form (10) when $T_e \lesssim T_i$, too. In this case, if the thermal motion of the beam electrons is neglected ($|\omega - k_{\|}v_0| \gg kv_e'$) the following expression is obtained for ϵ:

$$\varepsilon \equiv \left(\frac{M}{m}\right)^{1/2} \varepsilon_0 = \pm \frac{\Omega_e' [n^2 - \varepsilon_{22}^{(0)}]^{1/2}}{[\varepsilon_{33}^{(0)}(n^2 - \varepsilon_{22}^{(0)}) - \varepsilon_{23}^{(0)2}]^{1/2}}, \quad (14)$$

where the $\epsilon_{ij}^{(0)}$ are the components of the plasma dielectric tensor in the absence of the beam for $\omega = k_{\|}v_0$.

Since $\epsilon_{ij}^{(0)}$ has a nonvanishing imaginary part, it follows from (14) that an instability arises for $T_e \lesssim T_i$ in which the characteristic oscillations of the plasma are highly damped. The growth factor Im ϵ is proportional to $(n_0'/n_0)^{1/2}$ in this case. In order of magnitude terms Im $\epsilon \sim$ Re $\epsilon \sim (n_0'M/n_0m)^{1/2}$ for $v_0 \sim v_i \sim s \sim V_A$. This estimate holds, obviously, if the condition $kv_e' \ll |\epsilon| \ll |k_{\|}v_0|$ is satisfied, i.e., when $T_e'/T_e \ll n_0'/n_0 \ll m/M$. If $T_e \gg T_i$, Eq. (14) goes over to Eq. (11).

6. If the inequality $kv_i' \ll |\omega - k_{\|}v_0| \ll kv_e'$ holds, the thermal motion of the electrons in the beam must be taken into account. In this case the solution of (4) is of the form

$$\omega = k_{\|}v_0 + \varepsilon_0, \qquad |\varepsilon_0| \ll |k_{\|}v_0|, \quad (15)$$

where ϵ_0 is given by Eq. (14). As a rough approximation, we have Im $\epsilon_0 \sim$ Re $\epsilon_0 \sim (n_0'/n_0)^{1/2} kv_i$ for $T_e \sim T_i$ and $v_0 \sim s \sim V_A$. In this case Eq. (15) applies if the inequalities $T_i'/T \ll n_0'/n_0 \ll MT_e'/mT_e$ are satisfied.

If $k_{\|}v_0$ is close to the characteristic frequency of the magnetoacoustic waves in a nonisothermal plasma $kV_\pm$, then

$$\omega = kV_\pm + \varepsilon_0, \qquad |k(v_0 \cos\theta - V_\pm)| \ll |\varepsilon_0|, \quad (16)$$

where ϵ_0 is given by Eq. (13). As a rough approximation $\epsilon_0 \sim (n_0'/n_0)^{1/3} kV_\pm$ when $v_0 \sim s \sim V_A$. The expression in Eq. (16) applies if $T_i'/T_i \ll (n_0'/n_0)^{2/3} \ll MT_e'/mT_e$ (it is assumed that $v_0 \sim s \sim V_A$).

7. The preceding expressions are valid if the temperature of the ions in the beam is low enough so that $|\omega - k_{\|}v_0| \gg kv'$. Since ϵ is proportional to the small parameter $(n_0'/n_0)^{1/2}$, the inequality indicated above is not satisfied for sufficiently small values of n_0'/n_0. In this case the solution of (4) is of the form

$$\omega = \omega_\pm + i\gamma_\pm, \quad (17)$$

where $\omega_\pm$ is given by Eq. (9)

$$\gamma_\pm = -\frac{\sqrt{\pi}\omega_\pm \sin^2\theta\, (\xi_e + \xi_e' + \xi_i')}{4\zeta_\pm |\cos^2\theta - \zeta_\pm|(\zeta_+ - \zeta_-)}, \qquad \zeta_\pm = \left(\frac{V_\pm}{s}\right)^2. \quad (18)$$

Here, the quantity ξ_e ($\xi_e > 0$) determines the Cerenkov damping of the magnetoacoustic waves:

$$\xi_e = [(2\cos^2\theta - \zeta_\pm)^2 + \zeta_\pm^2]\, \omega_\pm / \sqrt{2}|k_{\|}|v_e. \quad (19)$$

The quantities ξ_e' and ξ_i' determine the damping (or growth) of the magnetoacoustic waves due to the Cerenkov absorption (or emission) of these waves by the electrons and ions of the beam:

$$\xi_e' = \frac{n_0's^2}{n_0s'^2}\left\{\left[\frac{2s'^2}{s^2}(\cos^2\theta - \zeta_\pm) + \zeta_\pm\right]^2 + \zeta_\pm^2\right\} \frac{\omega_\pm - k_{\|}v_0}{\sqrt{2}|k_{\|}|v_e'} \times \exp\left\{-\left(\frac{\omega_\pm - k_{\|}v_0}{\sqrt{2}k_{\|}v_e'}\right)^2\right\}, \quad (20)$$

$$\xi_i' = \frac{n_0's^2}{n_0v_i'^2}\left\{\left[\frac{2v_i'^2}{s^2}(\cos^2\theta - \zeta_\pm) - \zeta_\pm\right]^2 + \zeta_\pm^2\right\} \frac{\omega_\pm - k_{\|}v_0}{\sqrt{2}|k_{\|}|v_i'} \times \exp\left\{-\left(\frac{\omega_\pm - k_{\|}v_0}{\sqrt{2}k_{\|}v_i'}\right)^2\right\}. \quad (21)$$

Equations (17) — (21) show that a low-density beam with large thermal spreads in the ion and electron velocities will in general not excite magnetoacoustic oscillations in a plasma.

[1] A. I. Akhiezer and Ya. B. Faĭnberg, Doklady Akad. Nauk SSSR **68**, 555 (1949); JETP **21**, 1262 (1951).
[2] D. Bohm and E. P. Gross, Phys. Rev. **75**, 1851 (1949), ibid. **75**, 1864 (1949).
[3] G. V. Gordeev, JETP **27**, 19, 24 (1954).
[4] L. M. Kovrizhnykh, JETP **37**, 1692 (1959), Soviet Phys. JETP **10**, 1198 (1960).
[5] M. S. Kovner, JETP **40**, 527 (1961), Soviet Phys. JETP **13**, 369 (1961).
[6] I. B. Bernstein and R. M. Kulsrud, Phys. Fluids **3**, 937 (1960).
[7] V. P. Dokuchaev, JETP **39**, 413 (1960), Soviet Phys. JETP **12**, 292 (1961).
[8] K. N. Stepanov and A. B. Kitsenko, J. Tech. Phys. (U.S.S.R.) **31**, 167 (1961), Soviet Phys.-Tech. Phys. **6**, 120 (1961).
[9] K. N. Stepanov, Ukr. Phys. Journ. **6**, 678 (1959).

Translated by H. Lashinsky
314