PROPOSED RELATION BETWEEN THE POSITION OF A POLE OF THE SCATTERING AMPLITUDE AND THE VALUE OF THE RESIDUE AT THE POLE

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The question is investigated as to whether dispersion relations can in principle give a definite relation between the position of a pole and the residue at the pole, i.e., between the mass and the interaction constant. It is shown that restrictions on the possible values of the interaction constant [1-3,7] are a consequence of the use of models in the examples treated in the papers in question. It is proved that in the general case, if we start from only the dispersion relations and the unitarity relation for the forward scattering amplitude, there are no restrictions on the possible values of the interaction constant.

SEVERAL recent papers [1,2] have studied the question as to whether dispersion relations in principle give a definite connection between the position of a pole and its residue, i.e., between the mass and charge of particles. A nonrelativistic quantum-mechanical example [1] and a more general field example [2] show that it appears at first glance that there is in principle such a connection, or more exactly, that for a given position of the pole (a given mass) there is an upper bound on the residue (the charge). In a recent paper by Zachariasen [3] an interesting model of a quantum field theory is proposed, which is also free from internal contradictions only when there are definite restrictions on the possible values of the interaction constants (g^2 and λ) of the theory.

We shall show that these restrictions arise only because of the model nature of the examples considered, so that the results of [1-3] are of a model nature in a double sense, and do not follow at all from the dispersion relations in the general case. More exactly we shall show that if we start only from the dispersion relations for the forward scattering amplitude, the unitarity relation ("optical" theorem), and the supplementary condition associated with short-range action, [2] 1) then there are no restrictions on the possible values of the interaction constant g^2.

1. As is well known, [4] the causality condition together with the principle of spectral representation leads to the analytic character of the forward scattering amplitude f(E) in the complex E plane with cuts along the semiaxes $-\infty$, $-\mu$ and μ, ∞, where μ is the mass of the particles being scattered. For the usual case of scattering of mesons by nucleons [4] which we consider here, in the unphysical region $-\mu < E < \mu$, Im f(E) is proportional to a δ function:

$$\operatorname{Im} f(E) = g^2\delta(E - E_0), \tag{1}$$

where E_0 is the pole of the scattering amplitude f(E) which is fixed by the masses of the π meson and nucleon, and the negative quantity $-g^2$, i.e., the residue of f(E) at the pole $E = E_0$, is related in a definite way to the meson-nucleon interaction constant. We note that the presence of a single isolated pole and the absence of a continuous spectrum in the unphysical region is a consequence of the present known "spectrum" of elementary particles.

On the usual assumptions about the degree of the asymptotic behavior of $|f(E)|$ for $|E| \to \infty$ one gets dispersion relations connecting the real part Re f(E) and the imaginary part Im f(E) of the forward scattering amplitude [4]; in particular, the dispersion relation without subtractions2) is of the form

$$\operatorname{Re} f(E) = -\frac{g^2}{E - E_0} + \frac{1}{\pi} \mathrm{P} \int_{\mu}^{\infty} \frac{\operatorname{Im} f(E')}{E' - E}\, dE' + \frac{1}{\pi} \mathrm{P} \int_{-\infty}^{-\mu} \frac{\operatorname{Im} f(E')}{E' - E}\, dE'. \tag{2}$$

1)This is equivalent to the assumption that there are no scattering phase shifts for large values of l.

2)Hereafter when speaking of dispersion relations we shall have in mind just the dispersion relation (2), since the treatment of dispersion relations with subtractions is a natural extension.

The unitarity relation for the forward scattering amplitude f(E) reduces to the optical theorem [4]:

$$\operatorname{Im} f(E) = \frac{k}{4\pi}\sigma(E) = \frac{k}{2}\int_0^\pi \sin\theta\,|f(E,\theta)|^2 d\theta + \frac{k}{4\pi}\sigma_{\text{inel}}(E),$$
$$E \geqslant \mu, \quad k^2 \equiv E^2 - \mu^2, \quad f(E,0) \equiv f(E), \qquad (3)$$

where $f(E, \theta)$ is the scattering amplitude at the angle θ and $\sigma_{\text{inel}}(E)$ is the total cross section for inelastic processes (in particular, many-particle processes). For $E < -\mu$ we can use the condition of crossing symmetry [4]

$$f^*(-E) = f_{\text{an}}(E), \qquad (4)$$

where $f_{\text{an}}(E)$ is the forward scattering amplitude of the antiparticles.

We note a fact of importance for what follows. As Lehmann [5] has shown, $f(E, \theta)$ is an analytic function of θ for all finite $|E| < \infty$, so that the region of analyticity (the Lehmann ellipse) surely includes the physical region $0 \leq \theta \leq \pi$.

The meaning of the dispersion relations is in particular that one cannot prescribe independently the behaviors of Re f(E) and Im f(E) in a certain energy interval, but that the real part Re f(E) for a certain value of E is determined by the behavior of Im f(E) as a function of E over the entire range of variation of the energy. At the same time, and it is very important to emphasize this, the dispersion relations in themselves do not put any important restrictions on the choice of Im f(E), and thus provide a constructive method for fixing the forward scattering amplitude f(E) in terms of a given E_0 (position of the pole), given g^2 (the residue at the pole, which is equivalent to giving the interaction constant), and the value of Im f(E) in the physical region $-\infty < E < -\mu$ and $\mu < E < \infty$, and indeed for arbitrary Im f(E) [naturally Im f(E) must be such that the integrals in Eq. (2) converge].

The analytic property of the forward scattering amplitude, and consequently also the dispersion relations, are, like the unitarity relation, derived on the most general assumptions [4] regarding the form of the interaction, and consequently are valid for arbitrary values of the interaction constant. Thus the unitarity relation (3), and also the dispersion relation (2), are identities with respect to the value of the interaction constant, and consequently with respect to the value of the residue $-g^2$.

2. It is quite obvious that from the two identities with respect to the value of g^2 —the dispersion relation (2) and the unitarity relation (3)—one cannot derive any restrictions on the possible values of g^2, provided these identities are not mutually (internally) contradictory for certain values of g^2. Here mutual contradiction is understood in the sense that for certain values of g^2 one cannot construct any forward scattering amplitude f(E) (other than one that is identically zero) with residue at the pole $E = E_0$ equal to $-g^2$ and with an imaginary part Im f(E) in the physical region which satisfies the unitarity relation.

A possible contradiction between the dispersion relation (2) and the unitarity relation (3) in the general case could be caused by the following circumstance. The unitarity relation (3) connects the imaginary part Im f(E) of the forward elastic scattering amplitude with the absolute values $|f(E, \theta)|$ of the elastic scattering amplitude at arbitrary angles and with the cross sections of inelastic (in particular, of many-particle) processes. Since we are investigating possible contradictions only for the dispersion relation and the unitarity relation for the forward scattering amplitude f(E), the elastic scattering amplitudes $f(E, \theta)$ at nonzero angles ($\theta \neq 0$) and the amplitudes for inelastic processes must be regarded as "external" functions, which in general do not depend on the elastic forward scattering amplitude f(E). But since according to Eq. (3) Im f(E) can, in general, depend on $|f(E,0)|^2 = |f(E)|^2$, and consequently also on Re f(E), the unitarity relation (3) could have as a consequence a definite, local connection between Re f(E) and Im f(E), independent in its explicit form of the value of g^2 and of the dispersion relations.

At the same time the dispersion relations (2) also establish a definite connection between Re f(E) and Im f(E), which does depend explicitly on the value of g^2 and which is in general nonlocal. This could have the result that the dispersion relations (2) and the unitarity relation (3), in serving independently of each other to give connections between Re f(E) and Im f(E), would be inconsistent for certain values of g^2.

It is not hard to show, however, that in the general case the unitarity condition (3) does not lead at all to a local connection between Re f(E) and Im f(E). In fact, as can be seen from Eq. (3), Im f(E) can depend on $|f(E,0)| = |f(E)|$ only through the fact that $|f(E,0)|$ occurs in the integrand. But by a theorem of Lehmann [5] the integrand $\sin\theta\,|f(E,\theta)|^2$ has no singularity in the range of integration, so that the value of the integrand $\sin\theta\,|f(E,\theta)|^2$ at the one point $\theta = 0$ is immaterial to the value of the integral (the more so because of the factor $\sin\theta$), and consequently, generally speaking, Im f(E) does not depend on

$|f(E)|$. Thus in the general case no (local) connection between Re $f(E)$ and Im $f(E)$ arises from the unitary condition. Consequently, in the general case the dispersion relations and the unitarity relation are consistent, and thus cannot impose any restrictions on the possible value of the interaction constant g^2.

3. Of course, in special cases of models Im $f(E)$ can turn out to depend on the value of the integrand $|f(E, \theta)|^2$ at $\theta = 0$, which in turn leads by Eq. (3) to a local connection between Re $f(E)$ and Im $f(E)$, and consequently to a possible incompatibility of the unitarity relation and the dispersion relations for certain values of g^2. Then from the two incompatible conditions involving g^2 (the unitarity condition and the dispersion relation) one can also get limitations on the possible values of the coupling constant g^2. It is quite clear, however, from what has been said that the restrictions on g^2 so obtained relate solely to the model and really only show for what values of g^2 one can treat the special case of the model without contradiction.

According to Eq. (3) Im $f(E)$ can depend on the value of the integrand $|f(E, \theta)|^2$ at $\theta = 0$ only if the values of $|f(E, \theta)|^2$ in some finite range of θ (in particular in the neighborhood $\theta \sim 0$) are determined by the values of $|f(E, \theta)| = |f(E)|$,[3)] i.e., if important restrictions on the dependence of $f(E, \theta)$ on θ are postulated a priori. In particular, it would be sufficient for this to assume that in some arbitrarily small (but finite) region in E and θ

$$|f(E, \theta)| = |f(E, 0)| = |f(E)|. \tag{5}$$

But in virtue of Lehmann's theorem [5] and the fact that the partial amplitudes for elastic scattering are analytic in the energy (for finite energies), the relation (5) would hold for all E and θ, and thus the elastic scattering would reduce to pure s scattering.

It is precisely this special case that has been considered up to now.[1,3,7] In the paper by Castillejo and others [7] this model was used for the scattering of scalar mesons by stationary nuclei, and in the paper by Gribov and others [1] it was used for nonrelativistic scattering by a singular (delta-function) potential. In [7] a restriction on the value of the residue at the pole was obtained for the first time by means of the apparatus of R functions. It is now clear that this restriction is entirely due to the fact that for the model considered all of the elastic scattering reduced to pure s scattering.

Thus if the elastic scattering reduces to pure s scattering, then for the analytic forward scattering amplitude we indeed do get from the dispersion relations and the unitarity relation a restriction on the maximum value of the residue at the pole.[4)] Actually, however, this restriction on the possible value of the residue at the pole only means that it is only for $g^2 < g^2_{max}$ that elastic scattering for which the amplitude at $\theta = 0$ has analytic properties (satisfies the dispersion relations) can be treated without contradiction as pure s scattering.

In this connection we note that in the treatment of the problem of scattering by a potential the partial amplitude for s scattering has the necessary analytic properties (satisfies the dispersion relations) only if the finite interaction radius r_0 is equal to zero,[8] i.e., only for the model of the singular potential,[1] [5)] for which the entire elastic scattering reduces to pure s scattering.

4. As has already been pointed out, a local connection between Re $f(E)$ and Im $f(E)$, and consequently also possible restrictions on the value of g^2, can arise if we make drastic (model) assumptions about the dependence of $f(E, \theta)$ on θ, and in particular if we assume that the elastic scattering is completely described by a finite number of scattering phase shifts. This assumption actually means that in the neighborhood of $\theta = 0$ the amplitude $f(E, \theta)$ is a sufficiently smooth function (since it does not contain high phase shifts), and this in turn has the consequence that in the neighborhood of $\theta = 0$ the quantity $|f(E, \theta)|$ is to a large extent determined by its value at $\theta = 0$. But then, according to the unitarity relation (3), we arrive at a local connection between Re $f(E)$ and Im $f(E)$, and consequently, at possible restrictions on the maximum value of g^2, which are generalizations of those known for the case in which the finite set of phase shifts reduces to the s phase shift alone.[1,7]

It is very important to emphasize that in virtue of the often-used theorem of Lehmann [5] and of the analytic properties of the partial scattering

[3)]We note that if in some range of θ, $|f(E, \theta)|^2$ should depend explicitly on $|f(E)|$, then by the method of [6] one could also obtain restrictions on the asymptotic behavior of $|f(E)|$ for $|E| \to \infty$.

[4)]This restriction can also be obtained on the basis of the fact that when the elastic scattering reduces to pure s scattering there is an upper limit on $|f(E)|$ which does not depend on the value of g^2.[6]

[5)]From this same fact it follows at once that for scattering by a potential with a finite radius the main conclusions of the paper by Gribov and others [1] are incorrect. In [1] this conclusion is reached by indirect arguments.

amplitudes, the elastic scattering amplitude cannot be described exactly by a finite number of phase shifts in some restricted range of energies and by a different (finite or infinite) number of phase shifts in another range of energies; it must be described by the same number of phase shifts for all finite values of the energy. This assertion does not contradict the well known procedure (based, for example, on the uncertainty relation) of separating the phase shifts into the large (important) a_l with $l < kr_0$ and the small (unimportant) a_l with $l > kr_0$, in the case of a short-range interaction with a finite radius r_0.[6] We further emphasize that the separation into large and small phase shifts does not depend on the "strength" g^2 of the interaction (the coupling constant), because it depends only on the energy E and the interaction radius r_0, which by its very definition does not depend on the interaction constant.[7] It is not hard to convince oneself of this for the case of scattering by a potential, and also for scattering in quantum field theory. In fact, in the pole approximation, we have for the phase shifts a_l [10]

$$a_l = i\mu f^{(1)}(k^2)\, k^{-1} Q_l\,(1 + \mu^2/2k^2), \tag{6}$$

where Q_l are Legendre functions of the second kind, $\mu \sim 1/r_0$ is the mass of the mesons transmitting the interaction, and $f^{(1)}(k^2)$ is a dimensionless function proportional to g^2. From Eq. (6) and the asymptotic properties of the Q_l there follows, independently of the value of g^2, a separation of the phase shifts into the large (important) ones with $l \ll k/\mu = kr_0 \ll 1$,

$$a_l = \frac{i\mu f^{(1)}(k^2)}{k}\,\frac{l!\cdot 2^{l+1}}{(2l+1)!!}\left(\frac{k}{\mu}\right)^{2l+2} \tag{7}$$

and the "small" (unimportant) ones with $l \gg k/\mu = kr_0 \gg 1$,

$$a_l = i\mu f^{(1)}(k^2)\, k^{-1}\sqrt{\pi/2\mu l}\, e^{-\mu l/k}. \tag{8}$$

Thus the additional condition associated with the short-range nature of the force—the separation of the phase shifts into large (important) and small (unimportant) phase shifts—is an identity with respect to g^2, just as are the dispersion relations and the unitarity relation.

[6]It also follows from the uncertainty relation that the number of important phase shifts (in any case of nonvanishing phase shifts) for an interaction with a finite radius ($0 < r_0 < \infty$) increases with increasing energy. On the basis of Lehmann's theorem[5] it follows from this that an interaction with a finite radius must be described by an infinite number of phase shifts.

[7]The radius r_0 of the interaction is determined by the mass of the particles that transmit the interaction, and its "strength" (g^2) by their number.

In a paper by Ansel'm and others [2] an attempt is made to obtain restrictions on the maximum value of the interaction constant by starting from the dispersion relations for the forward scattering amplitude (in particular for scattering of π mesons by nucleons), the unitarity relation ("optical" theorem), and the short-range nature of the interaction. From what has been said above it is clear that if we start from these three identities in g^2, which do not internally contradict each other, we cannot obtain any restrictions on the possible values of g^2; consequently, the authors of [2] have essentially made assumptions that are particularly dependent on a model.

To derive restrictions on the maximum value of the interaction constant by the method of [2] it suffices to get a nontrivial ($C \neq 0$) lower bound on the quantity $\mathrm{Im}\, f(E)/|f(E)|^2$ in some arbitrary (finite) energy range,

$$\mathrm{Im} f(E) \,/\, |f(E)|^2 \geqslant C > 0, \tag{9}$$

which does not depend on g^2. In [2] an estimate of the type (9) is obtained in the following way. It is assumed that in a given energy range $E \in [E_1, E_2]$ the forward scattering amplitude $f(E)$ is completely described by a finite number l_{max} of phase shifts a_l; this number is given by

$$l_{max} = k_2 r_0, \tag{10}$$

where k_2 corresponds to the energy E_2 and r_0 is the radius of the interaction. Then

$$\mathrm{Im}\, f(E)/|f(E)|^2 \equiv F(a_l;\, l_{max}, k)$$

$$= \mathrm{Im}\left(\frac{1}{2ik}\sum_{l=0}^{l_{max}} a_l\,(2l+1)\right)\Big/\left|\frac{1}{2ik}\sum_{l=0}^{l_{max}} a_l\,(2l+1)\right|^2 \tag{11}$$

and by regarding $F(a_l)$ as a function of the variables a_l ($l = 0, 1, 2, \ldots, l_{max}$) we easily get a bound for F [2]:

$$\mathrm{Im}\, f(E)/|f(E)|^2 = F \geqslant C(k_2;\, l_{max}, k), \tag{12}$$

where C is the absolute minimum of $F(a_l)$ with respect to all possible a_l. Here it is essential that, since by hypothesis [2] l_{max} is determined by k_2 and r_0 only, and does not depend on g^2 [cf. Eq. (10)], the bound (12) contains a constant C that does not depend on g^2. Without writing out the explicit form of C, we merely note that when $l_{max} \to \infty$, $C \to 0$, and the bound (12) becomes a trivial one. Thus we again verify that to obtain a relation (12) with $C \neq 0$ it is necessary to assume that the forward scattering amplitude is completely (exactly) described by a finite number l_{max} of phase shifts a_l, which does not depend on g^2.

It is not hard to verify that the method of the paper of Ansel'm and others,[2] which we have just explained, cannot be applied in the general case. In fact, from the separation of the phase shifts into "large" and "small" (important and unimportant) phase shifts we can conclude nothing except that for $l < l_{max}$, where l_{max} is given by Eq. (10), the phase shifts are "large," and for $l > l_{max}$ they are "small." It by no means follows, however, (cf. footnote [6]) that, as is required for the method of [2] to apply, l_{max} gives a maximum (finite) number of phase shifts a_l which completely (exactly) describe the forward scattering amplitude, independent of the value of g^2. The only case in which this is true is the highly special case of a model in which the forward scattering amplitude f(E) is completely described by (the same) finite number of phase shifts a_l for all energies E ($\mu < E < \infty$). Only for this model case [with l_{max} fixed a priori, and not at all by Eq. (10)] is the bound (12) valid, and consequently only in this model case does one get on the basis of Eq. (12) a restriction on the possible values of g^2.

The meaning of this restriction is of course that only for $g^2 < g^2_{max}$ can we treat without contradiction the model case in which the forward scattering amplitude, while possessing analytic properties (i.e., satisfying dispersion relations) is completely described for all energies by a fixed finite number of phase shifts. In actual problems we must suppose that this model picture cannot apply. In fact, as can be seen, for example, from the formulas for phase shifts with sufficiently large l ($l \gg kr_0 \gg 1$)—in the pole approximation[10] of Eq. (7) or from a quasi-classical treatment of scattering by a potential[9]—the "unimportant" phase shifts a_l increase with increase of the interaction constant g^2. Therefore we cannot proceed independently of g^2 to neglect phase shifts a_l for large $l > kr_0$ (see also footnote [6]).

The only bound on $\mathrm{Im}\, f(E)/|f(E)|^2$ that is valid in this case is the trivial one (C = 0). In fact, from obvious physical considerations (positiveness of cross sections)

$$\frac{\mathrm{Im}\, f(E)}{|f(E)|^2} \geqslant \frac{\mathrm{Im}\left(\frac{1}{2ik}\sum_{l=0}^{L} a_l(2l+1)\right)}{\{[\mathrm{Re}\, f(E)]^2 + [\mathrm{Im}\, f(E)]^2\}}. \tag{13}$$

But it by no means follows from this that in Re f(E) we need keep only the finite number of phase shifts a_l ($l = 0, 1, 2, \ldots, L$) that is taken for the calculation of Im f(E) in the numerator.[8] For the estimate of $\mathrm{Im}\, f(E)/|f(E)|^2$ not to be in contradiction with the dispersion relations, even when it is assumed that only a finite number L of phase shifts are "important," one must express Re f(E) in the denominator of Eq. (13) in terms of Im f(E) by means of the dispersion relation (2). But then

$$\frac{\mathrm{Im}\, f(E)}{|f(E)|^2} \geqslant \mathrm{Im}\left(\frac{1}{2ik}\sum_{l=0}^{L} a_l(2l+1)\right) \Big/ \left\{\left[\mathrm{Im}\left(\frac{1}{2ik}\sum_{l=0}^{L} a_l(2l+1)\right)\right]^2 + \left[-\frac{g^2}{E-E_0} + \frac{1}{\pi} \mathrm{P}\int \mathrm{Im}\left(\frac{1}{2ik'}\sum_{l=0}^{L} a_l(k')(2l+1)\right) \times \frac{dE'}{E'-E}\right]^2\right\} \equiv \Phi(a_l, L, k, g^2, E_0), \tag{14}$$

where Φ already depends explicitly on g^2.

To obtain a lower bound on Φ we can proceed in analogy with [2], and find the absolute minimum of Φ with respect to the a_l for fixed L, k, g^2, E. Consequently, the bound on Φ also depends on g^2. It is not hard to see from Eq. (14) that the only bound on $\mathrm{Im}\, f(E)/|f(E)|^2$ that is independent of g^2 is the trivial one (C = 0).

5. Thus it has been shown that the dispersion relations for the forward scattering amplitude, the unitarity relation ("optical" theorem), and the condition of short-range forces [separation of the phase shifts into "large" (important) and "small" (unimportant) phase shifts] do not depend on the value of the interaction constant and are internally noncontradictory, and that consequently one cannot, using only these conditions, get any restrictions on the possible values of the interaction constant g^2.[9] A similar conclusion is also found when one considers also the dispersion relations (spectral representations) and unitarity relations for the (elastic) scattering amplitude at nonzero angles. One here makes essential use of the property of crossing symmetry of the theory (the presence of a cut on the left) (cf. investigation of the Lee model in a paper by Ter-Martirosyan[12]). It is true that it may be that if we examine the spectral representations and unitarity conditions for all amplitudes (including many-particle amplitudes), they will turn out to be compatible only for quite definite values of the interaction constant and the masses

[8]This assertion is not in contradiction with the fact that, for example in a paper by Pomeranchuk,[11] in determining restrictions on the asymptotic behavior of |f(E)| the same finite number of phase shifts has been taken for both Re f(E) and Im f(E), because in[11] g^2 is regarded as fixed.

[9]Owing to this there is no meaning in the hypothesis[2,13] that the strong interaction is the strongest of all possible interactions.

of the particles, but at present it would be unrealistic to discuss this possibility in the framework of the existing theory.

Thus the answer to the question stated at the beginning of this paper must be negative in the general case. Possible restrictions on the maximum value of g^2 can be obtained only for cases of very special models, and actually indicate for which values of g^2 these model cases can be treated without contradiction. The interaction constants, like the masses of the particles, are parameters in the present theory.

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Note added in proof (September 4, 1961). The results obtained in the present paper are also fully applicable to papers [14,15] in which the interaction constant is entirely determined in terms of the masses of the interacting particles.

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213