GRAVITATIONAL RADIATION BY A RELATIVISTIC PARTICLE

V. I. PUSTOVOĬT and M. E. GERTSENSHTEĬN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor June 16, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) 42, 163-170 (January, 1962)

Gravitational radiation from a charged relativistic particle moving in a magnetic field is calculated. The gravitational waves are due not only to the mass tensor of the particle itself, but also to the electromagnetic stresses caused by the charge, the contributions from which to the radiation are of the same order as that of the mass. The small additions to the metric tensor correspond to two processes of gravitational wave formation: to the usual type of charge and mass emission, and to resonance emission of gravitational waves by the electromagnetic field in the presence of a constant external magnetic field. The latter effect is considered by one of the authors elsewhere.[9] It is shown that in the ultrarelativistic case the energy dependence of the intensity of gravitational wave radiation is the same as that of an electromagnetic field.

IN view of the discussions of the existence of gravitational waves,[1,2,3], it is of interest to examine a gravitational analog of the problem of the radiating electron, which can be solved rigorously.

We consider a charged relativistic particle moving on some curved trajectory, which we assume to be specified. The trajectory of the particle is completely determined by the electromagnetic interaction, since the gravitational interaction is much smaller. We determine in the present paper the energy of the gravitational radiation, for which purpose we calculate the gravitational field in the "wave zone" and use the linearized Einstein equations.

1. GRAVITATIONAL FIELD IN THE "WAVE ZONE"

In the case of a weak field the metric tensor g^{ik} differs little from the Galilean tensor δ^{ik}, that is, $g^{ik} = \delta^{ik} + \psi^{ik} - \frac{1}{2}\delta^{ik}\psi_s^s$, where $|\psi| \ll 1$. A weak gravitational field is described by wave equations

$$\Box \psi^{ik} = -16\pi\varkappa c^{-4} (T_p^{ik} + T_f^{ik}), \qquad (1)$$

$$\partial\psi^{ik}/\partial x^k = 0, \qquad (1a)$$

$$\partial (T_p^{ik} + T_f^{ik})/\partial x^k = 0. \qquad (1b)$$

Here κ — gravitational constant; the Latin indices i, k, l, and s take on the values 1, 2, 3, 0; the Greek indices α and β take on the values 1, 2, and 3; $x^0 = x_0 = ict$.

The right-hand half of (1) contains the sum of the energy-momentum tensors of the particle and of the field, where

$$T_p^{ik} = \Sigma Mc^2 u^i u^k \sqrt{1 - \beta^2}\, \delta (\mathbf{r} - \mathbf{a}) \qquad (2)$$

is the energy-momentum tensor of the particles moving along the curves $\mathbf{a}(t)$ with 4-velocities u^k (summation is over all particles, $\beta \equiv v/c$), and

$$T_f^{ik} = (1/4\pi) (F^{il} F^{kl} - \tfrac{1}{4}\delta^{ik} F_{lm}^2) \qquad (3)$$

is the energy-momentum tensor of the electromagnetic field, which has two components, the particle field F_p^{ik} and the external field F_0^{ik}, i.e.,

$$F^{ik} = F_0^{ik} + F_p^{ik}. \qquad (4)$$

We shall consider from now on the motion of one mass M with charge e in a magnetic field (the analog of the problem of the radiating electron, see [5,6]).

We choose the origin at the center of inertia, which coincides with the center of rotation; then the energy-momentum tensor of the particle (2) has the form

$$T_p^{ik} = Mc^2 (1 - \beta^2)^{-1/2} q^{ik} (t)\, \delta (\mathbf{r} - \mathbf{a}(t)), \qquad (5)$$

where

$$q^{ik} (t) \equiv (1 - \beta^2)\, u^i (t)\, u^k (t). \qquad (6)$$

We now consider the electromagnetic field tensor T_f^{ik}, which is quadratic in the field and, consequently,

$$T_f^{ik} \sim F_0 \cdot F_0 + 2F_0 \cdot F_p + F_p \cdot F_p. \qquad (7)$$

The first term in (7) can be immediately discarded, since it is independent of the time and therefore does not contribute to the radiation. The last term describes the electromagnetic self energy of the particle, which is infinite in classical electrodynamics, but the infinity can be excluded by mass renormalization and we must take M to mean the experimental value of the mass. Thus, we need consider only the second term of (7), which, as can be readily seen,[4] has the form

$$T_p^{ik} = \frac{H^{(0)}}{4\pi}\begin{Vmatrix} H_3 & 0 & -H_1 & -iE_2 \\ 0 & H_3 & -H_2 & iE_1 \\ -H_1 & -H_2 & -H_3 & 0 \\ -iE_2 & iE_1 & 0 & -H_3 \end{Vmatrix}, \tag{8}$$

where $H^{(0)}$ is directed along the z axis, and H_1, H_2, H_3, E_1, and E_2 are the projections of the magnetic and electric fields on the x, y, and z axes respectively.[4]

It is easy to verify that the energy-momentum tensor of the particle and the retained terms of the energy-momentum tensor of the field (8) satisfy relation (1b).

In order to find the fields (8), we must solve the electrodynamics equations

$$\Box \mathbf{A}(t, \mathbf{r}) = -\frac{4\pi}{c}\mathbf{j}(t, \mathbf{r}), \tag{9}$$

where $\mathbf{A}(t, \mathbf{r})$ is the vector potential and $\mathbf{j}(t, \mathbf{r}) = e\mathbf{v}(t)\delta(\mathbf{r}-\mathbf{a}(t))$ is the current. We shall solve (2) and (9) by the Fourier method, expanding all the functions in accordance with the general formula

$$f(t, \mathbf{r}) = \sum_{m=-\infty}^{+\infty} e^{-im\omega_0 t} f(\omega, \mathbf{r}) = \sum_{m=-\infty}^{+\infty} e^{-i\omega t}\frac{1}{(2\pi)^3}\int f_{(m)}(\omega, k)\, e^{-i\mathbf{kr}}\, d^3\mathbf{k}, \tag{10}$$

where $\mathbf{k}$ is the wave vector, and we do not assume that $k = \omega n/c$ ($\mathbf{n} \equiv \mathbf{r}/r$) ($\omega_0 = \omega/m$ — angular velocity of rotation).

The terms of formula (10) with index $m = \pm 1$ describe dipole radiation, which does not exist for gravitational waves.[4] The vanishing of the term with $m = \pm 1$ is automatic, by virtue of the law of motion of the center of gravity. If there are several moving particles and only one is relativistic, then we can show that the dipole term vanishes, and the terms with $m \geq 2$ are hardly changed by an account of the motion of the nonrelativistic particles.

We note that the problem can be solved directly, for example, by using retarded potentials. However, if we consider the motion of one particle, the dipole term cannot be eliminated in the direct calculation. Direct calculation of the radiation of the field T_f^{ik} with the aid of retarded potentials is much more complicated than calculation with the aid of spectral expansion, because it is necessary to know T_f^{ik} and the electromagnetic field in the near zone. However, if we solve the problem by the Fourier method, the intermediate results obtained become physically more understandable, and this is precisely why we choose this method.

Substituting the expansion (10) for $\psi^{ik}(t, \mathbf{r})$ and the field sources in (1) we get

$$\psi_{(m)}^{ik}(\omega, \mathbf{k}) = \frac{16\pi\varkappa}{c^4}\frac{T_p^{ik}(\omega, \mathbf{k}) + T_f^{ik}(\omega, \mathbf{k})}{k^2 - \omega^2/c^2}. \tag{11}$$

If we substitute the value of $\psi_{(m)}^{ik}(\omega, \mathbf{k})$ in the Fourier inversion formula we obtain

$$\psi_{(m)}^{ik}(\omega, \mathbf{r}) = \frac{2\varkappa}{\pi^2 c^4}\int \frac{T_p^{ik}(\omega, \mathbf{k}) + T_f^{ik}(\omega, \mathbf{k})}{k^2 - \omega^2/c^2} e^{-i\mathbf{kr}} d^3\mathbf{k}. \tag{12}$$

The last integral can be transformed so as to eliminate integration over the components of the vector $\mathbf{k}$ which are perpendicular to $\mathbf{r}$. This can be done with the aid of the stationary-phase method (see [7]), i.e., we can show that the integral (12) assumes as $\mathbf{r} \to \infty$ the value ($\mathbf{R} = \mathbf{r} - \mathbf{a}$)

$$\psi_{(m)}^{ik}(\omega, \mathbf{k}) = -\frac{4\varkappa i}{\pi c^4 R}\int_{-\infty}^{+\infty} \frac{T_p^{ik}(\omega, k\mathbf{n}) + T_f^{ik}(\omega, k\mathbf{n})}{k^2 - \omega^2/c^2} e^{-ikR} k\, dk, \tag{13}$$

where the integration is only over the absolute value of the vector $\mathbf{k}$. We note that (13) is valid also for the case of multiple poles, when the functions $T^{ik}(\omega, k\mathbf{n})$ themselves have poles at the point $k = \omega/c$.

Let θ be the polar angle of the vector $\mathbf{R}$ to the z axis, and let φ be the azimuthal angle measured in the xy plane; it then follows from (5) and (10) that

$$T_p^{ik}(\omega, k\mathbf{n}) = \frac{Mc^2}{\sqrt{1-\beta^2}} e^{i(\varphi - \pi/2)m} I_p^{ik}(\omega, k\mathbf{n}), \tag{14}$$

where the nonvanishing components of $I_p^{ik}(\omega, k\mathbf{n})$ are [8]

$$\begin{aligned} I_p^{11}(\omega, k\mathbf{n}) &= \beta^2\Big(J_m(\xi)\cos^2\varphi - i\frac{d}{d\xi}\Big(\frac{m}{\xi}J_m(\xi)\Big)\sin 2\varphi + J_m''(\xi)\cos 2\varphi\Big), \\ I_p^{21}(\omega, k\mathbf{n}) &= \beta^2\Big(J_m(\xi)\cos\varphi\sin\varphi + i\frac{d}{d\xi}\Big(\frac{m}{\xi}J_m(\xi)\Big)\cos 2\varphi + J_m''(\xi)\sin 2\varphi\Big), \\ I_p^{01}(\omega, k\mathbf{n}) &= \beta\Big(J_m'(\xi)\sin\varphi + \frac{im}{\xi}J_m(\xi)\cos\varphi\Big), \\ I_p^{00}(\omega, k\mathbf{n}) &= -J_m(\xi), \end{aligned} \tag{15}$$

and the components I_p^{22} and I_p^{02} are obtained from

I_p^{11} and I_p^{01} by making the substitutions $\cos\varphi \to \sin\varphi$ and $\sin\varphi \to -\cos\varphi$. Here $J_m(\xi)$ is the Bessel function and $\xi \equiv ka\sin\theta$. It is easy to obtain an expression for $T_f^{ik}(\omega, kn)$ in the same manner.

From (9) we can determine the Fourier component of the vector potential, and thus determine the intensities of the electric and magnetic fields:

$$\mathbf{E}(\omega, k\mathbf{n}) = \frac{4\pi i}{c(k^2-\omega^2/c^2)}\left\{\frac{i\omega}{c}\mathbf{j}(\omega, k\mathbf{n}) - \frac{ck^2}{\omega}\mathbf{n}(\mathbf{n}, \mathbf{j}(\omega, k\mathbf{n}))\right\},$$

$$\mathbf{H}(\omega, k\mathbf{n}) = \frac{4\pi i}{c(k^2-\omega^2/c^2)}k[\mathbf{n}, \mathbf{j}(\omega, k\mathbf{n})] \qquad (16)^*$$

where the Fourier components of the current are

$$j^1(\omega, k\mathbf{n}) = iev\exp\{i(\varphi-\pi/2)m\}I_p^{01}(\omega, k\mathbf{n}),$$

$$j^2(\omega, k\mathbf{n}) = iev\exp\{i(\varphi-\pi/2)m\}I_p^{02}(\omega, k\mathbf{n}). \qquad (17)$$

We now determine $\psi_{(m)}^{ik}(\omega, kn)$. For this purpose we substitute (14) in (13), transform the integral, and obtain directly by residues†

$$\psi_{(m)}^{ik}(\omega, k\mathbf{n}) = \frac{4\varkappa M}{c^2R\sqrt{1-\beta^2}}\exp\left\{i\left(\varphi-\frac{\pi}{2}-\frac{\omega_0R}{c}\right)m\right\}I^{ik}(\omega, k\mathbf{n}), \qquad (18)$$

where

$$I^{ik}(\omega, k\mathbf{n}) = I_p^{ik}(\omega, k\mathbf{n}) + I_f^{ik}(\omega, k\mathbf{n}) \qquad (19)$$

and the $I_f^{ik}(\omega, kn)$ are in turn equal to

$$I_f^{11} = I_f^{22} = -I_f^{33} = -I_f^{00} = \frac{\beta}{2}\cos\theta\left(\frac{1}{m}J_m(\xi) + \frac{\xi}{m}J_m''(\xi) - \frac{i\omega_0R}{c}J_m'(\xi)\right),$$

$$I_f^{31} = \frac{\beta}{2}\cos\theta\Big(-\frac{1}{m}J_m'(\xi)\cos\varphi - \frac{\xi}{m}J_m''(\xi)\cos\varphi + iJ_m''(\xi)\sin\varphi + \frac{i\omega_0R}{c}J_m'(\xi)\cos\varphi + \frac{R\sin\varphi}{a\cos\theta}J_m(\xi)\Big),$$

$$I_f^{01} = \frac{i\beta}{2}\Big(-\frac{\xi}{m}J_m''(\xi)\cos\varphi + i\cos^2\theta J_m'(\xi)\sin\varphi - \frac{ic\sin\varphi(1+\sin^2\theta)}{a\omega\sin\theta}J_m(\xi) + \frac{i\omega_0R}{c}J_m'(\xi)\cos\varphi + \frac{R\sin\varphi\cos^2\theta}{a\sin\theta}J_m(\xi)\Big), \qquad (20)$$

and the components I_f^{32} and I_f^{02} being obtained from I_f^{31} and I_f^{01} by means of the same substitution as used above. We employed here the relation $H^{(0)} = Mc^2\omega_0^2/e\sqrt{1-\beta^2}$, and in this case $\xi = m\beta\sin\theta$ throughout.

*$[\mathbf{n}, \mathbf{j}] = \mathbf{n}\times\mathbf{j}$.

†The residue is taken only at the pole $k = \omega/c$, which corresponds to the outgoing wave.

We can verify by direct calculation that the field components (18) satisfy condition (1a), i.e.,

$$k_l\psi_{(m)}^{ls}(\omega, k\mathbf{n}) = 0 \qquad (k_0 = i\omega/c).$$

The integral of $T_f^{ik}(\omega, kn)$ has a double pole, and in taking the residue the integrand (13) must therefore be differentiated with respect to k, which gives rise to a factor that increases with increasing R. We shall show below that this corresponds to a secondary effect, namely to resonant excitation of gravitational waves by the electromagnetic field.[9]

Expressions (15), (18), (19), and (20) determine the gravitational field of a relativistically moving mass M with charge e at a given distance R in the "wave zone."

2. ENERGY FLUX IN THE "WAVE ZONE"

In the "wave zone" the field represents a plane traveling wave, the energy carried by which per unit time is (see [10])

$$\frac{d\mathfrak{E}}{dt} = c\oint\langle t_\alpha^0 n^\alpha\rangle r^2\sin\theta\, d\theta\, d\varphi, \qquad (21)$$

$$t_\alpha^0 = -\frac{c^4}{32\pi\varkappa}\left(\frac{\partial\psi^{ik}}{\partial x^0}\frac{\partial\psi_{ik}}{\partial x^\alpha} - \frac{1}{2}\frac{\partial\psi_s^s}{\partial x^0}\frac{\partial\psi_l^l}{\partial x^\alpha}\right) \qquad (22)$$

— the energy momentum tensor of the gravitational field (the brackets in (21) denote averaging over the time of revolution of the particle).

Equation (22) contains the longitudinal field components ψ^{00} and $\psi^{0\alpha}$, which, as is well known, do not correspond to radiation. The longitudinal components of the field can be eliminated by coordinate and field transformations (see Fock's book[11]). There is no need for such a transformation, however, since the final expression for the energy flux is the same.

Expressions (15), (18), (19), and (20) determine the components $\psi_m^{ik}(t, \mathbf{r})$ of the Fourier expansion in the time. Therefore, if we substitute (10) in (21) and (22) and integrate over the time of revolution of the particle, we obtain

$$-\frac{d\mathfrak{E}}{dt} = \frac{c^3\omega_0^2}{16\pi\varkappa}\oint r^2\sin\theta d\theta d\varphi\sum_{m=2}^{+\infty}m^2\{(\bar{\psi}_{(m)}^{ik})^2 - (\hat{\psi}_{(m)}^{ik})^2 - \tfrac{1}{2}(\bar{\psi}_{(m)}^{ss})^2 + \tfrac{1}{2}(\hat{\psi}_{(m)}^{ss})^2\}. \qquad (23)$$

where

$$\bar{\psi}_{(m)}^{ik} \equiv \tfrac{1}{2}(\psi_{(m)}^{ik} + \psi_{(-m)}^{ik}), \qquad \hat{\psi}_{(m)}^{ik} \equiv \tfrac{1}{2}(\psi_{(m)}^{ik} - \psi_{(-m)}^{ik}). \qquad (24)$$

Let us substitute (18) in (23); then

$$-\frac{d\mathfrak{E}}{dt} = \frac{2\varkappa M^2\beta^2c}{a^2(1-\beta^2)}\int\sin\theta d\theta\sum_{m=2}^{+\infty}m^2\Phi^{(m)}(\theta), \qquad (25)$$

with

$$\Phi^{(m)}(\theta) \equiv \Phi_p^{(m)}(\theta) + 2\Phi_{fp}^{(m)}(\theta) + \Phi_f^{(m)}(\theta)$$

and furthermore

$$\Phi_p^{(m)}(\theta) = \left\{\frac{(1-\xi_1^2)(\beta^2-\xi_1^2)}{\xi_1^4} + \frac{\beta^2}{\xi^2} + \frac{(1-\beta^2)^2}{4}\right\} J_m^2(\xi)$$
$$+ \frac{(\xi_1^2-4)}{m\xi_1^3} J_m'(\xi) J_m(\xi)$$
$$+ \left(\frac{\beta^2-\xi_1^2}{\xi_1^2} - \frac{\beta^2}{\xi^2}\right) J_m'^2(\xi) \qquad \left(\xi_1 \equiv \frac{\xi}{m}\right), \tag{26}$$

$$\Phi_f^{(m)}(\theta) = \frac{\beta^2}{2}(\sin^2\theta\cos^2\theta J_m'^2(\xi) + \beta^2\sin^4\theta J_m''^2(\xi))$$
$$+ \frac{R^2\beta^2}{2}\left(\frac{\cos^2\theta}{a^2} J_m^2(\xi) + \frac{\omega_0^2}{c^2}\sin^2\theta J_m'^2(\xi)\right), \tag{27}$$

$$\Phi_{fp}^{(m)}(\theta) = -\frac{1}{2}\left(\beta(1+\beta)\sin\theta + \frac{\beta(1+\sin^2\theta)}{\sin\theta}\right)\frac{1}{m} J_m'(\xi) J_m(\xi)$$
$$+ \beta^2\cos^2\theta J_m'^2(\xi) - \frac{1}{2}\beta^2(1-\beta^2)\sin^2\theta J_m''(\xi) J_m(\xi). \tag{28}$$

In (27) and (28) we left out the terms that are small in the ultrarelativistic case. It is seen from (27) and (28) that there are no terms proportional to the first power of R.

We now use the relations (see Watson's book[8])

$$\sum_{m=1}^{+\infty} m^2 J_m^2(mx) = \frac{x^2(4+x^2)}{16(1-x^2)^{7/2}}, \qquad \sum_{m=1}^{+\infty} m^2 J_m'^2(mx) = \frac{4+3x^2}{16(1-x^2)^{5/2}},$$
$$\sum_{m=1}^{+\infty} m J_m'(mx) J_m(mx) = \frac{x}{4(1-x^2)^{5/2}} \quad (|x| \leqslant 1). \tag{29}$$

The sum is taken over m ≥ 2 in (25) and over m ≥ 1 in (29). In the ultrarelativistic case, however, the term with m = 1 is small compared with the entire sum, and there is no need for eliminating it, since the maximum of the spectral density of the radiation occurs in the region of frequencies for which

$$m \sim (\mathfrak{E}/\mathfrak{E}_0)^{3/2}.$$

With the aid of (29) and the equation for the Bessel functions it is easy to obtain for the ultrarelativistic case

$$\int_0^\pi \sin\theta d\theta \sum_{m=1}^{\infty} m^2 \Phi_p^{(m)}(\theta) = \frac{11}{6}\left(\frac{\mathfrak{E}}{\mathfrak{E}_0}\right)^2, \tag{30}$$

$$\int_0^\pi \sin\theta d\theta \sum_{m=1}^{\infty} m^2 \Phi_f^{(m)}(\theta) = \frac{7}{16}\left(\frac{\mathfrak{E}}{\mathfrak{E}_0}\right)^2 + \frac{1}{3}\frac{R^2}{a^2}\left(\frac{\mathfrak{E}}{\mathfrak{E}_0}\right)^3, \tag{31}$$

$$\int_0^\pi \sin\theta d\theta \sum_{m=1}^{\infty} m^2 \Phi_{fp}^{(m)}(\theta) = \frac{1}{12}\left(\frac{\mathfrak{E}}{\mathfrak{E}_0}\right)^2, \tag{32}$$

where $\mathfrak{E}_0 = Mc^2$ is the self energy of the particle.

Substituting (30) — (32) (without the term proportional to R^2) in (25) we obtain a formula for the total radiation of gravitational waves from the system:

$$-\frac{d\mathfrak{E}}{dt} = \frac{39}{8}\frac{\varkappa M^2 c}{a^2}\left(\frac{\mathfrak{E}}{\mathfrak{E}_0}\right)^4, \tag{33}$$

which is equivalent to the gravitational force of the radiation friction

$$-\frac{d\mathfrak{E}}{dx} = \frac{39}{8}\frac{\varkappa M^2}{a^2}\left(\frac{\mathfrak{E}}{\mathfrak{E}_0}\right)^4. \tag{34}$$

In the ultrarelativistic case, as is well known, radiation has a local character and the particle radiates predominantly forward. By virtue of the local character of the radiation, the value of a in (33) and (34) is equal to the instantaneous value of the radius of curvature.

The forward directivity of the radiation enables us to interpret illustratively, although not rigorously, the dependence of the radiation intensity on the particle energy for different fields. For the radiation of a scalar field with a three-dimensional source density independent of the velocity, the sum of the Bessel functions in the formal summation over the harmonics yields a factor proportional to $(\mathfrak{E}/\mathfrak{E}_0)^6$, i.e., the radiation of such a scalar field increases with energy as $(\mathfrak{E}/\mathfrak{E}_0)^6$.

The transverse electromagnetic field, the source of which is the current, is likewise independent of the velocity in the ultrarelativistic case, and therefore the summation over their harmonics leads to a factor $(\mathfrak{E}/\mathfrak{E}_0)^4$. The reduction in the degree is due to the transverse character of the electromagnetic field, which makes forward radiation difficult. The amplitude of the wave is proportional to the matrix element $\langle \mathbf{j}, \mathbf{A} \rangle$ and for forward radiation, when **k** is parallel to **j**, the matrix element vanishes by virtue of the transverse nature of the field.

The gravitational waves are transverse-transverse,[10] and by virtue of the "double" transversality the summation over the harmonics yields only $(\mathfrak{E}/\mathfrak{E}_0)^2$, but since the source of the gravitational field itself, namely the energy-momentum tensor of the matter, is proportional to $\mathfrak{E}/\mathfrak{E}_0$, the radiation energy turns therefore out to be proportional to $(\mathfrak{E}/\mathfrak{E}_0)^4$, i.e., the dependence on the energy is the same as for the electromagnetic field.

The ratio of the gravitational losses to the electric losses is independent of the energy in the ultrarelativistic case:

$$\left(\frac{d\mathfrak{E}}{dt}\right)_{\text{grav}} \Big/ \left(\frac{d\mathfrak{E}}{dt}\right)_{\text{el}} = \frac{13}{4}\frac{\varkappa M^2}{e^2}. \tag{35}$$

Formulas (34) and (35) differ from the results of Havas[12] only in a coefficient. Our coefficient is approximately 30 percent greater, because the problem considered by Havas,[12] that of radiational gravitational friction of an isolated mass with specified motion, is incorrectly formulated. A weak gravitational field satisfies the wave equation (1) only subject to the additional condition (1a), from which follows the condition (1b) on the field sources, a condition valid only for closed systems. What radiates is not only the accelerated mass, but also the stresses that cause the acceleration. In Havas' paper[12] these stresses are omitted and only the radiation of the mass is taken into account, a fact corresponding to formula (30). The coefficient of formula (33) is expected to change somewhat when an ultrarelativistic particle moves in a field of a different structure.

We note that at low energies the bulk of the gravitational radiation is due to the tensor T_p^{ik}, and the radiation of the gravitational waves by the electromagnetic stresses T_f^{ik} is much smaller.

We now consider the term proportional to R^2. The total radiation intensity corresponding to this term is

$$P_{grav} = \frac{2}{3}\frac{\varkappa M_c^2 R^2}{a^4}\left(\frac{\mathfrak{E}}{\mathfrak{E}_0}\right)^6 = \frac{\varkappa R^2 H^{(0)2}}{c^4} P_{el}, \qquad (36)$$

where P_{el} is the electromagnetic power radiated by the particle and R is the distance from the center of the orbit to the point of observation, or the path of electromagnetic radiation of the particle. The part of gravitational radiation considered here is the result of the interaction between the field of the electromagnetic wave, which is detached from the particle, and the fixed field $H^{(0)}$ (for more details see [9]). The coefficient of P_{el} in (36) shows what part of the electromagnetic power radiated by the particle is converted into gravitational radiation if the electromagnetic wave traverses a distance R in the field $H^{(0)}$. In this case the energy of the gravitational radiation is taken from the energy of the electromagnetic radiation, and consequently the expression for the radiation force is not changed by inclusion of this term.

The gravitational radiation fluxes of the particle and of the stresses do not interfere with the gravitational flux due to the electromagnetic wave propagating in the field $H^{(0)}$. This can be seen directly from (27) and (28), where there are no interference terms (proportional to the first power of R).

We see from (36) that the radiated power is proportional to the square R^2 of the distance traversed in the magnetic field $H^{(0)}$, and consequently the amplitude of this gravitational wave is proportional to the distance R. This is valid only for a homogeneous magnetic field. In the case of a random magnetic field the gravitational waves excited in different sections will be incoherent, and thus the power rather than the amplitude is additive. In this case R^2 in formula (36) must be replaced by RR^0, where R^0 is the correlation radius of the "fixed" field (see [9]).

The authors are grateful to Prof. V. L. Ginzburg for guidance and to Prof. L. É. Gurevich for valuable remarks.

[1] L. Infeld and A. E Scheidegger, Can. J. Math. 3, 195 (1951).

[2] J. N. Goldberg, Phys. Rev. 99, 1873 (1955).

[3] A. E. Scheidegger, Phys. Rev. 99, 1883 (1955).

[4] L. D. Landau and E. M. Lifshitz. Teoriya polya (Field Theory), Fizmatgiz, 1960.

[5] L. Artsimovich and I. Pomeranchuk, J. Phys. (U.S.S.R.) 9, 265 (1945).

[6] V. Lopukhin and V. Ugarov, Usp. Fiz. Nauk 39, 398 (1948).

[7] V. L. Ginzburg, Teoriya rasprostraneniya radiovoln v ionosfere (Theory of Propagation of Radio Waves in the Ionosphere), GITTL, 1949.

[8] G. N. Watson, A Treatise on the Theory of Bessel Functions, Cambridge Univ. Press, 1944.

[9] M. E. Gertsenshtein, JETP 41, 113 (1961), Soviet Phys. JETP 14, 84 (1962).

[10] A. S. Eddington, The Mathematical Theory of Relativity, Cambridge Univ. Press, 1930.

[11] V. A. Fock, Teoriya prostranstva, vremeni i tyagoteniya (Theory of Space, Time, and Gravitation), GITTL, 1955.

[12] P. Havas, Phys. Rev. 108, 1351 (1957).

Translated by J. G. Adashko
25