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COMPLEX ANGULAR MOMENTA AND THE RELATION BETWEEN THE CROSS SECTIONS OF VARIOUS PROCESSES AT HIGH ENERGIES

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Recently a connection has been found between the analytical properties of the amplitudes as functions of the angular momenta, and their asymptotic behavior at high energies.[1,2] It is assumed that the asymptotic behavior of the scattering amplitudes of any particles in the diffraction region is determined by the moving pole $j(t)$ of a partial wave in the annihilation channel.[3-6] Several important properties of strong interactions at high energies (s) follow from this assumption. In particular, the amplitude of the elastic scattering of strongly interacting particles must have the form $f(t)s^{j(t)}$ (s and t are the usual Mandelstam variables). The total cross section is constant if $j(0)$ assumes the maximum possible value, equal to 1.[7] The elastic scattering cross section must tend slowly to zero (as $1/\ln s$). The diffraction cone must narrow with increasing energy; this behavior corresponds to the scattering on a system whose transparency and radius increase with the energy.[3] Such behavior seems to be in agreement with the experimental data recently obtained.[8] Up to now[2-6] only the properties of $j(t)$ were discussed. It was emphasized, in particular, that $j(t)$ is the same for the various reactions. We show that the unitarity conditions on complex j, obtained by one of us (V.G.),[2] lead to a great number of relations for the functions $f(t)$, corresponding to the various reactions.

Particular cases of these relations are simple connections between total interaction cross sections for various particles at high energies (s). For example,

$$\sigma_{AB}/\sigma_{AC} = \sigma_{DB}/\sigma_{DC},$$

where σ_{AB} is the total interaction cross section for the particles A and B at the energy $s \to \infty$. Hence, for instance, we have

$$\sigma_{\pi N}{}^2 = \sigma_{\pi\pi}\sigma_{NN}, \quad \sigma_{KN}{}^2 = \sigma_{KK}\sigma_{NN},$$

$$\sigma_{\Lambda N}{}^2 = \sigma_{NN}\sigma_{\Lambda\Lambda}, \quad \sigma_{K\Lambda}{}^2 = \sigma_{KK}\sigma_{\Lambda\Lambda}, \tag{1}$$

$$\sigma_{\Lambda N}/\sigma_{NN} = \sigma_{\pi\Lambda}/\sigma_{\pi N} = \sigma_{K\Lambda}/\sigma_{KK}. \tag{2}$$

Thus, using the values $\sigma_{NN} \approx 40$ mb, $\sigma_{\pi N} \approx 25$ mb, $\sigma_{KN} \simeq 22$ mb, we obtain $\sigma_{\pi\pi} \approx 16$ mb, $\sigma_{KK} \approx 12$ mb.

To derive these relations, let us write the unitarity conditions for the amplitudes of the partial waves in the annihilation (t) channel $4\mu^2 < t < 16\mu^2$, where μ is the mass of the π meson. The choice of this interval and the exclusive choice of the π-meson states are due to the fact that we can write the exact unitarity condition only in that region.

For simplicity let us consider the spinless particles, π and K mesons. Let f_j, g_j, and h_j be the amplitudes of the partial waves of the reactions $\pi + \pi \to \pi + \pi$, $K + \bar{K} \to \pi + \pi$, and $K + \bar{K} \to K + \bar{K}$, in a given isotopic spin state [to each value of the isotopic spin corresponds a particular $j(t)$]. Then the unitarity condition has the form[2]

$$(1/2i)(f_j - f_{j^*}{}^*) = (k/\omega) f_j f_{j^*}{}^*,$$

$$(1/2i)(g_j - g_{j^*}{}^*) = (k/\omega) g_j f_{j^*}{}^*,$$

$$(1/2i)(h_j - h_{j^*}{}^*) = (k/\omega) g_j g_{j^*}{}^*, \tag{3}$$

where k and ω are the momentum and energy of the π meson:

$$\omega = \tfrac{1}{2}t^{1/2}, \quad k = \tfrac{1}{2}(t - 4\mu^2)^{1/2}.$$

If we express f_j, g_j, h_j through $f_{j^*}{}^*$, $g_{j^*}{}^*$, $h_{j^*}{}^*$:

$$f_j = \frac{f_{j^*}{}^*}{1-2i(k/\omega)f_{j^*}{}^*},$$

$$g_j = \frac{g_{j^*}{}^*}{1-2i(k/\omega)f_{j^*}{}^*},$$

$$h_j = h_{j^*}{}^* + 2i\frac{k}{\omega}\frac{(g_{j^*}{}^*)^2}{1-2i(k/\omega)f_{j^*}{}^*}, \tag{4}$$

then it can be seen that all the amplitudes have a pole at a $j(t)$ for which $(k/\omega)f_{j^*}{}^* = (1/2i)$. For j close to $j(t)$ we have

$$f_{j^*}{}^* = (\omega/2ik)\{1-(1/\beta)[j-j(t)]\}. \tag{5}$$

Substituting (5) into (4), we obtain

$$f_j = \frac{\beta(\omega/2ik)}{j-j(t)}, \quad g_j = \frac{\beta g_{j^*(t)}{}^*}{j-j(t)}, \quad h_j = 2i(k/\omega)\beta\frac{(g_{j^*(t)}{}^*)^2}{j-j(t)}. \tag{6}$$

Hence it follows that the residues of these amplitudes, $r_{\pi\pi}$, $r_{\pi K}$, and r_{KK}, satisfy the simple relation

$$r_{\pi K}{}^2(t) = r_{\pi\pi}(t)r_{KK}(t).$$

This relation corresponds precisely to the connection between the probabilities of the various processes in the case where they pass through one Breit-Wigner level.

Since $r_{\pi\pi}$, r_{KK}, and $r_{\pi K}$ are analytic functions of t, this relation is valid at any t. The total cross section at high energies is determined by the pole in the isotopic spin zero state and is connected with $r(0)$ by the relations

$$\sigma_{\pi\pi} = 12\pi^2(1/\mu^2)r_{\pi\pi}(0),$$

$$\sigma_{\pi K} = 12\pi^2(1/m\mu)r_{\pi K}(0),$$

$$\sigma_{KK} = 12\pi^2(1/m^2)r_{KK}(0), \tag{7}$$

where m is the K-meson mass. Hence

$$\sigma_{\pi K}{}^2 = \sigma_{\pi\pi}\sigma_{KK}.$$

If we considered the spinless particles π, A, B, and C, we should obtain in just the same way a relation of the form

$$r_{\pi\pi}/r_{\pi A} = r_{B\pi}/r_{BA} = r_{C\pi}/r_{CA},$$

and others. Thus relations of the type (2) follow.

The nucleon spin, as detailed analysis shows, does not change the relation between the total cross sections obtained for spinless particles. Unfortunately, one cannot now compare the relation given above with experiment due to the instability of all the strongly interacting particles other than the nucleon.

If we include, however, the photon,[9] we obtain, in the same manner, the relation between the total cross sections of the photon-nucleon, the nucleon-nucleon, and the photon-photon interactions:

$$\sigma_{\gamma N} = \sigma_{\gamma\gamma}\sigma_{NN}. \tag{8}$$

$\sigma_{\gamma\gamma}$ may be found from the experimental study of the interaction of the photon with the Coulomb field of nuclei. The latter, for small nuclear recoil, is separable from the purely nuclear interaction.[10]

Let us list a number of other consequences following from the same unitarity relations:

(1) We have considered up to now the processes whose asymptotic form is determined by the pole having (according to the terminology of Chew and Frautschi) the vacuum quantum numbers. If we consider the processes whose asymptotic form is determined by the other poles, we can obtain a large number of relations between their amplitudes using the unitarity condition: for example, the relations between the values of the amplitudes of the processes $\pi^- + p \to \pi^0 + n$, $\gamma + N \to \pi + N$, and $\gamma + \pi \to 2\pi$ (the latter is observed from the process $\pi \to \pi + \pi$ on the Coulomb field of nuclei).

(2) The spin structures of the amplitudes for various processes are closely connected with each other. For example, the spin correlation experiments for nucleon-nucleon, π-nucleon, and K-nucleon scattering are determined by a single parameter.

(3) Interesting questions arise if we extend the relations mentioned above to the nuclei.[11] A number of observable relations arise here, whose regions of validity are not, however, absolutely clear, since the nuclear processes have anomalous thresholds.

In conclusion we should like to point out that according to (8) the residue in the pole of the partial wave for π-π scattering is

$$r_{\pi\pi} \sim 1/12\pi^2 \approx 10^{-2},$$

since

$$\sigma_{\pi\pi} \sim 1/\mu^2.$$

The authors feel that the smallness of the terms appearing here is connected with a slow change of the position of the vacuum pole $j(t)$, since the imaginary part of $j(t)$ $[j''(t)]$ in the region $t > 4\mu^2$ is proportional to $r_{\pi\pi}(t)$ if $j''(t)$ is small.[12] [$j(t)$ changes essentially when t changes to a value of the order $m^2 \approx 40\mu^2$.[5,8]]

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