SOME CONSEQUENCES OF THE MOVING POLE HYPOTHESIS FOR PROCESSES AT HIGH ENERGIES

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RECENTLY two of us [1] (V. G. and I. P.) derived, using the hypothesis that the behavior of elastic-scattering amplitudes at high energies is determined by the extreme right-hand pole in the complex l-plane, several relations between the asymptotic values of the total cross sections of different processes. We derive here the same relations with the aid of the theory of the matrix for the reaction in the annihilation channel (R-matrix), and point out several consequences of this hypothesis.

1. Following [1], we consider the partial amplitudes in the annihilation channel $T^l_{ik}(t)$, where l —orbital momentum (generally speaking, complex), i and k —initial and final state, and t —square of the total energy. We confine ourselves to two-particle initial and final states and to spinless particles. Then, introducing the statistical weight ρ into the definition of the T-matrix and R-matrix (see, for example, [2]), namely $T' = \pi\rho^{1/2}T\rho^{1/2}$ and $R' = \pi\rho^{1/2}R\rho^{1/2}$, we obtain (we drop the index l and the primes in T′ and R′)

$$(1 - iR)T = R. \tag{1}$$

In the region below the thresholds of all real processes, ρ is pure imaginary and (1) assumes the form

$$(1 + R)T = R. \tag{2}$$

Solving (2), we have

$$T_{ik} = [\Delta\delta_{ik} - (-1)^{i+k} M_{ik}] / \Delta, \tag{3}$$

where Δ is the determinant of the matrix 1 + R, and M_{ik} are the minors of Δ. To establish a connection between the amplitudes of different processes in the annihilation channel we use a relation well known from determinant theory

$$M_{ik}M_{lm} - M_{im}M_{lk} = \Delta N_{iklm}, \tag{4}$$

where N_{iklm} —determinant obtained from Δ by crossing out the rows i and l and the columns k and m.

The asymptotic behavior of the scattering amplitudes at high energies is expressed in terms of the values of T_{ik} near the pole. Then Δ tends to zero as $\Delta \sim l - l_0(t)$ and $T_{ik} \approx -(-1)^{i+k}M_{ik}/\Delta$. Using (4), we obtain

$$T_{ik} / T_{im} = T_{lk} / T_{lm} \equiv \Gamma_k / \Gamma_m, \tag{5}$$

i.e., the ratio T_{ik}/T_{im} is the same for all initial states i. Using the connection between the amplitudes in the annihilation channel near the pole $l = l_0(t)$ and the amplitudes in the scattering channel at high energies, we can establish with the aid of (5), as was done in [1], relationships between the amplitudes of different reactions at high energies.

2. Let us consider inelastic scattering with production of two particles in the final state, at least one of which is unstable. To be specific we confine ourselves to the scattering of nucleons by nucleons with production of pion-nucleon resonance N* in a state with isotopic spin $T = 1/2$ (for example, resonance $D_{3/2}$ with mass $m^* = 1.51$ BeV or resonance $F_{5/2}$ with mass $m^* = 1.69$ BeV). We assume that in spite of the presence in the final state of an unstable particle, which modifies the Mandelstam representation, the behavior of the amplitudes at high energies is determined as before by the extreme right-hand pole in the l-plane. (Inclusion of the spin dependence does not change the derivation—see [3].) Then, as can be readily seen, the system N + N* can go over into a "quasi-vacuum" state with isotopic spin T = 0, total momentum j = 0, 2, 4, . . . , and positive parity. Among these states is one whose moving pole $l_0(t)$ determines the total and elastic cross sections at high energies. It follows therefore that the amplitude f(s, t) of the process N + N → N + N* should have the following form at high energy [4]

$$f(s, t) = r(t)\, s^{l_0(t)}, \qquad l_0(0) = 1 \tag{6}$$

and the cross section of this process is proportional to

$$\sigma \sim \text{const} / [c + \ln(s/4m^2)]. \tag{7}$$

The function r(t) is proportional to the residue of the partial amplitude in the annihilation channel at the pole point $l = l_0(t)$. If we assume that at small values of t the value of r(t) changes appreciably in the interval $0 < t \le \alpha$, then $c \approx \alpha/l'_0(0)$.

We see from (7) that the cross sections of similar inelastic processes will decrease slowly with energy. From the experimental data [5-7] we can estimate the constant c for NN-scattering.

This constnnt turns out to be quite large,[7] $c_{NN\rightarrow NN} \approx 4$, so that at energies up to $E_{lab} \approx 50-100$ BeV the cross sections of the elastic and inelastic processes that proceed through the annihilation channel via the "quasi-vacuum" states are practically constant.

The experimental data on NN scattering[5] show, in accord with the statement made above, that in the range 10—27 BeV there is a noticeable cross section for the production of the $D_{3/2}$ and $F_{5/2}$ resonances. No $P_{3/2}$ resonance with isotopic spin $T = 3/2$ is seen to be created here, a fact attributed in our theory to the lack of "quasi-vacuum" states in the annihilation channel for this process. If we do not assume this agreement between theory and experiment to be accidental, it serves as an argument in favor of assuming that modification of the Mandelstam representation in anomalous cases is of no significance in the given scheme.

In this connection, we note a circumstance of importance from the experimental point of view. Since the cross sections for the creation of resonant states in processes proceeding via the annihilation channel through the quasi-vacuum states do not decrease in practice with increasing energy, it becomes possible to search for these resonances in reactions at higher energies. It is obvious that such resonances will be created effectively both in forward and in backward scattering.

3. Let us consider the scattering of nucleons by nuclei. We assume that in the anomalous cases when all the particles involved are stable in spite of the modification of the Mandelstam representation, the behavior of the amplitudes at high energies is determined by the extreme right-hand pole in the l-plane and, in addition, relation (5) holds true.

One of the consequences of this assumption is the following relation between the total cross sections σ_{NA}, σ_{NN}, and σ_{AA} (nucleon-nucleus, nucleon-nucleon, and nucleus-nucleus, respectively) at high energies:

$$\sigma_{NN}\sigma_{AA} = \sigma_{NA}^2. \tag{8}$$

The usual dependence of the cross section on the atomic number, $\sigma_{NA} \sim A^{2/3}$ and $\sigma_{AA} \sim A^{2/3}$, does not satisfy this relation. This may be physically due to the fact that in this theory the radius of the nucleon increases logarithmically with increasing energy, and becomes greater than the nuclear radius at sufficiently high energy.

One cannot exclude the possibility of having $\sigma_{NA} \sim A$ and $\sigma_{AA} \sim A^2$, which does not contradict relation (8) within the framework of the physical picture at high energies (this idea is due to I. Yu. Kobzarev). The energies above which the relation $\sigma_{NA} \sim A^{2/3}$ is violated are difficult to estimate at the present time.

Note added in proof (May 3, 1962). At high energies, the resonances $D_{3/2}$ and $F_{5/2}$ should arise effectively in both NN-scattering and πN-scattering. As follows from (5), the ratio of the differential cross sections for the creation of such a resonance in NN and πN scattering is equal to the ratio of the differential cross sections of elastic NN and πN scattering:

$$d\sigma_{NN\rightarrow NN^*}(s,t)/d\sigma_{\pi N\rightarrow \pi N^*}(s,t) = d\sigma_{NN\rightarrow NN}(s,t)/d\sigma_{\pi N\rightarrow \pi N}(s,t).$$

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PROBABILITY OF THE $\pi^+ \rightarrow \pi^0 + e^+ + \nu$ AND $\pi^+ \rightarrow \gamma + e^+ + \nu$ DECAY

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THE study of the β decay of the π meson

$$\pi^+ \rightarrow \pi^0 + e^+ + \nu \tag{1}$$

permits us to determine directly whether the vector current is conserved in weak interactions.[1] The present interest in this process, which was